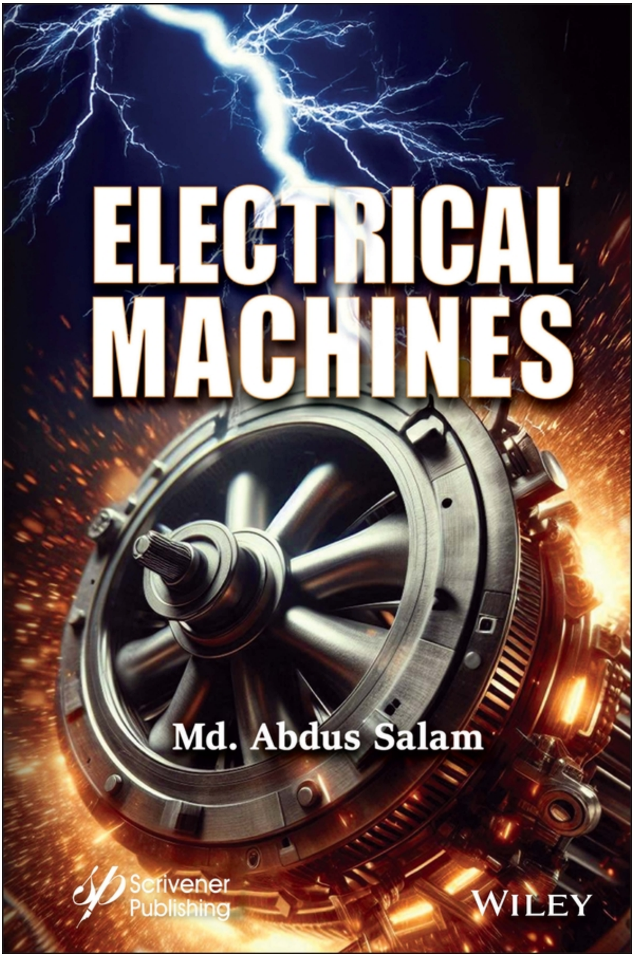

ELECTRICAL MACHINES

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Dedication

To my parents, my wife (Asma Ara Bagum), my children (Syeed Hasan, Yusra Salam, and Sundus Salam), and my daughter-in-law (Naomi Hoque) for their kindness and endless support.

Additionally, I would like to thank my instructors and well-wishers who have supported my professional development over the years.

Md. Abdus Salam

Preface

Electrical machines such as DC motors, DC generators, induction motors, synchronous machines, and special motors are widely used in different applications including the manufacturing industry. As technology continues to advance, the importance of understanding the principles and applications of electrical machines becomes increasingly critical. Electrical machines play a pivotal role in numerous aspects of our daily lives, from powering our homes and industries to driving innovations in renewable energy. Therefore, the basic principles of electrical machines along with their characteristics need to be known to proceed to higher semesters as well as to work in the practical field.

Welcome to a new textbook, *Electrical Machines: Fundamentals and Analysis.* This textbook has been thoroughly written to provide students, professors and engineers with a comprehensive, hands-on and accessible guide to the fascinating scope of electrical machines.

In addition, this textbook is designed to offer a balanced blend of theoretical concepts and practical applications, ensuring that readers not only grasp the underlying principles but also develop the skills necessary to apply them in real-world scenarios and applications, especially in the manufacturing industry.

Key Features

Fundamentals Concepts

The book begins with a solid foundation in the fundamentals of electrical circuits which will help readers gain a clear understanding of the essential theories that govern electrical machines' operation and characteristics.

Problem-Solving Approach

A step-by-step problem-solving approach is incorporated into each theory, encouraging active engagement and reinforcing knowledge. A variety of examples and exercises are included to enhance problem-solving skills.

Emerging Trends

Recognizing the dynamic and vibrant nature of the field, this textbook explores emerging trends and advancements in electrical machines, including discussions on the integration of smart technologies and software to start and control electrical machines.

Visual Learning

Rich and coloured illustrations, diagrams, and practical examples are employed to facilitate visual learning and enhance the clarity of complex concepts.

Practical Applications

Illustrations, diagrams, and practical examples are employed to facilitate visual learning. Real-world applications are included throughout the textbook, illustrating how electrical machines are used in different industries and inspiring engineers to see the direct impact of their knowledge.

Supplementary Resources

Based on adoption of this textbook, additional online resources, including tutorials, simulations, and supplementary materials will be made available to further support the learning process.

In the undergraduate course curriculum, universities normally offer two courses in electrical machines, each three credit hours. Most universities have already been evaluated or are in the process of evaluating their undergraduate programs in electrical and computer engineering to meet the Accreditation Board for Engineering and Technology (ABET) requirements.

This textbook strives to be your thorough companion, whether you are a student starting your journey into electrical engineering, a practicing engineer looking for a refresher, or just someone who is intrigued about the inner workings of the machines that power our modern world. Accept the fascinating realm of electrical machinery and start your educational journey now!

Aids for Instructors

Instructors who adopt this book as a text may obtain the solution manual as a supplement copy by contacting the publishers.

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The improvement of the book is an ongoing process. Therefore, I would be grateful to academicians, students and professional engineers for their constructive comments on this edition of the book through my email: ma.abdus@gmail.com.

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1
Analysis of Electrical Power

1.1 Introduction

AC generators, transformers, and other related high-voltage devices are required to generate, transmit and distribute alternating voltage and current. These types of equipment are rated by Mega Watt (MW) and Mega Voltage-Ampere (MVA). Electrical appliances, such as DVD players, televisions, microwaves, light bulbs, refrigerators, ceiling fans and many more are rated in a Watt (W). Careful attention is required in designing these kinds of equipment to address their power ratings, which in turn addresses the varying electricity bills. The electricity bills depend on the use of energy, and the unit of the energy is kWh. Power and time are also required to calculate the energy. The power is classified as electrical power, mechanical power and fluid power. Electrical power is usually generated and transferred over a long distance using an AC generator, transformer, transmission and distribution lines. Mechanical power is transferred to a short distance using the pulley, chain and conveyor belt, whereas fluid power is generated and distributed to a short distance using the pump, motor, valves and piping system. Therefore, knowledge power analysis plays an important role in designing and calculating the energy cost of a system. This chapter will discuss instantaneous power, average power, complex power, power factor, maximum power transfer theorem and power factor correction.

1.2 Power

The rate of receiving and delivering energy is known as power. Electrical power consists of DC power and AC power. DC power is calculated based on the supply voltage and draws current by the device from the source, whereas the calculation of AC power depends on the voltage, current and power factor. Fluid power is also calculated from the fundamental definition of power. Mathematically, the power is defined as,

$$P = \frac{dW}{dt} = \frac{W}{t} \tag{1.1}$$

$$P = \frac{dW}{dq}\frac{dq}{dt} \tag{1.2}$$

The voltage is defined as the work done per unit charge and is expressed as,

$$V = \frac{dW}{dq} \tag{1.3}$$

The current is defined as the rate of change of charge and it is expressed as,

$$I = \frac{dq}{dt} \tag{1.4}$$

Substituting equations (1.3) and (1.4) into equation (1.2) yields,

$$P = VI \tag{1.5}$$

Again, a force F is applied to an object to move a distance d. The work done is expressed as,

$$W = F \times d \tag{1.6}$$

Substituting equation (1.6) into equation (1.1) yields,

$$P = \frac{F \times d}{t} = F \times \frac{d}{t} \tag{1.7}$$

$$P = \frac{F \times d}{t} = F \times v \;\; (\text{lb-ft/s}) \text{ or } (\text{N-m/s}) \tag{1.8}$$

But, 550 lb-ft/s is equal to one HP and 1000 N-m/s is equal to one kW. Then the fluid power in FPS and MKS (SI) units are expressed as,

$$P_{HP} = \frac{F \times v}{550} \tag{1.9}$$

$$P_{\mathrm{kW}} = \frac{F \times v}{1000} \tag{1.10}$$

To derive the expression of power in an AC circuit, consider an AC voltage source $v(t)$ connected in series with an impedance as shown in Fig. 1.1. Let us consider that the expression of voltage is,

$$v(t) = V_m \sin \omega t \tag{1.11}$$

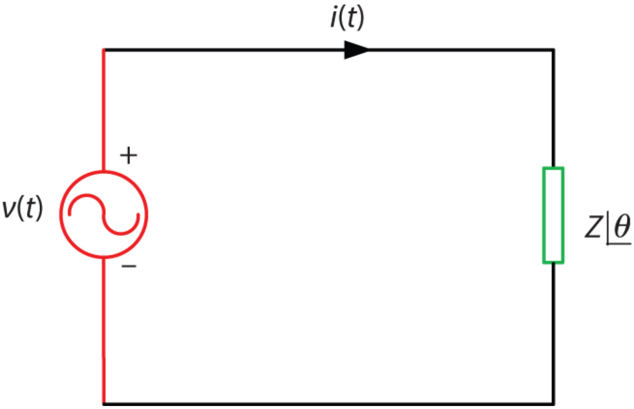


Fig. 1.1 A circuit with an impedance.

The series current $i(t)$ varies with time and the expression of the current is written as,

$$i(t) = \frac{v(t)}{Z\angle\theta} \tag{1.12}$$

Substituting equation (1.11) into equation (1.12) yields,

$$i(t) = \frac{V_m \sin \omega t}{Z\angle\theta} \tag{1.13}$$

$$i(t) = \frac{V_m\angle 0^\circ}{Z\angle\theta} = \frac{V_m}{Z}\angle -\theta \tag{1.14}$$

$$i(t) = I_m\angle -\theta = I_m \sin(\omega t - \theta) \tag{1.15}$$

The instantaneous power also varies with time, and it is defined as the product of the voltage and the current. The expression of instantaneous power is,

$$p(t) = v(t) \times i(t) \tag{1.16}$$

Substituting equations (1.11) and (1.15) into equation (1.16) yields,

$$p(t) = V_m \sin \omega t \times I_m \sin(\omega t - \theta) \tag{1.17}$$

$$p(t) = \frac{V_m I_m}{2} 2 \sin \omega t \times \sin(\omega t - \theta) \tag{1.18}$$

$$p(t) = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \theta) - \cos(\omega t + \omega t - \theta)\right] \tag{1.19}$$

$$p(t) = \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} \cos(2\omega t - \theta) \tag{1.20}$$

Equation (1.20) provides the expression for the instantaneous power for a series AC circuit.

Example 1.1

The excitation voltage and the impedance of a series circuit are given by v(t) = 15 sinωt V and $Z = 5\angle 10^\circ\ \Omega$, respectively. Calculate the instantaneous power.

Solution

The value of the current is calculated as,

$$i(t)=\frac{15\angle 0^\circ}{5\angle 10^\circ}=3\angle -10^\circ \text{ A} \tag{1.21}$$

The instantaneous power is calculated as,

$$p(t)=\frac{15\times 3}{2}2\sin\omega t\times\sin(\omega t-10^\circ) \tag{1.22}$$

$$p(t)=\frac{45}{2}\cos 10^\circ-\frac{45}{2}\cos(2\omega t-10^\circ) \tag{1.23}$$

$$p(t)=22.16-22.5\cos(2\omega t-10^\circ)\text{W} \tag{1.24}$$

Practice Problem 1.1

The current and impedance of a series circuit are given by $i(t)$ = 10sin(ωt + 15°)A and $Z=2\angle 20^\circ\ \Omega$, respectively. Calculate the instantaneous voltage and power.

1.3 Average Power and Reactive Power

The average power is related to the maximum value of the sinusoidal voltage and current, which are shown in equations (1.11) and (1.15), respectively. The average power for a periodic waveform over one cycle can be derived as,

$$P=\frac{1}{T}\int_0^T p(t) \tag{1.25}$$

Substituting equation (1.20) into the equation (1.25) yields,

$$P=\frac{V_m I_m}{2T}\int_0^T\cos\theta\, dt-\frac{V_m I_m}{2T}\int_0^T\cos(2\omega t-\theta)\,dt \tag{1.26}$$

$$P=\frac{V_m I_m}{2T}\cos\theta\left[T\right]-\frac{V_m I_m}{2T}\int_0^T\cos(2\omega t-\theta)\,dt \tag{1.27}$$

The second term of equation (1.27) is a cosine waveform. The average value of any cosine waveform over one cycle is zero. Therefore, from equation (1.27), the final expression of the average power can be represented as,

$$P = \frac{V_m I_m}{2} \cos\theta \qquad (1.28)$$

Similarly, the expression of the average reactive power can be written as,

$$Q = \frac{V_m I_m}{2} \sin\theta \qquad (1.29)$$

The term cos θ in equation (1.28) is the power factor of the circuit, and it is determined by the phase angle θ of the circuit impedance, where θ is the phase difference between the voltage and current phases i. e., $\theta = \theta_v - \theta_i$. The average power is often known as the true power or real power. The units of average power and reactive power are watts (W) and volt-ampere reactive (Var), respectively. The average power from equation (1.28) can be represented in terms of rms values of the voltage and current as,

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta \qquad (1.30)$$

Substituting $V_{rms} = \frac{V_m}{\sqrt{2}}$ and $I_{rms} = \frac{I_m}{\sqrt{2}}$ into equation (1.30) yields,

$$P = V_{rms} I_{rms} \cos\theta \qquad (1.31)$$

Similarly, from equation (1.29), the reactive power can be represented as,

$$Q = V_{rms} I_{rms} \sin\theta \qquad (1.32)$$

The current flows back and forth between the source and the network due to a specific magnitude of the reactive power. The reactive power does not dissipate any energy in the load. However, in practice, it produces energy losses in the line. In addition to that, it is required to create a magnetic field to operate AC and DC machines. Therefore, extra attention needs to be taken in designing the power system networks.

For a purely resistive circuit, the voltage and the current are in phase. This means that the phase angle between them is zero,

$$\theta = 0^\circ \tag{1.33}$$

Substituting equation (1.33) into equation (1.28) yields,

$$P_R = \frac{V_m I_m}{2} \cos 0^\circ \tag{1.34}$$

$$P_R = \frac{V_m I_m}{2} \tag{1.35}$$

Equation (1.35) can be rearranged by substituting $V_m = I_m R$ or $I_m = V_m / R$ as,

$$P_R = \frac{I_m^{\ 2} R}{2} = \frac{V_m^{\ 2}}{2R} \tag{1.36}$$

The phase difference between the voltage and current for the inductance and capacitance is,

$$\theta = \pm 90^\circ \tag{1.37}$$

Substituting equation (1.37) into equation (1.28) yields the average power for either an inductance or a capacitance,

$$P_L = P_C = \frac{V_m I_m}{2} \cos 90^\circ = 0 \tag{1.38}$$

The reactive power is usually stored in a circuit and it can be expressed for the inductor and capacitor as,

$$Q_L = I_L{}^2 X_L = \frac{V_L{}^2}{X_L} \tag{1.39}$$

$$Q_C = I_C{}^2 X_C = \frac{V_C{}^2}{X_C} \tag{1.40}$$

From equations (1.28) and (1.38), it can be concluded that the resistive load absorbs power whereas inductive or capacitive loads do not absorb any power.

Example 1.2

Fig. 1.2 shows an electrical series circuit with resistance, inductive and capacitive reactance. Find the average power supplied by the source and the power absorbed by the resistor.

Solution

The net impedance is calculated as,

$$Z_t = 2 + j5 - j9 = 4.47\angle{-63.43^\circ}\ \Omega \tag{1.41}$$

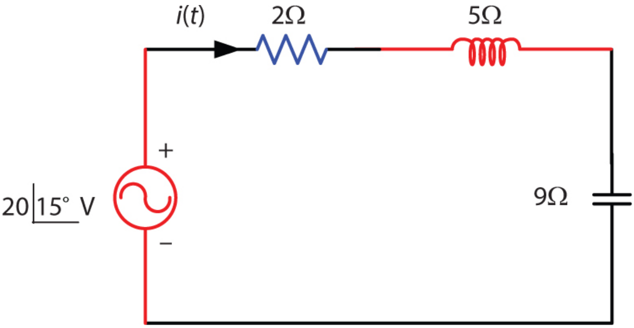


Fig. 1.2 Circuit for example 1.2.

The source current in phasor form is calculated as,

$$I = \frac{20\angle{15^\circ}}{4.47\angle{-63.43^\circ}} = 4.47\angle{78.43^\circ}\ \text{A} \tag{1.42}$$

The average power supplied by the source is calculated as,

$$P_s = \frac{20 \times 4.47}{2}\cos(15^\circ - 78.43^\circ) = 20\,\text{W} \tag{1.43}$$

The average power absorbed by the resistor is calculated as,

$$P_R = \frac{4.47^2 \times 2}{2} = 20\,\text{W} \tag{1.44}$$

Practice Problem 1.2

Fig. 1.3 shows a series-parallel circuit with resistance, inductive and capacitive reactance. Calculate the average power supplied by the source and the power absorbed by the resistors.

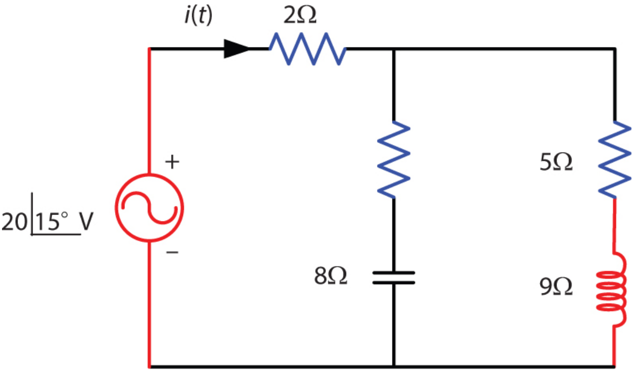


Fig. 1.3 Circuit for practice problem 1.2.

1.4 Apparent Power

The apparent power is the product of the voltage and the current. It is also defined as the vector sum of real power and reactive power. Real power is used to do useful work and can transmit over a long distance. Reactive power is the power that flows back into the source from the energy storage elements inductor and capacitor. Reactive power helps to regulate voltages on the transmission system to flow the real power. In addition to that, reactive power flow wastes the energy and capability of the machines and causes voltage drops. A leading reactive power (current leads the voltage) is supplied to correct the lagging power flow which tends to bring the current in phase with the voltage. In this condition (current in phase with the voltage), system losses decrease, increase the system capability and rise the voltage drops. The units of the reactive power are Var, kVar and Mvar. Therefore, the apparent power is related to the sinusoidal voltage and current. Let us consider that the expressions for sinusoidal voltage and current are,

$$v(t) = V_m \sin(\omega t + \theta_v) \tag{1.45}$$

$$i(t) = I_m \sin(\omega t + \theta_i) \tag{1.46}$$

The phasor forms of these voltage and current components are,

$$V = V_m \underline{|\theta_v} \tag{1.47}$$

$$I = I_m \underline{|\theta_i} \tag{1.48}$$

According to equation (1.28), the average power is,

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \tag{1.49}$$

The apparent power is also defined as the product of the rms voltage and rms current and its unit is Volt-Amps (VA). The largest units of apparent power are kVA and MVA. The apparent power is denoted by the letter S and it is expressed as,

$$S = \frac{V_m I_m}{2} = V_{rms} I_{rms} \tag{1.50}$$

Substituting equation (1.50) into equation (1.49) yields,

$$P = S \cos(\theta_v - \theta_i) \tag{1.51}$$

Similarly, the reactive power is expressed as,

$$Q = S\sin(\theta_v - \theta_i) \quad (1.52)$$

In addition, the apparent power can be determined by the vector sum of the real power (P) and the reactive power (Q). In this case, the expression of apparent power becomes,

$$S = P + jQ \quad (1.53)$$

Power Triangle: A right-angle triangle that shows the vector relationship between active power, reactive power and apparent power) with a lagging and leading power factor is shown in Fig. 1.4. The power triangles with an inductance and the capacitance loads will be lagging and leading, respectively.

In Fig. 1.4, the top right-angle triangle represents the lagging power factor and the bottom right-angle triangle represents the leading power factor.

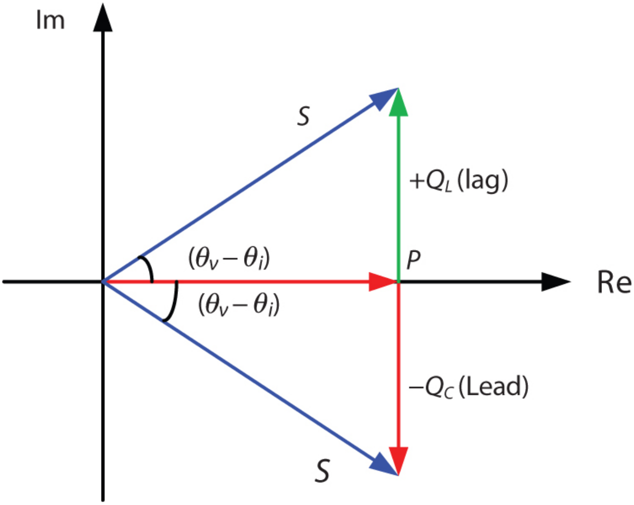


Fig. 1.4 Power triangles.

Example 1.3

An industrial load draws a current of $i(t) = 16\sin(314t + 25^\circ)$ A from an alternating voltage source of $v(t) = 220\sin(314t + 60^\circ)$ V. Determine the apparent power, circuit resistance and inductance.

Solution

The apparent power is calculated as,

$$S = \frac{V_m I_m}{2} = \frac{220 \times 16}{2} = 1.76\,\text{kVA} \tag{1.54}$$

The circuit impedance is calculated as,

$$Z = \frac{220\angle 60^\circ}{16\angle 25^\circ} = 11.26 + j7.89\,\Omega \tag{1.55}$$

The value of the circuit resistance is,

$$R = 11.26\,\Omega \tag{1.56}$$

The circuit inductance is calculated as,

$$L = \frac{7.89}{314} = 0.025\,\text{H} \tag{1.57}$$

Practice Problem 1.3

An industrial load draws a current of $i(t) = 10\sin(100t + 55^\circ)$ A from an alternating voltage source of $v(t) = 120\sin(100t + 10^\circ)$ V. Calculate the apparent power, circuit resistance and capacitance.

1.5 Complex Power

Complex power is the combination of real power and reactive power. The reactive power is necessary to create the magnetic field and sustain for generation of AC voltage using a synchronous generator. The complex power is defined as the half of the phasor form of voltage and the conjugate of the phasor current. Mathematically, the product of half of the phasor voltage and the conjugate of the phasor current is known as complex power. The complex power is represented by the letter S_C and is expressed as,

$$S_c = \frac{1}{2} V I^* \tag{1.58}$$

Substituting equations (1.57) and (1.58) into equation (1.58) yields,

$$S_c = \frac{1}{2} V_m \angle\theta_v \times I_m \angle{-\theta_i} \tag{1.59}$$

$$S_c = V_{rms} \angle\theta_v \times I_{rms} \angle{-\theta_i} \tag{1.60}$$

Based on equation (1.60), the complex power is again defined as the product of the rms voltage phasor and the conjugate of the rms current phasor. Mathematically, it is expressed as,

$$S_c = \mathbf{V}_{rms} \times \mathbf{I}_{rms}{}^* \tag{1.61}$$

Equation (1.51) can be rearranged as,

$$S_c = V_{rms} I_{rms} \angle\theta_v - \theta_i \tag{1.62}$$

$$S_c = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i) \tag{1.63}$$

Consider the circuit as shown in Fig. 1.5 to explain the complex power. The impedance of this circuit is,

$$Z = R + jX \tag{1.64}$$

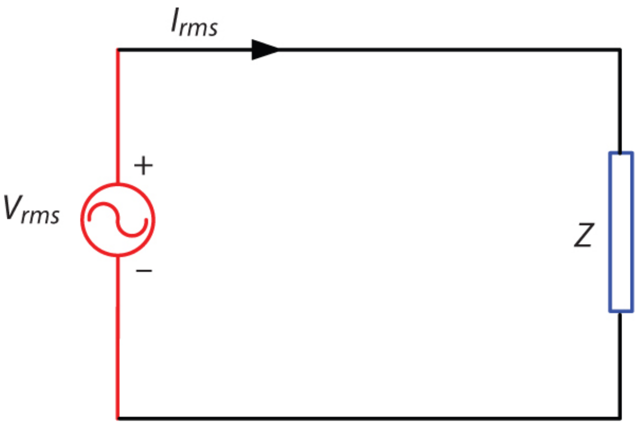


Fig. 1.5 A simple AC circuit.

Equation (1.64) can be represented by the impedance triangle as shown in Fig. 1.6.

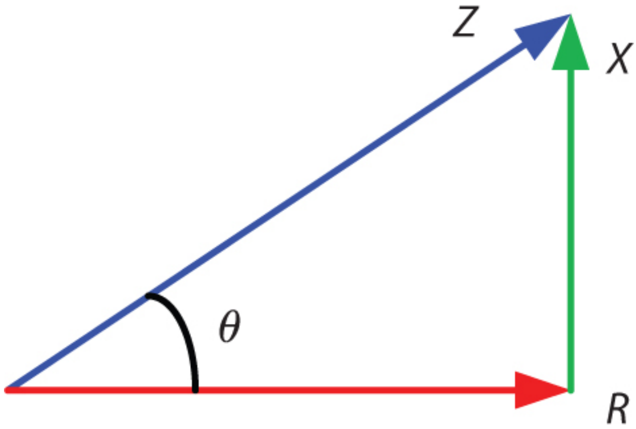


Fig. 1.6 Impedance triangle.

The rms value of the current is written as,

$$I_{rms} = \frac{V_{rms}}{Z} \tag{1.65}$$

Substituting equation (1.65) into equation (1.61) yields,

$$S = V_{rms}\frac{V_{rms}{}^{*}}{Z^{*}} = \frac{V_{rms}{}^{2}}{Z} \tag{1.66}$$

Equation (1.61) again can be represented as,

$$S_c = I_{rms} Z I_{rms}{}^{*} = I_{rms}{}^{2} Z \tag{1.67}$$

Substituting equation (1.64) into equation (1.67) yields,

$$S_c = I_{rms}{}^{2}(R + jX) = I_{rms}{}^{2}R + jI_{rms}{}^{2}X = P + jQ \tag{1.68}$$

where P and Q are the real and the imaginary parts of the complex power, and in this case, the expressions of P and Q can be written as,

$$P = \mathrm{Re}(S_c) = I_{rms}{}^2 R \tag{1.69}$$

$$Q = \mathrm{Im}(S_c) = I_{rms}{}^2 X \tag{1.70}$$

The real power, reactive power and apparent power of equation (1.67) are shown in Fig. 1.7. The complex power for a resistive branch can be written as,

$$S_{cR} = P_R + jQ_R = I_{rms}{}^2 R \tag{1.71}$$

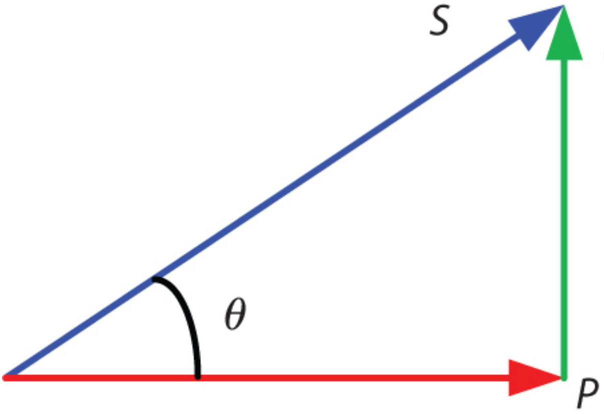


Fig. 1.7 Power triangle.

From equation (1.71), the real power and the reactive power for the resistive branch can be expressed as,

$$P_R = I_{rms}{}^2 R \tag{1.72}$$

$$Q = 0 \tag{1.73}$$

The complex power of an inductive branch is written as,

$$S_{cL} = P_L + jQ_L = jI_{rms}{}^2 X_L \tag{1.74}$$

From equation (1.73), the real power and the reactive power for an inductive branch can be separated as,

$$P_L = 0 \tag{1.75}$$

$$Q_L = I_{rms}^{\;2} X_L \tag{1.76}$$

The complex power for a capacitive branch is expressed as,

$$S_{cC} = P_C + jQ_C = jI_{rms}^{\;2} X_C \tag{1.77}$$

From equation (1.77), the real power and the reactive power for a capacitive branch can be separated as,

$$P_c = 0 \tag{1.78}$$

$$Q_C = I_{rms}^{\;2} X_C \tag{1.79}$$

For a reactive power, the following points are summarized:

$Q = 0$ for resistive load, i.e., unity power factor,

$Q > 0$ for inductive load, i.e., lagging power factor,

$Q < 0$ for capacitive load, i.e., leading power factor.

Example 1.4

A series AC circuit is shown in Fig. 1.8. Calculate the source current, apparent, real and reactive powers. The expression of the alternating voltage source is $v(t) = 16\sin(10t + 25°)$V.

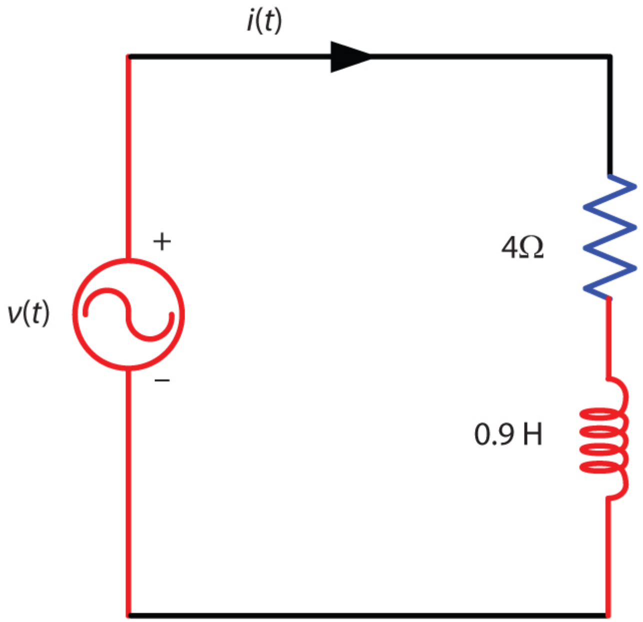


Fig. 1.8 Circuit for example 1.4.

Solution

The rms value of the source voltage is calculated as,

$$V_{rms} = \sqrt{2}\times 16\angle 25^\circ = 22.63\angle 25^\circ \text{ V} \tag{1.80}$$

The inductive reactance is,

$$X_L = 10\times 0.09 = 9\Omega \tag{1.81}$$

The circuit impedance is calculated as,

$$Z = 4 + j9 = 9.85\angle 66.04^\circ \ \Omega \tag{1.82}$$

The source current is calculated as,

$$I_{rms} = \frac{22.63\angle 25^\circ}{9.85\angle 66.04^\circ} = 2.30\angle -41.04^\circ \text{ A} \tag{1.83}$$

The complex power is calculated as,

$$S = V_{rms} I_{rms}^{*} = 22.63\underline{|25^\circ} \times 2.30\underline{|-41.04^\circ} = 50.02\,\text{W} - j14.38\ \text{Var} \tag{1.84}$$

The apparent power is determined as,

$$S_{ap} = |S| = 52.05\,\text{VA} \tag{1.85}$$

The real power is calculated as,

$$P = \text{Re}(S) = 52.02\,\text{W} \tag{1.86}$$

The reactive power is determined as,

$$Q = \text{Im}(S) = 14.38\,\text{Var} \tag{1.87}$$

Example 1.5

A 220 V rms delivers power to a load. The load absorbs an average power of 10 kW at a leading power factor of 0.9. Determine the complex power and the impedance of the load.

Solution

The power factor angle is calculated as,

$$\cos\theta = 0.9 \tag{1.88}$$

$$\theta = 25.84^\circ \tag{1.89}$$

The reactive component is calculated as,

$$\sin\theta = \sin 25.84^\circ = 0.44 \tag{1.90}$$

The magnitude of the complex power is calculated as,

$$|S| = \frac{P}{\cos\theta} = \frac{10}{0.9} = 11.11\text{kVA} \tag{1.91}$$

The reactive power is determined as,

$$Q = |S|\sin\theta = 11.11 \times 0.44 = 4.89\,\text{kVar} \tag{1.92}$$

The complex power is calculated as,

$$S = P + jQ = 10\,\text{kW} - j4.89\,\text{kVar} \tag{1.93}$$

The rms voltage can be determined as,

$$P = V_{rms} I_{rms} \cos\theta = 10000 \tag{1.94}$$

$$I_{rms} = \frac{10000}{220 \times 0.9} = 50.51\,\text{A} \tag{1.95}$$

The value of the impedance is calculated as,

$$|Z| = \frac{|V_{rms}|}{|I_{rms}|} = \frac{220}{50.51} = 4.36\,\Omega \tag{1.96}$$

$$Z = 4.36\underline{|-25.84^\circ}\,\Omega \tag{1.97}$$

Practice Problem 1.4

A series RC circuit is shown in Fig. 1.9. Find the source current, apparent, real and reactive powers. The expression of alternating voltage source is $v(t) = 10\sin(2t + 12^\circ)$ V.

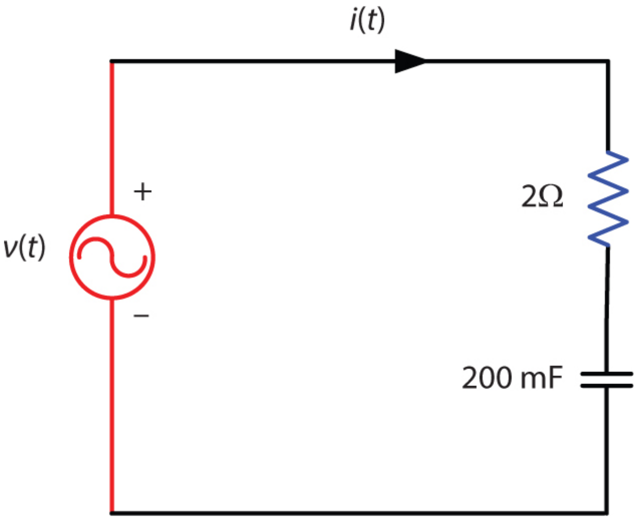


Fig. 1.9 Circuit for practice problem 1.4.

Practice Problem 1.5

An electrical load absorbs an average power of 12 kW from a source of 230 V rms at a lagging power factor of 0.95. Calculate the complex power and the impedance of the load.

1.6 Complex Power Balance

Two electrical loads are connected in parallel with a voltage source as shown in Fig. 1.10. According to the conservation of energy, the real power delivered by the source will be equal to the total real power absorbed by the loads [7-8]. Similarly, the complex power delivered by the source will be equal to the total complex power absorbed by the loads. According to KCL, the rms value of the source current is equal to the sum of the rms values of the branch currents I_1 and I_2, i.e.,

$$I = I_1 + I_2 \tag{1.98}$$

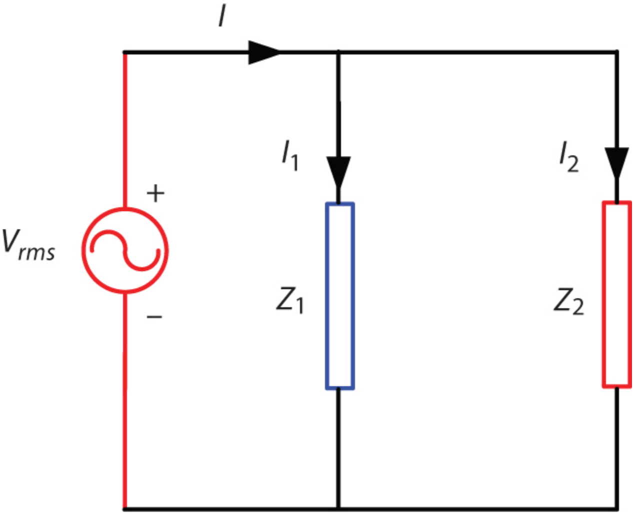


Fig. 1.10 Circuit with two parallel impedances.

The total complex power is defined as the product of the rms value of the source voltage and the conjugate of the current supplied by the source, and it is expressed as,

$$S = V_{rms} I^* \tag{1.99}$$

Substituting equation (1.98) into equation (1.99) yields complex power of the parallel circuit,

$$S_p = V_{rms}[I_1 + I_2]^* \tag{1.100}$$

$$S_p = V_{rms}I_1^* + V_{rms}I_2^* \tag{1.101}$$

$$S_p = S_1 + S_2 \tag{1.102}$$

The electrical loads are again connected in series with a voltage source as shown in Fig. 1.11. According to KVL, the rms value of the source voltage is equal to the sum of the rms values of the load voltages and it is written as,

$$V_{rms} = V_1 + V_2 \tag{1.103}$$

Substituting equation (1.103) into equation (1.99) yields the complex power of the series circuit is,

$$S_s = (V_1 + V_2)I^* \tag{1.104}$$

$$S_s = V_1 I^* + V_2 I^* \tag{1.105}$$

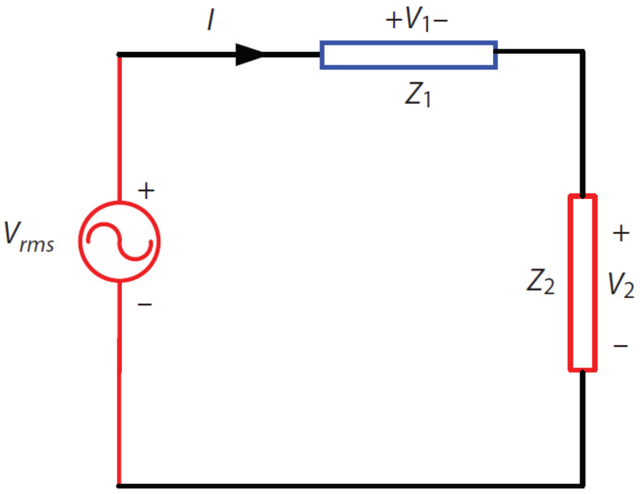


Fig. 1.11 Circuit with series impedances.

$$S_s = S_1 + S_2 \tag{1.106}$$

From equations (1.102) and (1.106), it is observed that the total complex power delivered by the source is equal to the sum of the individual complex power, absorbed by the loads when it is connected in parallel and series.

Example 1.6

A series-parallel circuit is supplied by a source of 60 V rms as shown in Fig. 1.12. Find the complex power for each branch and the total complex power.

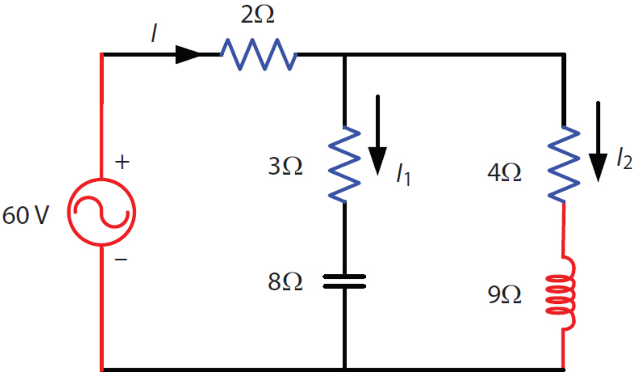


Fig. 1.12 Circuit for example 1.6.

Solution

The circuit impedance is calculated as,

$$Z_t = 2 + \frac{(4 + j9)(3 - j8)}{4 + j9 + 3 - j8} = 13.87\underline{\left|-9.88^\circ\right.}\ \Omega \tag{1.107}$$

The source current is determined as,

$$I = \frac{60}{13.87\underline{\left|-9.88^\circ\right.}} = 4.33\underline{\left|9.88^\circ\right.}\ \text{A} \tag{1.108}$$

The branch currents can be calculated as,

$$I_1 = 4.33\angle 9.88^\circ \times \frac{4+j9}{7+j1} = 6.03\angle 67.79^\circ \text{ A} \tag{1.109}$$

$$I_2 = 4.33\angle 9.88^\circ \times \frac{3-j8}{7+j1} = 5.23\angle -67.69^\circ \text{ A} \tag{1.110}$$

The voltage across the parallel branches is,

$$V_p = 60 - 2\times 4.33\angle 9.88^\circ = 51.49\angle -1.65^\circ \text{ V} \tag{1.111}$$

The complex power in the branches are calculated as,

$$S_1 = 51.49\angle -1.65^\circ \times 6.03\angle -67.79^\circ = 310.48\angle -69.44^\circ \text{ VA} \tag{1.112}$$

$$S_2 = 51.49\angle -1.65^\circ \times 5.23\angle 67.69^\circ = 269.29\angle 66.04^\circ \text{ VA} \tag{1.113}$$

The voltage drops across the 2Ω resistor is calculated as,

$$V_{2\Omega} = 2\times 4.33\angle 9.88^\circ = 8.66\angle 9.88^\circ \text{ V} \tag{1.114}$$

The complex power for 2Ω resistor is calculated as,

$$S_{2\Omega} = 8.66\angle 9.88^\circ \times 4.33\angle -9.88^\circ = 37.50 \text{ VA} \tag{1.115}$$

The total complex power is calculated as,

$$S_t = 310.48\angle -69.44^\circ + 269.29\angle 66.04^\circ + 37.50 = 259.76\angle -9.89^\circ \text{ VA} \tag{1.116}$$

Alternatively, the total complex power can be calculated as,

$$S = 60\times 4.33\angle -9.88^\circ = 259.8\angle -9.88^\circ \text{ VA} \tag{1.117}$$

Practice Problem 1.6

A series-parallel circuit is supplied by a rms voltage source as shown in Fig. 1.13. Determine the complex power for each branch and the total complex power.

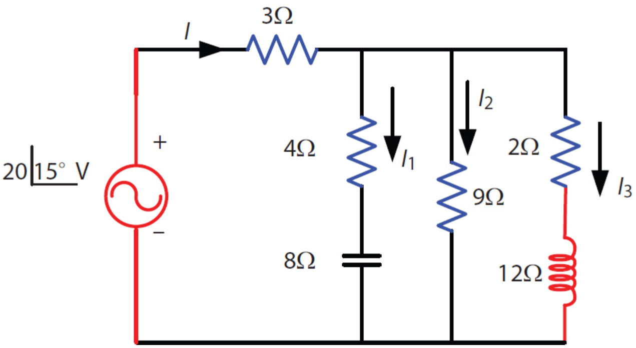


Fig. 1.13 Circuit for practice problem 1.6.

1.7 Power Factor and Reactive Factor

In an AC circuit, power is calculated by multiplying a factor with the rms values of current and voltage. This factor is known as the power factor. The power factor is defined as the cosine of the difference in phase angles between the voltage and the current, whereas the reactive factor is defined as the sine of the difference in the phase angles between the voltage and the current. The power factor is also defined as the cosine of the phase angle of the load impedance. Close to unity power factor represents an efficient power transfer from the source to a load, whereas a low power factor identifies an inefficient transmission of power. Low power factor usually affects power generation devices. Mathematically, the power factor is written as,

$$pf = \cos(\theta_v - \theta_i) \tag{1.118}$$

The reactive factor is written as,

$$rf = \sin(\theta_v - \theta_i) \tag{1.119}$$

From impedance and power triangles as shown in Figs. 1.6 and 1.7, the power factor can be written as,

$$\text{pf} = \cos\theta = \frac{R}{Z} = \frac{\text{kW}}{\text{kVA}} \tag{1.120}$$

The angle θ is positive if the current lags the voltage, and in this case, the power factor is considered as lagging. Whereas the angle

θ is negative if the current leads the voltage, and in this case, the power factor is considered as leading. The leading power factor is usually considered for capacitive loads. The industrial loads are inductive and have a low lagging power factor.

A low power factor has many disadvantages, which are outlined below:

i. kVA rating of electrical machines is increased,

ii. larger conductor size is required to transmit or distribute electric power at a constant voltage,

iii. copper losses are increased, and

iv. voltage regulation is small.

1.8 Power Factor Correction

In the electrical domain, heavy and medium-sized industry applications contain inductive loads which draw a lagging current from the source. As a result, the reactive power for these applications is increased. In this scenario, the transformer rating and the conductor size need to be increased to carry out the additional reactive power.

In order to cancel this reactive component of power, an opposite type of reactance needs to be included in the circuit. Let us consider that a single-phase inductive load is connected across a voltage source as shown in Fig. 1.14 and this load draws a current with a lagging power factor of cos θ_1.

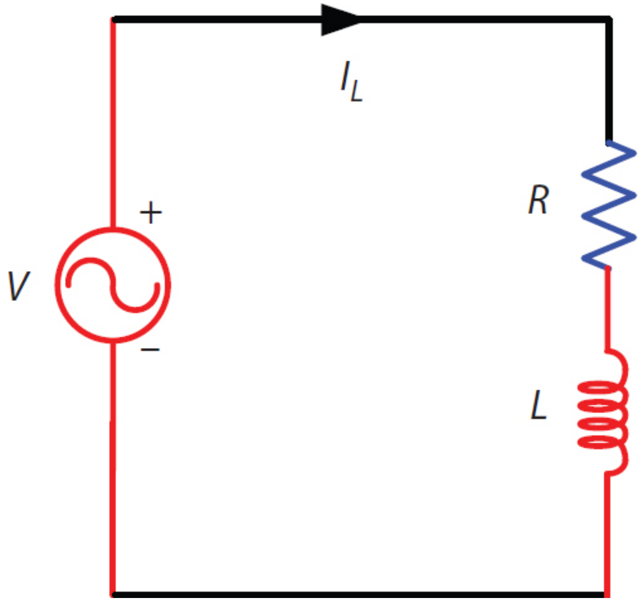


Fig. 1.14 A single-phase inductive circuit.

Fig. 1.15 shows a circuit where the capacitor is connected in parallel with the load to improve the power factor. The capacitor will draw current from the source that leads the source voltage by 90°. The line current is the vector sum of the currents in the inductive load and the capacitor. The current in the inductive load circuit lags the supply voltage by θ_1, and the current in the capacitor leads the voltage by 90° as shown in the vector diagram in Fig. 1.16.

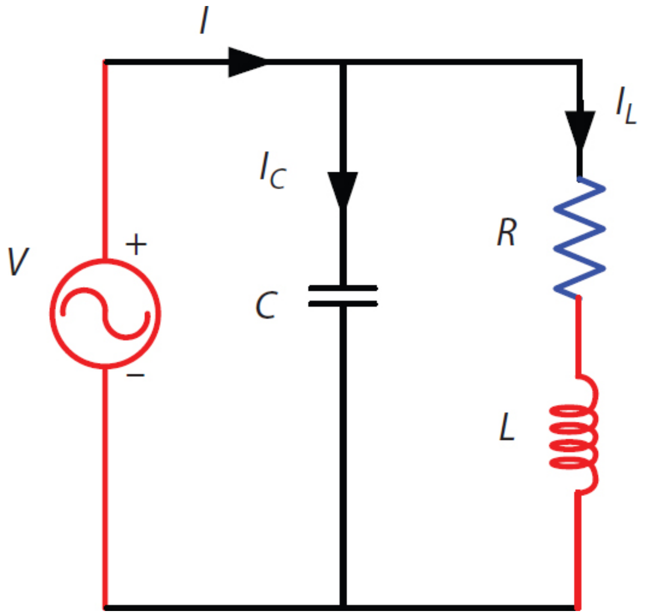


Fig. 1.15 A capacitor is in parallel with inductive load.

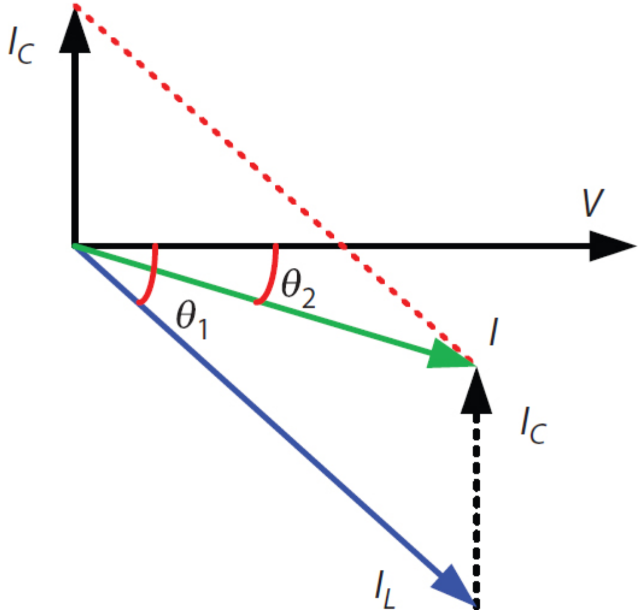

I_C
V
θ_2
θ_1
I
I_C
I_L

Fig. 1.16 A vector diagram with different currents.

The exact value of the capacitor needs to be identified to improve the power factor from $\cos\phi_1$ to $\cos\phi_2$ without changing the real power. A power triangle is drawn using the inductive load and the capacitor as shown in Fig. 1.17.

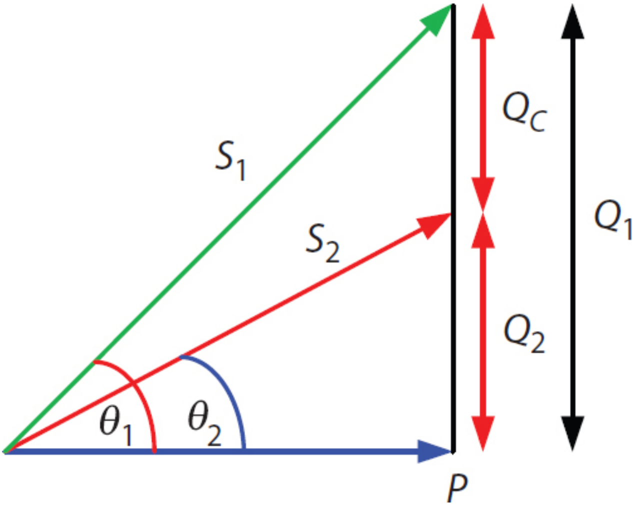


Fig. 1.17 Power triangles for inductive load and capacitor.

The reactive power of the original inductive load is written as,

$$Q_1 = P\tan\phi_1 \tag{1.121}$$

The expression of new reactive power is written as,

$$Q_2 = P\tan\phi_2 \tag{1.122}$$

The reduction in reactive power due to parallel capacitor is expressed as,

$$Q_C = Q_1 - Q_2 \tag{1.123}$$

Substituting equations (1.121) and (1.122) into equation (1.133) yields,

$$Q_C = P(\tan\phi_1 - \tan\phi_2) \tag{1.124}$$

The reactive power due to the capacitor can be calculated as,

$$Q_C = \frac{V_{rms}^{\ 2}}{X_C} = \omega C V_{rms}^{\ 2} \quad (1.125)$$

Substituting equation (1.124) into equation (1.125) yields the expression for the capacitor,

$$\omega C V_{rms}^{\ 2} = P(\tan\phi_1 - \tan\phi_2) \quad (1.126)$$

$$C = \frac{P(\tan\phi_1 - \tan\phi_2)}{\omega V_{rms}^{\ 2}} \quad (1.127)$$

Example 1.7

A load of 6 kVA, 50 Hz, 0.75 lagging power factor is connected across a voltage source of 120 V rms as shown in Fig. 1.18. A capacitor is connected across the load to improve the power factor to 0.95 lagging. Determine the capacitance of the connected capacitor.

Solution

The initial power factor angle is calculated as,

$$\cos\phi_1 = 0.75 \quad (1.128)$$

$$\phi_1 = 41.41^\circ \quad (1.129)$$

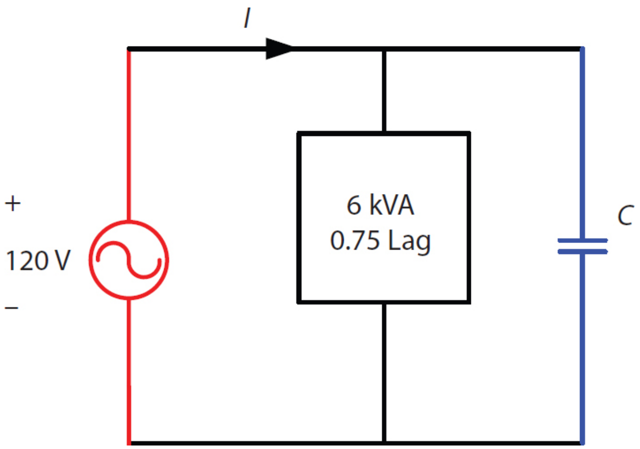


Fig. 1.18 Circuit for example 1.7.

The final power factor angle is calculated as,

$$\cos\phi_2 = 0.95 \tag{1.130}$$

$$\phi_2 = 18.19^\circ \tag{1.131}$$

The real power is calculated as,

$$P = 6 \times 0.75 = 4.5\,\mathrm{kW} \tag{1.132}$$

The value of the parallel capacitance can be calculated as,

$$C = \frac{4.5 \times 1000(\tan 41.41^\circ - \tan 18.19^\circ)}{2\pi \times 50 \times 120^2} = 0.55\,\mathrm{mF} \tag{1.133}$$

Example 1.8

A load of 10 kVA, 50 Hz, 0.8 lagging power factor is connected across the voltage source shown in Fig. 1.19. A capacitor is connected across the load to improve the power factor to 0.90 lagging. Calculate the value of the capacitance and line loss with and without the capacitor.

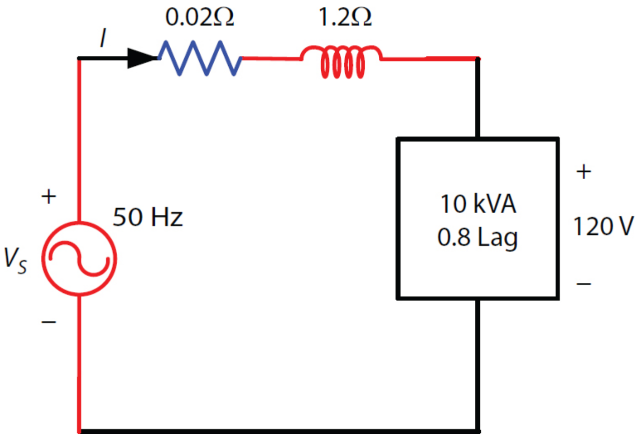


Fig. 1.19 Circuit for example 1.8.

Solution

The initial power factor angle is calculated as,

$$\cos\phi_1 = 0.8 \tag{1.134}$$

$$\phi_1 = 36.87^\circ \tag{1.135}$$

The final power factor angle is calculated as,

$$\cos\phi_2 = 0.9 \tag{1.136}$$

$$\phi_2 = 25.84^\circ \tag{1.137}$$

The power of the load is calculated as,

$$P = 10 \times 0.8 = 8\,\text{kW} \tag{1.138}$$

The capacitor is calculated as,

$$C = \frac{8 \times 1000(\tan 36.87^\circ - \tan 25.84^\circ)}{2\pi \times 50 \times 120^2} = 0.47\,\text{mF} \tag{1.139}$$

The line current before adding the capacitor is calculated as,

$$I_1 = \frac{8000}{0.8 \times 120} = 83.33\,\text{A} \tag{1.140}$$

The power loss in the line before adding the capacitor is calculated as,

$$P_1 = 83.33^2 \times 0.02 = 138.88\,\text{W} \tag{1.141}$$

The apparent power with a power factor of 0.9 lagging is calculated as,

$$S = \frac{8000}{0.9} = 8888.89\,\text{VA} \tag{1.142}$$

The line current after adding the capacitor is calculated as,

$$I_2 = \frac{8888.89}{120} = 74.07\,\text{A} \tag{1.143}$$

The power loss in the line after adding the capacitor is calculated as,

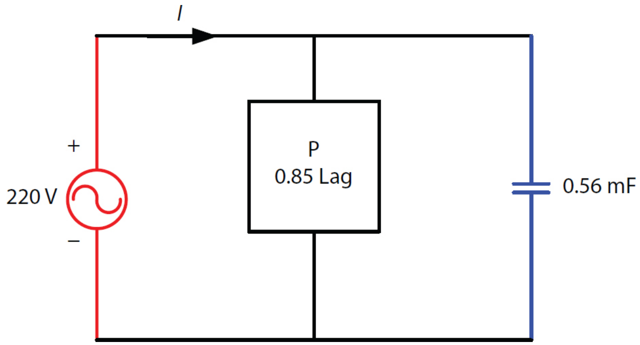
$$P_2 = 74.07^2 \times 0.02 = 109.73\,\mathrm{W} \tag{1.144}$$

Practice Problem 1.7

A load of 0.85 lagging power factor is connected across a voltage source of 220 V rms as shown in Fig. 1.20. A 0.56 mF capacitor is connected across the load to improve the power factor to 0.95 lagging. Find the value of the load, *P*.

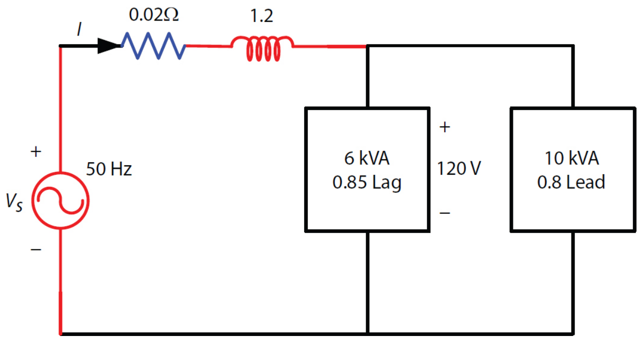
Practice Problem 1.8

Two loads are connected to a source through a line as shown in Fig. 1.21. Determine the value of the voltage source.



I
+
220 V
−
P
0.85 Lag
0.56 mF

Fig. 1.20 Circuit for practice problem 1.7.



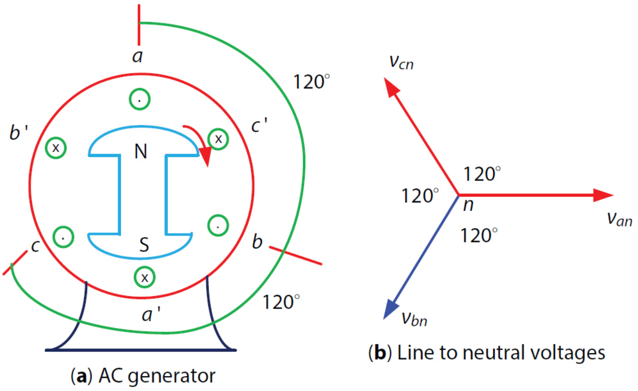
0.02Ω
1.2
I
+
V_S
50 Hz
−
6 kVA
0.85 Lag
+
120 V
−
10 kVA
0.8 Lead

Fig. 1.21 Circuit for practice problem 1.8.

1.9 Three-Phase Voltage Generation

Fig. 1.22 shows a two-pole three-phase AC generator for three-phase voltage generation. Coils *aa*’, *bb*’ and *cc*’ represent the whole coils in a three-phase system as shown in Fig. 1.22(a). The rotor of the AC machine is energized by the DC source, which creates the magnetic field. This rotor is attached to the turbine through a soft coupling, and this turbine rotates the rotor. According to Faraday’s law of electromagnetic induction, three-phase voltages V_{an}, V_{bn} and V_{cn} will be generated across the generator terminals.

The magnitudes of these voltages are constant and are displaced from each other by 120 electrical degrees as shown in Fig. 1.22(b). The waveforms of the generated voltages are shown in Fig. 1.23.



(**a**) AC generator

(**b**) Line to neutral voltages

Fig. 1.22 Schematic of AC generator and phase voltages.

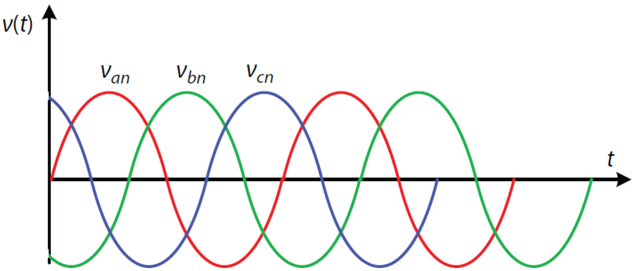


Fig. 1.23 Three-phase voltage waveforms.

The expression of the generated voltages can be represented as,

$$(1.145)\quad v_{an} = V_{an} \sin \omega t$$

$$(1.146)\quad v_{bn} = V_{bn} \sin(\omega t - 120^\circ)$$

$$(1.147)\quad v_{cn} = V_{cn} \sin(\omega t - 240^\circ)$$

where V_{an}, V_{bn} and V_{cn} are the magnitudes of the line to neutral or phase voltages. These voltages are constant in magnitude, and it can be expressed as,

$$(1.148)\quad |V_{an}| = |V_{bn}| = |V_{cn}| = V_p$$

The phasor of the generated voltages can be written as,

$$(1.149)\quad V_{an} = V_P\underline{|0^\circ}$$

$$(1.150)\quad V_{bn} = V_P\underline{|-120^\circ}$$

$$(1.151)\quad V_{cn} = V_P\underline{|-240^\circ} = V_P\underline{|120^\circ}$$

The sum of phasor voltages and the sum of sinusoidal voltages are zero, and these can be expressed as,

$$(1.152)\quad V = V_{an} + V_{bn} + V_{cn} = V_P\underline{|0^\circ} + V_P\underline{|-120^\circ} + V_P\underline{|-240^\circ} = 0$$

$$(1.153)\quad v = v_{an} + v_{bn} + v_{cn} = V_{an} \sin \omega t + V_{bn} \sin(\omega t - 120^\circ) + V_{cn} \sin(\omega t - 240^\circ) = 0$$

1.10 Phase Sequence

The phase sequence is very important for the interconnection of the three-phase transformer, motor and other high-voltage equipment. The three-phase systems are numbered either by the numbers 1, 2 and 3 or by the letters *a*, *b,* and *c*. Sometimes, these are labelled by the colours red, yellow and blue or *RYB* in short. The generator is said to have a positive phase sequence when the generated voltages reach their maximum or peak values in the sequential order of *abc*, whereas the generator is said to have a negative phase sequence when the generated voltages reach their maximum or peak values in the sequential order of *acb*. Fig. 1.24 shows the positive and negative phase sequences.

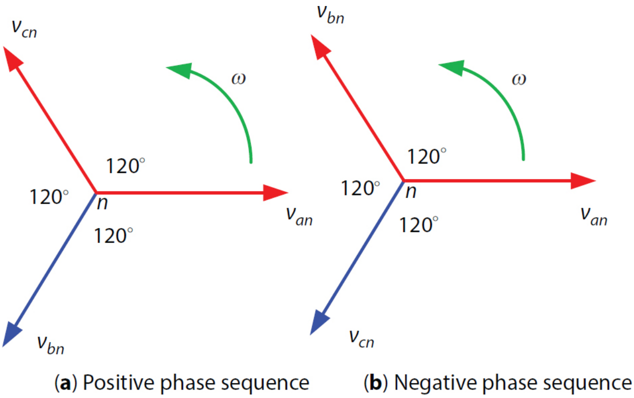


Fig. 1.24 Phase sequence identification.

Here, the voltage V_{an} is considered to be the reference voltage while the direction of rotation is considered to be anticlockwise. In the positive phase sequence, the crossing sequence of voltage rotation is identified by V_{an} - V_{bn} - V_{cn}, whereas for the negative phase sequence, it is identified as V_{an} - V_{cn} - V_{bn}.

1.11 Wye Connection

Three-phase transformer, AC generator and induction motor are connected either in a wye or in a delta connection. In the wye connection, one terminal of each coil is connected to form a common or neutral point, and other terminals are to the three-phase supply. The voltage between any line and neutral is known as the phase voltage, and the voltage between any two lines is called the line voltage. The line voltage and phase voltage are usually represented by V_L and V_P, respectively. The important points of this connection are the line voltage is equal to the square root 3 times the phase voltage, the line current is equal to the phase current, and the current (I_n) in the neutral wire is equal to the phasor sum of the three-line currents. For a balanced three-phase load, the neutral current is zero i. e., $I_n = 0$. The wye-connected generator and load are shown in Fig. 1.25.

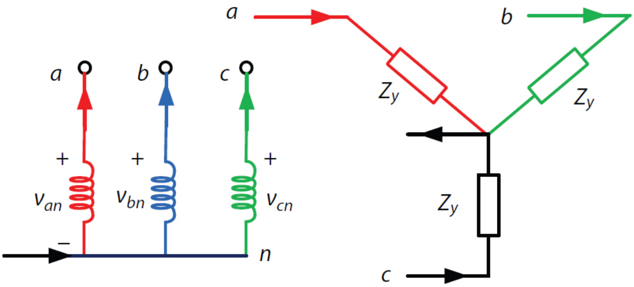

a
b
c
+
+
+
V_{an}
V_{bn}
V_{cn}
−
n
a
b
Z_y
Z_y
Z_y
c

Fig. 1.25 Wye-connected generator and load.

1.12 Analysis of Wye Connection

A three-phase wye-connected AC generator shown in Fig. 1.26 is considered for analysis. Here, V_{an}, V_{bn} and V_{cn} are the phase voltages. Whereas V_{ab}, V_{bc} and V_{ca} are the line voltages. Applying KVL to the circuit to find the line voltage between lines a and b yields,

$$V_{an} - V_{bn} - V_{ab} = 0 \tag{1.154}$$

$$V_{ab} = V_{an} - V_{bn} \tag{1.155}$$

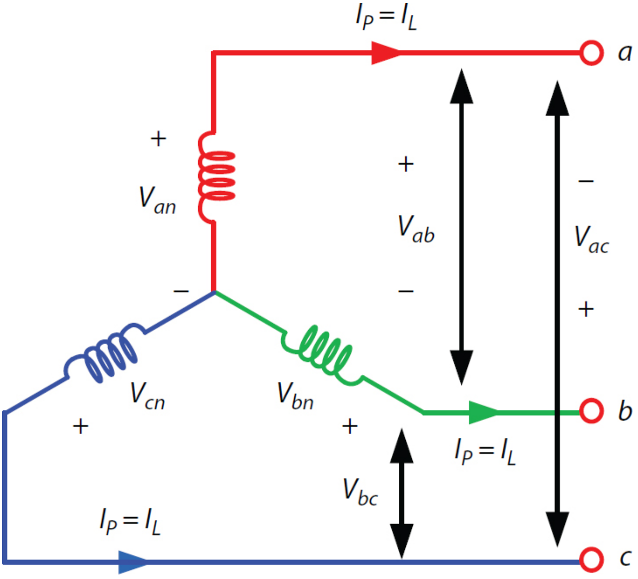

$I_P = I_L$
a
+
V_{an}
−
+
V_{ab}
−
−
V_{ac}
+
V_{cn}
V_{bn}
+
+
b
$I_P = I_L$
V_{bc}
$I_P = I_L$
c

Fig. 1.26 Wye-connected generator.

Substituting equations (1.149) and (1.150) into equation (1.155) yields,

$$V_{ab} = V_P\angle 0^\circ - V_P\angle -120^\circ \tag{1.156}$$

$$V_{ab} = \sqrt{3}\,V_P\angle 30^\circ \tag{1.157}$$

Applying KVL between lines *b* and *c* yields the expression of line voltage as,

$$V_{bc} = V_{bn} - V_{cn} \tag{1.158}$$

Substituting equations (1.150) and (1.151) into equation (1.158) yields,

$$V_{bc} = V_P\angle -120^\circ - V_P\angle -240^\circ = \sqrt{3}\,V_P\angle -90^\circ \tag{1.159}$$

Applying KVL between lines *c* and *a* yields the expression of the line voltage as,

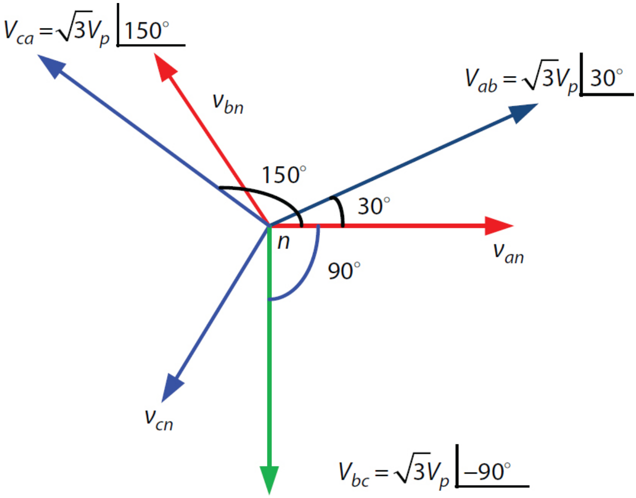
$$V_{ca} = V_{cn} - V_{an} \tag{1.160}$$

Substituting equations (1.149) and (1.151) into equation (1.160) yields,

$$V_{ca} = V_P\angle -240^\circ - V_P\angle 0^\circ = \sqrt{3}\,V_P\angle 150^\circ \tag{1.161}$$

Line voltages with angles are drawn as shown in Fig. 1.27. From equations (1.157), (1.159) and (1.161), it is seen that the magnitude of the line voltage is equal to the square root 3 times the magnitude of the phase voltage. The general relationship between the line voltage and the phase voltage can be written as,

$$V_L = \sqrt{3}\,V_P \tag{1.162}$$



$V_{ca} = \sqrt{3}V_p \angle 150^\circ$
v_{bn}
$V_{ab} = \sqrt{3}V_p \angle 30^\circ$
150°
30°
n
v_{an}
90°
v_{cn}
$V_{bc} = \sqrt{3}V_p \angle -90^\circ$

Fig. 1.27 Phasor diagram with line and phase voltages.

From Fig. 1.26, it is also observed that the phase current is equal to the line current and it is written as,

$$I_L = I_P \tag{1.163}$$

Alternative approach: A vector diagram with phase voltages is drawn using the lines *a* and *c* as shown in Fig. 1.28. A perpendicular line is drawn from point *A*, which divides the line *BD* equally. From the triangle *ABC*, the following expression relation can be written,

$$\cos 30^\circ = \frac{BC}{AB} \tag{1.164}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{|V_{nc}|} \tag{1.165}$$

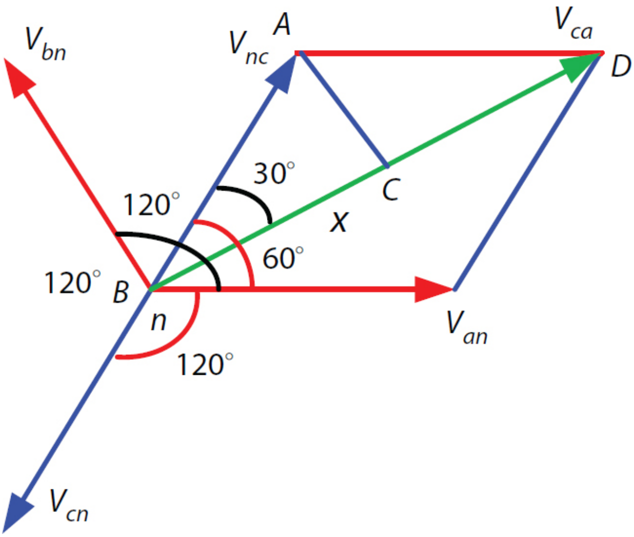


Fig. 1.28 A vector diagram.

According to Fig. 1.28, the following expression can be written,

$$BD = 2AC \tag{1.166}$$

$$V_{ca} = 2x \tag{1.167}$$

Substituting equation (1.165) into equation (1.167) yields,

$$V_{ca} = 2 \times \frac{\sqrt{3}}{2} \left| V_{nc} \right| \tag{1.168}$$

$$V_{ca} = \sqrt{3} V_{nc} \tag{1.169}$$

In general, the following equation can be written as,

$$V_L = \sqrt{3} V_p$$

Example 1.9

The phase voltage is given by $V_{an} = 230\underline{|10^{\circ}}\ \text{V}$. For the *abc* phase sequence, determine V_{bn} and V_{cn}.

Solution

The phase voltage for line a is calculated as,

$$V_{an} = 230\angle 10^\circ \text{ V} \tag{1.170}$$

The phase voltage for line b is calculated as,

$$V_{bn} = 230\angle 10^\circ - 120^\circ = 230\angle -110^\circ \text{ V} \tag{1.171}$$

The phase voltage for line c is calculated as,

$$V_{cn} = 230\angle 10^\circ - 240^\circ = 230\angle -230^\circ \text{ V} \tag{1.172}$$

Practice Problem 1.9

The phase voltage is given by $V_{bn} = 200\angle 10^\circ \text{ V}$. For the *abc* phase sequence, calculate V_{an} and V_{cn}.

Example 1.10

A wye-connected generator generates a voltage of 180 V rms as shown in Fig. 1.29. For the *abc* phase sequence, write down the phase and line voltages.

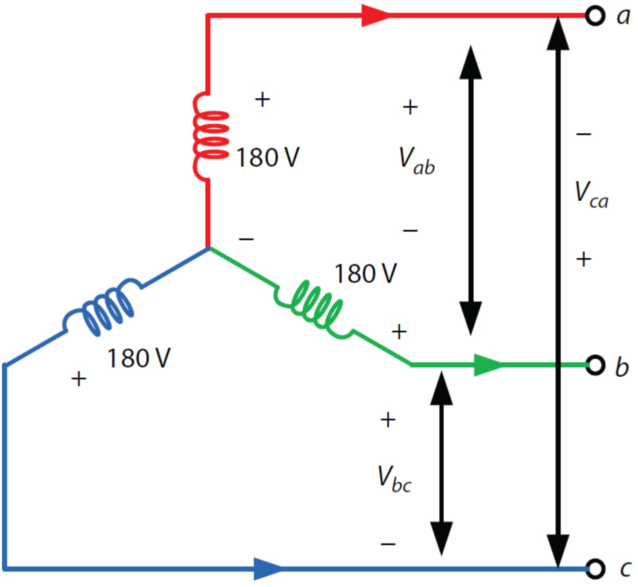


Fig. 1.29 Circuit for example 1.10.

Solution

The phase voltages are,

$$V_{an} = 180\angle 0^\circ \text{ V} \tag{1.173}$$

$$V_{bn} = 180\angle -120^\circ \text{ V} \tag{1.174}$$

$$V_{cn} = 180\angle -240^\circ \text{ V} \tag{1.175}$$

The line voltages are calculated as,

$$V_{ab} = \sqrt{3} \times 180\angle 30^\circ = 311.77\angle 30^\circ \text{ V} \tag{1.176}$$

$$V_{bc} = \sqrt{3} \times 180\angle -120^\circ + 30^\circ = 311.77\angle -90^\circ \text{ V} \tag{1.177}$$

$$V_{ca} = \sqrt{3} \times 180\angle -240^\circ + 30^\circ = 311.77\angle -210^\circ \text{ V} \tag{1.178}$$

Practice Problem 1.10

A wye-connected generator generates the line-to-line voltage of 200 V rms as shown in Fig. 1.30. For the *abc* phase sequence, write down the phase voltages.

1.13 Delta Connection

The coils in a delta-connected circuit are arranged in such a way that a looking structure is formed. The delta-connection is formed by connecting point a_2 of the a_1a_2 coil to point b_1 of the b_1b_2 coil, point b_2 of *the* b_1b_2 coil to point c_1 of the c_1c_2 coil and point c_2 of the c_1c_2 coil to the point a_1 of the a_1a_2 coil. In this connection, the phase voltage is equal to the line voltage, and the line current is equal to the square root 3 times the phase current. Fig. 1.31 shows the delta-connected generator and load.

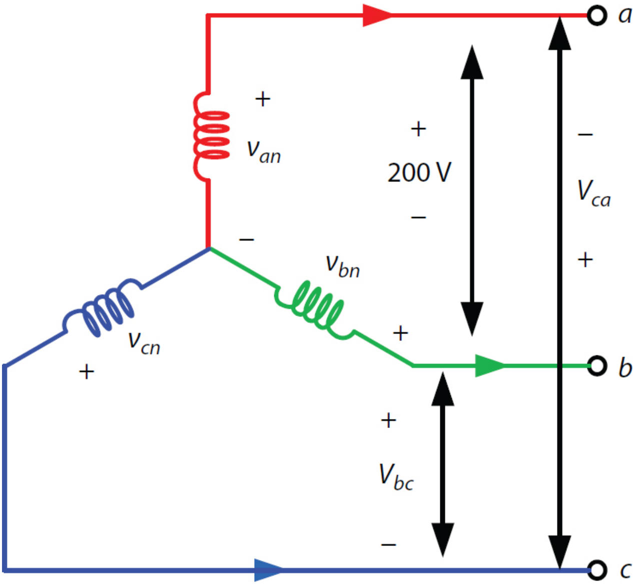

a
+
v_{an}
−
+
200 V
−
−
V_{ca}
+
v_{bn}
+
v_{cn}
+
b
+
V_{bc}
−
c

Fig. 1.30 Circuit for practice problem 1.10.

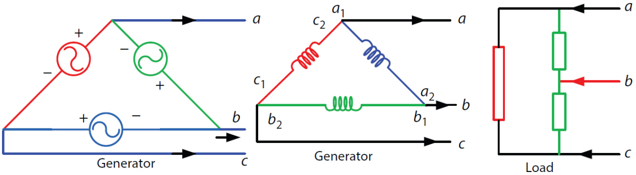

+
−
−
+
+
−
a
b
c
Generator
a_1
c_2
a
c_1
a_2
b
b_2
b_1
c
Generator
a
b
c
Load

Fig. 1.31 Delta-connected generator and load.

1.14 Analysis of Delta Connection

A three-phase delta-connected load is shown in Fig. 1.32. In this connection, I_{ab}, I_{bc} and I_{ca} are the phase currents and I_a, I_b and I_c are the line currents. For the *abc* phase sequence, the phase currents can be written as,

$$I_{ab} = I_p \angle 0^\circ \tag{1.179}$$

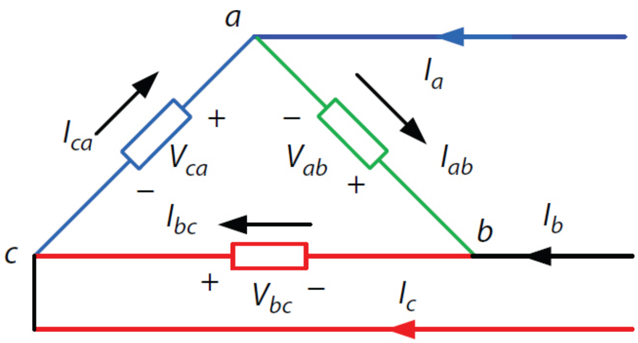


Fig. 1.32 Delta-connected load.

$$I_{bc} = I_P\angle{-120^\circ} \tag{1.180}$$

$$I_{ca} = I_P\angle{-240^\circ} \tag{1.181}$$

Applying KCL at the node *a* of the circuit in Fig. 1.32 yields,

$$I_a = I_{ab} - I_{ca} \tag{1.182}$$

Substituting equations (1.179) and (1.181) into equation (1.182) yields,

$$I_a = I_P\angle 0^\circ - I_P\angle{-120^\circ} = \sqrt{3}\ I_P\angle{-30^\circ} \tag{1.183}$$

Applying KCL at the node *b* of the circuit in Fig. 1.32 yields,

$$I_b = I_{bc} - I_{ab} \tag{1.184}$$

Substituting equations (1.180) and (1.179) into equation (1.184) yields,

$$I_b = I_P\angle{-120^\circ} - I_P\angle 0^\circ = \sqrt{3}\ I_P\angle{-150^\circ} \tag{1.185}$$

Applying KCL at the node *c* of the circuit in Fig. 1.32 yields,

$$I_c = I_{ca} - I_{bc} \tag{1.186}$$

Substituting equations (1.181) and (1.180) into equation (1.186) yields,

$$I_c = I_P\underline{|-240^\circ} - I_P\underline{|-120^\circ} = \sqrt{3}\, I_P\underline{|\,90^\circ} \tag{1.187}$$

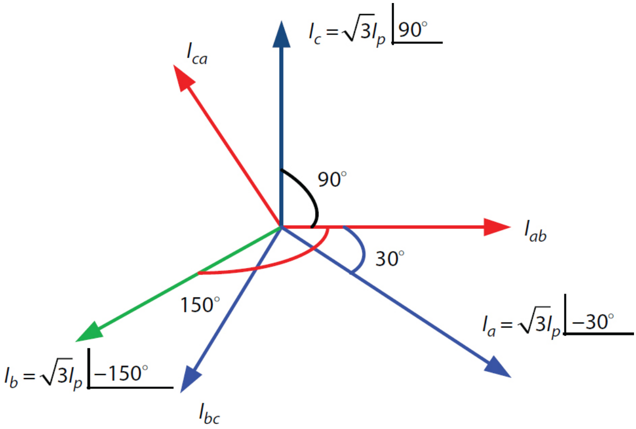
From equations (1.183), (1.185) and (1.178), it is found that the magnitude of the line current is equal to the square root 3 times the phase current. The general relationship between the line current and the phase current is,

$$I_L = \sqrt{3}\, I_P \tag{1.188}$$

According to Fig. 1.32, it is observed that the phase voltage is equal to the line voltage, i. e.,

$$V_L = V_P \tag{1.189}$$

The line and phase currents with their phase angles are drawn as shown in Fig. 1.33, where the phase current I_{ab} is arbitrarily chosen as a reference.



$I_c = \sqrt{3} I_p \underline{|90^\circ}$
I_{ca}
90°
I_{ab}
30°
150°
$I_a = \sqrt{3} I_p \underline{|-30^\circ}$
$I_b = \sqrt{3} I_p \underline{|-150^\circ}$
I_{bc}

Fig. 1.33 Phasor diagram using line and phase currents.

1.15 Analysis of Three-Phase Power

Consider a balanced three-phase wye-connected generator that delivers power to the balanced three-phase wye-connected load as shown in Fig. 1.34. The total power of the three-phase system is calculated by considering the instantaneous voltages and currents. The instantaneous voltages are,

$$v_{AN} = V_m \sin \omega t \tag{1.190}$$

$$v_{BN} = V_m \sin(\omega t - 120^\circ) \tag{1.191}$$

$$v_{CN} = V_m \sin(\omega t - 240^\circ) \tag{1.192}$$

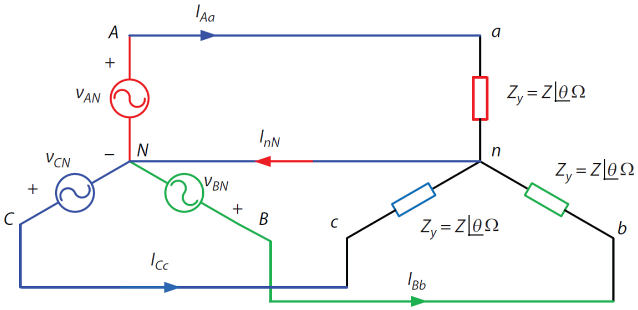

I_{Aa}
A
a
$+$
v_{AN}
$Z_y = Z\angle\theta\ \Omega$
I_{nN}
$-$
N
n
v_{CN}
$Z_y = Z\angle\theta\ \Omega$
$+$
v_{BN}
$+$
C
B
c
$Z_y = Z\angle\theta\ \Omega$
b
I_{Cc}
I_{Bb}

Fig. 1.34 Wye-wye system for power calculation.

The phase currents of the three-phase wye-connected load can be expressed as,

$$i_{Aa} = \frac{v_{AN}}{Z_y} = \frac{V_m \sin \omega t}{Z\angle\theta} = I_m \sin(\omega t - \theta) \tag{1.193}$$

$$i_{Bb} = \frac{v_{BN}}{Z_y} = \frac{V_m \sin(\omega t - 120^\circ)}{Z\angle\theta} = I_m \sin(\omega t - \theta - 120^\circ) \tag{1.194}$$

$$i_{Cc} = \frac{v_{CN}}{Z_y} = \frac{V_m \sin(\omega t - 240^\circ)}{Z\angle\theta} = I_m \sin(\omega t - \theta - 240^\circ) \tag{1.195}$$

The instantaneous power for phase *a* can be expressed as,

$$p_a(t) = \frac{1}{T}\int_0^T v_{AN} i_{Aa}\, dt \tag{1.196}$$

Substituting equations (1.190) and (1.193) into equation (1.196) yields,

$$p_a(t) = \frac{V_m I_m}{T}\int_0^T \sin\omega t \times \sin(\omega t - \theta)dt \tag{1.197}$$

$$p_a(t) = \frac{V_m I_m}{2T}\int_0^T 2\sin\omega t \times \sin(\omega t - \theta)dt \tag{1.198}$$

$$p_a(t) = \frac{V_m I_m}{2T}\int_0^T [\cos\theta - \cos(2\omega t - \theta)]dt \tag{1.199}$$

$$p_a(t) = \frac{V_m I_m}{2T} \times \cos\theta \times T - 0 \tag{1.200}$$

$$p_a(t) = \frac{V_m I_m}{\sqrt{2} \times \sqrt{2}}\cos\theta = V_P I_P \cos\theta \tag{1.201}$$

where V_p and I_p are the rms values of phase voltage and phase current.

Similarly, the expressions of the instantaneous power for the phase *b* and phase *c* can be written as,

$$p_b(t) = V_P I_P \cos\theta \tag{1.202}$$

$$p_c(t) = V_P I_P \cos\theta \tag{1.203}$$

Therefore, the average three-phase power P can be calculated as,

$$P = p_a(t) + p_b(t) + p_c(t) \tag{1.204}$$

Substituting equations (1.201), (1.202) and (1.203) into equation (1.205) yields,

$$P_t = 3V_p I_p \cos\theta \tag{1.205}$$

Similarly, the expression of three-phase reactive power can be expressed as,

$$Q_t = 3V_p I_p \sin\theta \tag{1.206}$$

Therefore, the per phase average (P_p) and reactive (Q_p) power can be written as,

$$P_p = V_p I_p \cos\theta \tag{1.207}$$

$$Q_p = V_p I_p \sin\theta \tag{1.208}$$

The complex power per phase S_p is represented as,

$$S_p = P_p + jQ_p \tag{1.209}$$

Substituting equations (1.207) and (1.208) into equation (1.209) yields,

$$S_p = V_p I_p \cos\theta + jV_p I_p \sin\theta \tag{1.210}$$

Equation (1.210) can be expressed as,

$$S_p = V_p I_p \underline{\left|\theta\right.} \tag{1.211}$$

From equation (1.211), it is seen that the per-phase complex power is equal to the product of the voltage per phase and the phase current with an angle.

Y-connection: Substituting equations (1.162) and (1.163) into equation (1.205) yields,

$$P_{tY} = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\theta \tag{1.212}$$

$$P_{tY} = \sqrt{3}\, V_L I_L \cos\theta \tag{1.213}$$

Substituting equations (1.162) and (1.163) into equation (1.206) yields,

$$Q_{tY} = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \sin\theta \tag{1.214}$$

$$Q_{tY} = \sqrt{3}\, V_L I_L \sin\theta \tag{1.215}$$

Delta connection: Again, substituting equations (1.188) and (1.189) into equation (1.207) yields,

$$P_{t\Delta} = 3 \times \frac{I_L}{\sqrt{3}} \times V_L \times \cos\theta \tag{1.216}$$

$$P_{t\Delta} = \sqrt{3}\, V_L I_L \cos\theta \tag{1.217}$$

Substituting equations (1.188) and (1.189) into equation (1.208) yields,

$$Q_{t\Delta} = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \sin\theta \tag{1.218}$$

$$Q_{t\Delta} = \sqrt{3}\, V_L I_L \sin\theta \tag{1.219}$$

In general, the total real and reactive power can be expressed as,

$$P_t = \sqrt{3}\, V_L I_L \cos\theta \tag{1.220}$$

$$Q_t = \sqrt{3}\, V_L I_L \sin\theta \tag{1.221}$$

The total complex power can be written as,

$$S_t = P_t + jQ_t \tag{1.222}$$

Substituting equations (1.220) and (1.221) into equation (1.222) yields,

$$S_t = \sqrt{3}\, V_L I_L \cos\theta + j\sqrt{3}\, V_L I_L \sin\theta \tag{1.223}$$

$$S_t = \sqrt{3}\, V_L I_L \angle\theta \tag{1.224}$$

The three-phase system uses less amount of copper wire than a single-phase system for the same line voltage and same power factor to transmit the same amount of power over a fixed distance. From Fig. 1.35, the real power for a single-phase two-wire system is,

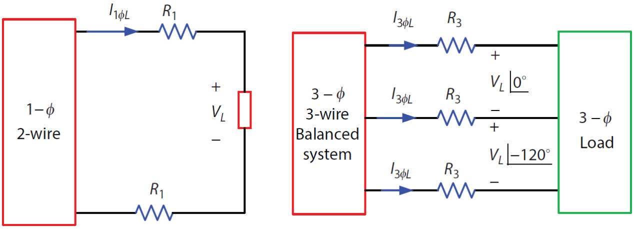

$I_{1\phi L}$
R_1
$1-\phi$
2-wire
+
V_L
−
R_1
$I_{3\phi L}$
R_3
$3-\phi$
3-wire
Balanced
system
$I_{3\phi L}$
R_3
$I_{3\phi L}$
R_3
+
$V_L \angle 0^\circ$
−
+
$V_L \angle -120^\circ$
−
$3-\phi$
Load

Fig. 1.35 Single-phase and three-phase systems with loads.

$$P_{1\phi 2w} = V_L I_{1\phi L} \cos\theta \tag{1.225}$$

From Fig. 1.35, the real power for a three-phase three-wire system is,

$$P_{3\phi 3w} = \sqrt{3}\, V_L I_{3\phi L} \cos\theta \tag{1.226}$$

Equations (1.225) and (1.226) will be equal for transmitting or delivering the same amount of power over a fixed distance. It can be expressed as,

$$V_L I_{1\phi L} \cos\theta = \sqrt{3}\, V_L I_{3\phi L} \cos\theta \tag{1.227}$$

$$I_{1\phi L} = \sqrt{3}\, I_{3\phi L} \tag{1.228}$$

The power loss in the single-phase wire is,

$$P_{1\phi 2wloss} = 2 I_{1\phi L}{}^2 R_1 \tag{1.229}$$

$$P_{3\phi 3wloss} = 3 I_{3\phi L}{}^2 R_3 \tag{1.230}$$

From equations (1.229) and (1.230), the ratio of power loss of a single-phase system to a three-phase system can be derived as,

$$\frac{P_{1\phi 2wloss}}{P_{3\phi 3wloss}} = \frac{2{I_{1\phi L}}^2 R_1}{3{I_{3\phi L}}^2 R_3} \tag{1.231}$$

For equal losses ($P_{1\phi 2\text{wloss}} = P_{3\phi 3\text{wloss}}$), equation (1.231) can be modified as,

$$1 = \frac{2{I_{1\phi L}}^2 R_1}{3{I_{3\phi L}}^2 R_3} \tag{1.232}$$

$$\frac{3{I_{3\phi L}}^2}{2{I_{1\phi L}}^2} = \frac{R_1}{R_3} \tag{1.233}$$

Substituting equation (1.228) into equation (1.233) yields,

$$\frac{3{I_{3\phi L}}^2}{2\times 3{I_{3\phi L}}^2} = \frac{R_1}{R_3} \tag{1.234}$$

$$\frac{R_1}{R_3} = \frac{1}{2} \tag{1.235}$$

The following ratio can be written as,

$$\frac{\text{Copper for } 3\phi \text{ system}}{\text{Copper for } 1\phi \text{ system}} = \frac{\text{number of wires in } 3\phi \text{ system}}{\text{number of wires in } 1\phi \text{ system}} \times \frac{R_1}{R_3} \tag{1.236}$$

Substituting equation (1.235) and the number of wires for both systems in equation (1.236) yields,

$$\frac{\text{Copper for } 3\phi \text{ system}}{\text{Copper for } 1\phi \text{ system}} = \frac{3}{2} \times \frac{1}{2} \tag{1.237}$$

$$\text{Copper for } 3\phi \text{ system} = \frac{3}{4} \times \text{Copper for } 1\phi \text{ system} \tag{1.238}$$

From equation (1.238), it is seen that the copper required for a three-phase system is equal to three-fourths of the copper required for a single-phase system.

Example 1.11

A balanced three-phase wye-wye system is shown in Fig. 1.36. For the *ABC* phase sequence, calculate the line current, power supplied to each phase, power absorbed by each phase and the total complex power supplied by the source.

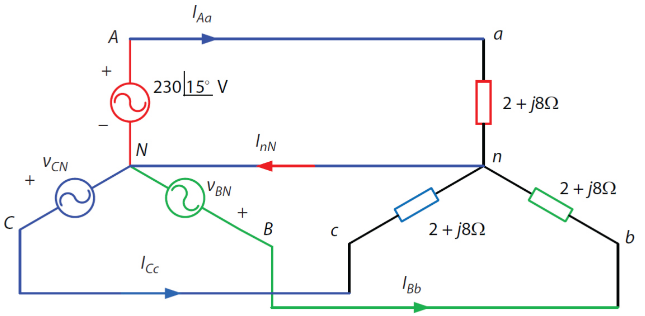


Fig. 1.36 Circuit for example 1.11.

Solution

The line currents are calculated as,

$$I_{Aa} = \frac{230\angle 15^\circ}{2 + j8} = 27.89\angle -60.96^\circ \text{ A} \tag{1.239}$$

$$I_{Bb} = I_{Aa}\angle -120^\circ = 27.89\angle -180.96^\circ \text{ A} \tag{1.240}$$

$$I_{Cc} = I_{Aa}\angle +120^\circ = 27.89\angle 59.04^\circ \text{ A} \tag{1.241}$$

Power supplied to each phase is calculated as,

$$P_A = V_p I_p \cos\theta = V_{AN} I_{Aa} \cos(\theta_v - \theta_i) = 230 \times 27.89 \times \cos(15 + 60.96) = 1556.20 \text{ W} \tag{1.242}$$

Per phase power absorbed by the load is calculated as,

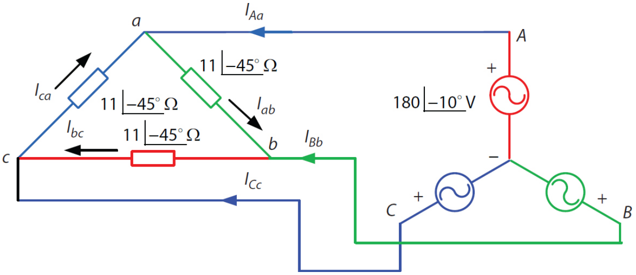
$$P_{L1\phi} = 27.89^2 \times 2 = 1555.70\,\text{W} \tag{1.243}$$

The total complex power supplied by the source is calculated as,

$$S_t = 3V_{An}I_{Aa}^* = 3\times 230\underline{|15^\circ} \times 27.89\underline{|60.96^\circ} = 4668.60 + j18669.21\,\text{VA} \tag{1.244}$$

Practice Problem 1.11

A balanced three-phase wye-delta system is shown in Fig. 1.37. For the *ABC* phase sequence, calculate the line current, power supplied to each phase, power absorbed by each phase and the total complex power supplied by the source.



a
I_{Aa}
A
I_{ca}
$11\underline{|-45^\circ}\,\Omega$
$11\underline{|-45^\circ}\,\Omega$
I_{ab}
$180\underline{|-10^\circ}\,\text{V}$
+
I_{bc}
$11\underline{|-45^\circ}\,\Omega$
b
I_{Bb}
c
−
I_{Cc}
C
+
+
B

Fig. 1.37 Circuit for practice problem 1.11.

1.16 Basic Measurement Equipment

In laboratory experiments, students usually verify fundamental electrical theories through different types of measurement of voltage, current, resistance and power. The voltmeter, ammeter, ohmmeter and wattmeter are used to measure those parameters. Nowadays, almost every educational laboratory is equipped with digital meters. An ohmmeter is used to check the circuit continuity, identify short circuits and open circuits and find out the specific lead of a multi-lead cable. The voltmeter is used to measure the voltage of a circuit, and it always connects in parallel with a specific element. The voltmeter has a high input impedance. An ammeter is an instrument that is used to measure the current in the circuit. An ammeter has low input impedance, and it connects in series with an element of the circuit. Under the energized condition, the ammeter cannot disconnect from the circuit, whereas the voltmeter can be disconnected from the circuit. A wattmeter has two coils, namely a voltage coil and a current coil. The voltage coil is connected across the element, and the current coil is connected in series. The symbols of voltmeter, ammeter, wattmeter and ohmmeter are shown in Fig. 1.38.

There are many advanced electrical meters available for use in the practical field and can be obtained at a low price from hardware stores. Fluke Corporation introduced different types of the digital multimeter as shown in Fig. 1.39, which includes advanced safety features during practical measurements. These meters have proprietary functions that prevent accidents resulting from breakers that suddenly trip due to incorrect connections. These meters also have high accuracy, reliability, extensive additional functionality and a broad range of measurement activities.

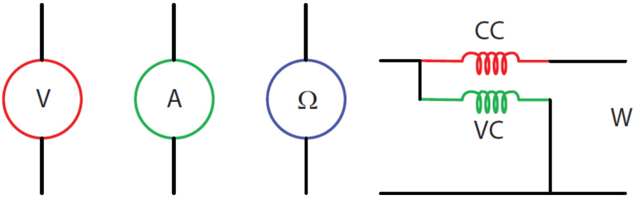


Fig. 1.38 Symbols of basic electrical meters.

FLUKE 87 V MAX TRUE RMS MULTIMETER
19998
MIN MAX
RANGE
Auto HOLD
PEAK MIN MAX
REL
Hz %
OFF
10A MAX FUSED
400mA FUSED
COM
IP67
FLUKE 289 TRUE RMS MULTIMETER
120.12 VAC
Maximum 121.32
Average 120.06
Minimum 97.37
Save
Stop
F1
F2
F3
F4
HOLD
RANGE
MIN MAX
Info
LoZ
COM

Fig. 1.39 Digital multimeters courtesy of Fluke Corporation.

Exercise Problems

1.1 The excitation voltage and the impedance of a series circuit are given by v(t) = 8 sin(10t) V and $Z = 5\angle 10^\circ\ \Omega$, respectively. Calculate the instantaneous power.

1.2 The excitation current and the impedance of a series circuit are given by $i(t) = 4\sin(100t - 20^\circ)$A and $Z = 5\angle 10^\circ\ \Omega$, respectively. Determine the instantaneous power.

1.3 Calculate the average power supplied by the source and the power absorbed by the resistors as shown in Fig. P1.1.

1.4 Determine the average power supplied by the source and the power absorbed by the 8Ω resistor shown in Fig. P1.2.

1.5 Calculate the average power supplied by the source and the power absorbed by the 3Ω resistor as shown in Fig. P1.3.

1.6 Determine the average power supplied by the source and the power absorbed by the 2Ω resistor shown in Fig. P1.4.

1.7 Find the total average power absorbed by all the resistors in the circuit shown in Fig. P1.5.

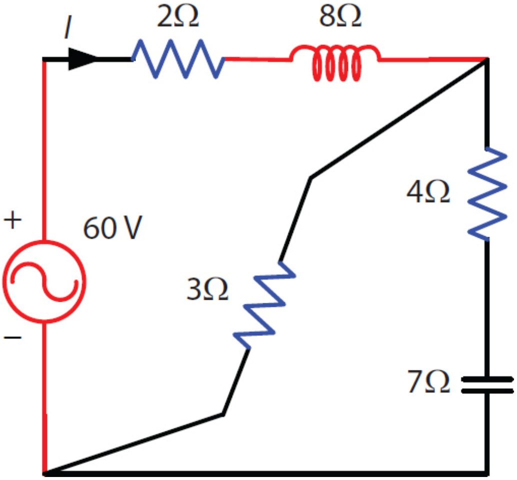

I
2Ω
8Ω
4Ω
+
60 V
3Ω
−
7Ω

Fig. P1.1 Circuit for problem 1.3.

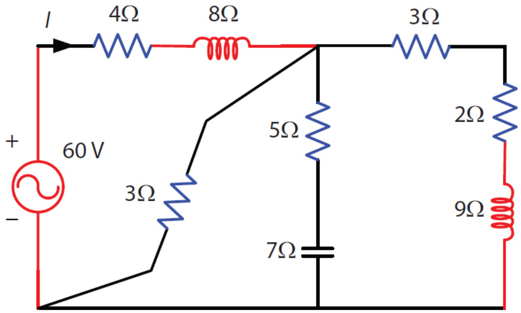

I
4Ω
8Ω
3Ω
2Ω
5Ω
+
60 V
3Ω
−
9Ω
7Ω

Fig. P1.2 Circuit for problem 1.4.

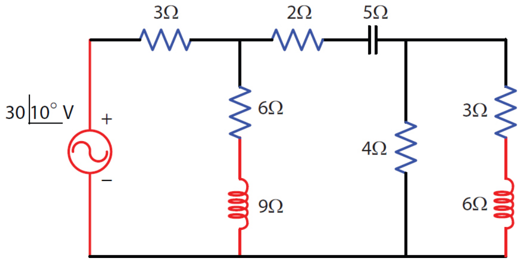

3Ω
2Ω
5Ω
30∠10° V
+
6Ω
3Ω
4Ω
−
9Ω
6Ω

Fig. P1.3 Circuit for problem 1.5.

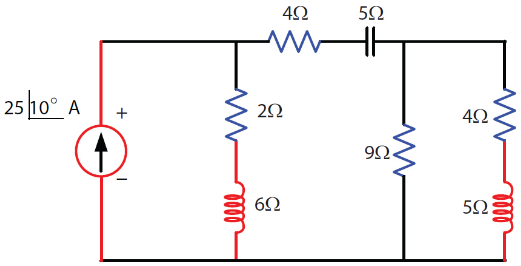


Fig. P1.4 Circuit for problem 1.6.

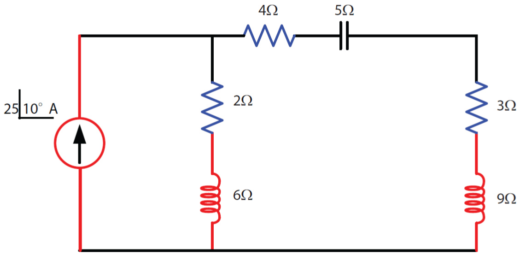


Fig. P1.5 Circuit for problem 1.7.

1.8 An industrial load is connected across an alternating voltage source $v(t) = 230\sin(314t + 20°)$V that draws a current of $i(t) = 15\sin(314t + 45°)$A. Determine the apparent power, circuit resistance and capacitance.

1.9 The rms values of voltage and current are given by $V = 20\angle{-15°}\text{ V}$, and $I = 3\angle 25°\text{ A}$. Calculate the complex power, real power and reactive power.

1.10 The rms value of voltage is given by $V = 34\angle 25°\text{ V}$, and the impedance is $Z = 6\angle{-15°}\,\Omega$. Determine the complex power, real power and reactive power.

1.11 A series-parallel circuit is supplied by a rms source of 60 V as shown in Fig. P1.6. Find the total complex power.

1.12 Calculate the total complex power of the circuit shown in Fig. P1.7.

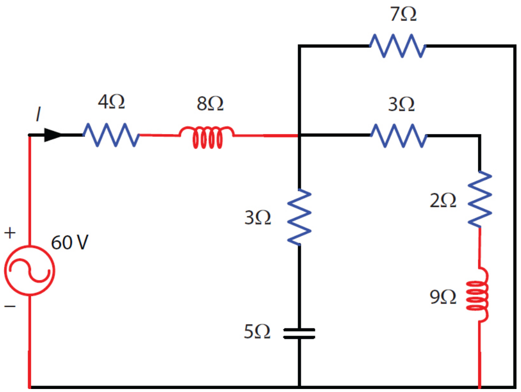


Fig. P1.6 Circuit for problem 1.11.

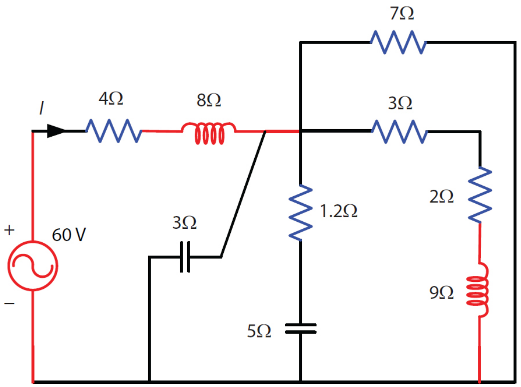


Fig. P1.7 Circuit for problem 1.12.

1.13 A 10 kVA, 50 Hz, 0.6 lagging power factor load is connected across an rms voltage source of 220 V as shown in Fig. P1.8. A capacitor is connected across the load to improve the power factor to 0.85 lagging. Find the value of the capacitor.

1.14 Two loads with different power factors are connected with the source through a transmission line as shown in Fig. P1.9. Determine the source current and the source voltage.

1.15 A voltage source delivers power to the three loads shown in Fig. P1.10. Find the source current and the source voltage.

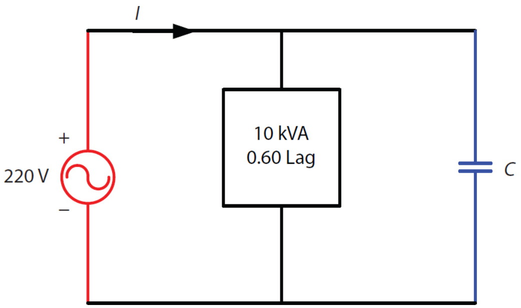

I
+
220 V
−
10 kVA
0.60 Lag
C

Fig. P1.8 Circuit for problem 1.13.

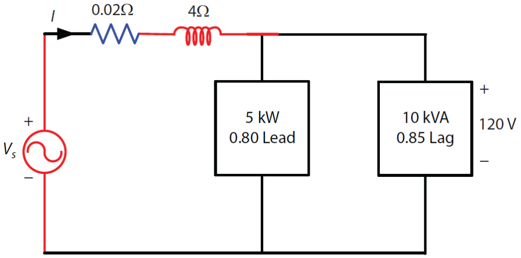

I
0.02Ω
4Ω
+
V_s
−
5 kW
0.80 Lead
10 kVA
0.85 Lag
+
120 V
−

Fig. P1.9 Circuit for problem 1.14.

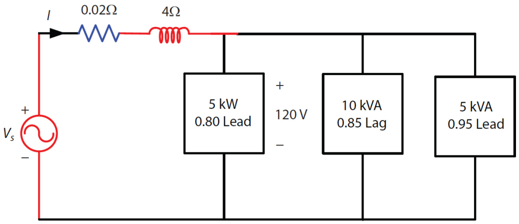

I
0.02Ω
4Ω
+
V_s
−
5 kW
0.80 Lead
+
120 V
−
10 kVA
0.85 Lag
5 kVA
0.95 Lead

Fig. P1.10 Circuit for problem 1.15.

1.16 The line voltage of a three-phase wye-connected generator is found to be 440 V. For the *abc* phase sequence, calculate the phase voltages.

1.17 The phase voltage of a three-phase wye-connected generator is given by $V_{an} = 100\angle -10^\circ$ V. Determine the voltages V_{bn} and V_{cn} for the *abc* phase sequence.

2

Magnetic Circuits

2.1 Introduction

Knowledge of electricity, magnetism, and electromagnetic induction is important to understand the working principles of electrical machines. These machines are generators, transformers, motors, electromagnetic relays, and inductors that work on the principle of magnetic force produced by the current in the conductors.

Many scientists observed that the magnetic field could be created by an electric current in a conductor. However, in 1819, Danish physicist Hans Christian Oersted was the first scientist to connect electricity and magnetism successfully. He tested his hypothesis by placing a compass near a current-carrying conductor. When an electric current flows in the conductor, the compass needle is deflected perpendicular to the conductor. The compass needle went back to its original position when the conductor carried no current. He also observed that the strength of the magnetic field gets weaker farther away from the conducting wire or conductor.

In this chapter, the main features of magnetism will be discussed and analyzed to enhance the understanding of electrical machines.

2.2 Magnetic Force and Field

A moving charge or collection of moving charges around a current-carrying conductor creates a magnetic field. This magnetic field exerts magnetic forces on other moving charges. The force of a magnetic field exerts on a charge q that moves with a velocity v is known as the magnetic component of Lorentz force. The magnetic component of Lorentz force is expressed as,

$$F_m = q\,v \times B \tag{2.1}$$

where,

F_m is the Lorentz force perpendicular to the direction of the magnetic field,

F_m is perpendicular to the direction of the velocity v,

F_m is perpendicular to the plane of $v \times B$.

The charge q moves at an angle ϕ to the magnetic field and experiences a magnetic force F_m as shown in Fig. 2.1. The magnetic force is expressed as,

$$F_m = qv_{\perp}B = q\,v \times B \tag{2.2}$$

From equation (2.2), the magnitude of the force is written as,

$$F_m = q\,vB\sin\phi \tag{2.3}$$

Here, ϕ is the angle between the magnetic field (B) and velocity (v) and it is less than 180°.

where,

q is the charge in C,

v is the velocity in m/s,

B is the magnetic flux density in Wb/m2.

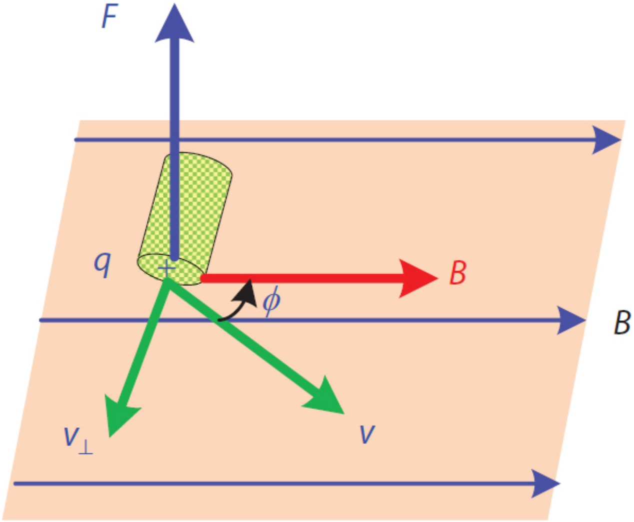


Fig. 2.1 Velocity of charge moves with an angle ϕ.

The Lorentz force is maximum when the velocity of the charge is perpendicular to the magnetic field as shown in Fig. 2.2. In this condition, the maximum force is determined as,

$$F_{max} = qvB\sin 90^\circ = qvB \tag{2.4}$$

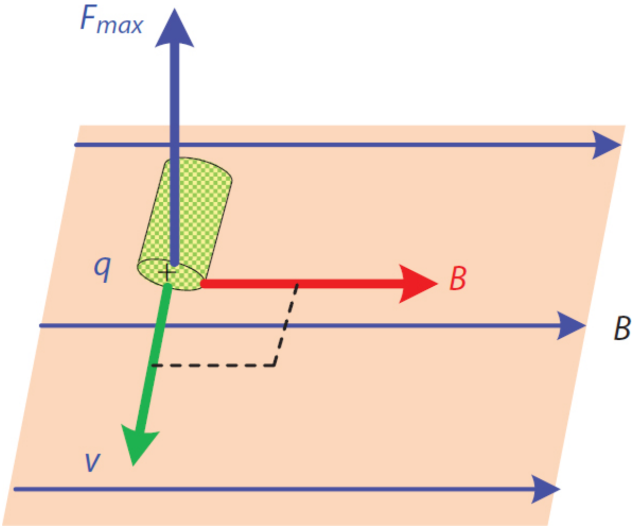


Fig. 2.2 Velocity of charge moves with an angle of 90º.

The charge q moves parallel to the magnetic fields that experience a zero magnetic force as shown in Fig. 2.3.

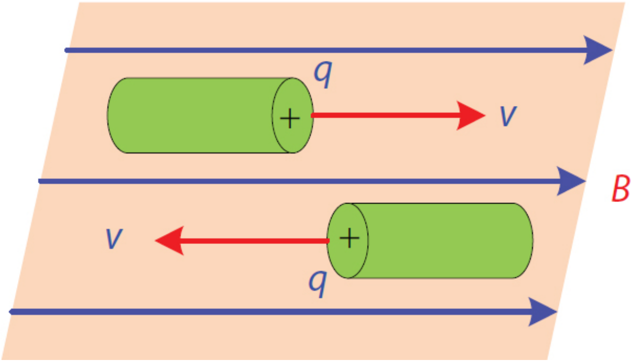


Fig. 2.3 Charge moves parallel to the magnetic fields.

The permanent magnet contains the north pole at one end and the south pole at the other end that can attract and hold small pieces of iron and steel. The magnetic field lines usually come out from the north pole and enter the south pole as shown in Fig. 2.4.

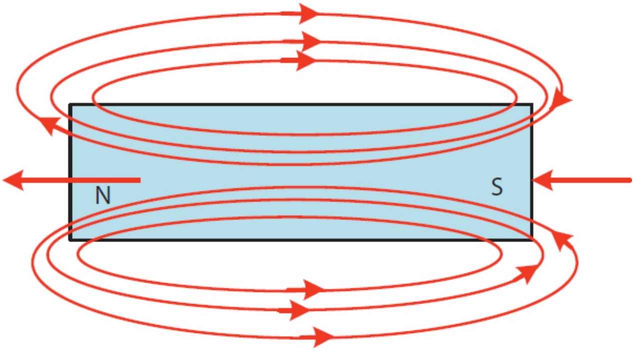


Fig. 2.4 Magnetic field lines travel.

Unlike poles of different magnets attract each other as shown in Fig. 2.5(a), whereas the like poles of different magnets repel each other as shown in Fig. 2.5(b).

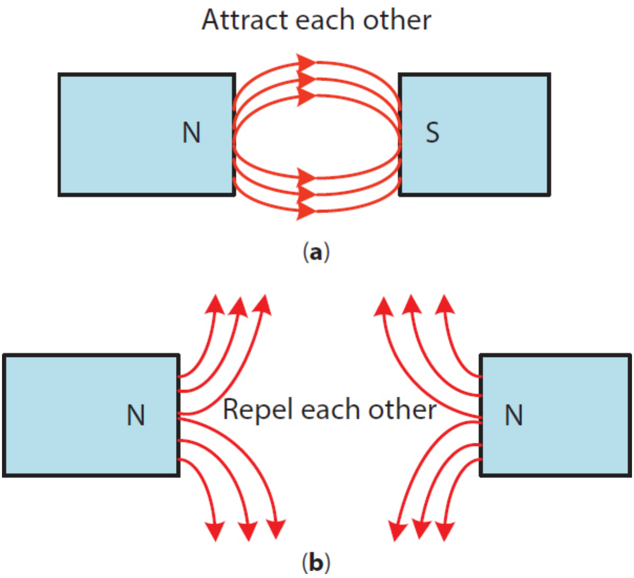


Fig. 2.5 Magnetic field lines for like and unlike poles.

Some points need to be known about the magnetic field lines. The magnetic field lines do not intersect with each other. The magnetic field lines are continuous and form a closed loop in the air. Magnetic field lines come out from the north pole and come into the south pole. Magnetic field lines follow the least reluctance path.

Hendrik Antoon Lorentz (July 18, 1853–February 4, 1928) was a Dutch physicist who discovered the theories related to electromagnetic field radiation. According to his name, this law is known as Lorentz law. The statement of the Lorentz law is “The force is the sum of the electric force and magnetic force when a charge particle ***q*** moves with a velocity ***v*** in the electric and magnetic field”.

A charge particle q in an electric field always observes a force. The electric field is defined as the force that is represented as,

$$E = \frac{F_e}{q} \tag{2.5}$$

$$F_e = qE \tag{2.6}$$

Then the Lorentz force law is expressed as,

$$F = F_e + F_m \tag{2.7}$$

Substituting equations (2.1) and (2.6) in equation (2.7) yields the Lorentz force as,

$$F = qE + qv \times B = q(E + v \times B) \tag{2.8}$$

2.3 Magnetic Circuit

The circuit followed by a magnetic in a closed path is known as a magnetic circuit. The schematic of a simple magnetic circuit is shown in Fig. 2.6. The dashed red line represents the average length of the magnetic circuit. The coil is connected to the alternating voltage source and the current flow in the circuit. This current will create the magnetic flux and the magnetic flux is directly proportional to the current. The maximum value of this magnetic flux will move through the closed magnetic circuit. The current is sinusoidal and the flux is also sinusoidal.

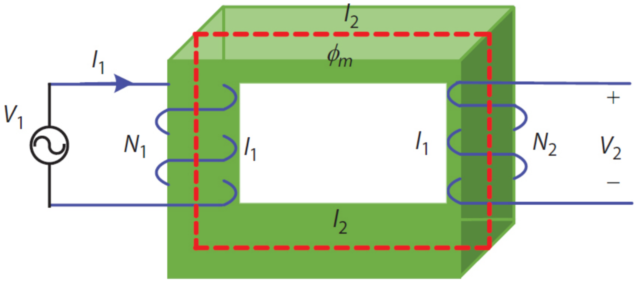


Fig. 2.6 Schematic of a magnetic circuit.

2.4 Magnetic Flux and Magnetic Flux Density

The total number of magnetic field lines passing through a given area is known as magnetic flux. The magnetic flux provides information on how much magnetic flux is passing through a given area. Magnetic flux is very important in electromagnetism and it is also known as magnetic induction. Mathematically, the magnetic flux is defined as the product of the magnetic flux density and the cross-sectional area. The magnetic flux is represented by the Greek letter ϕ and it is measured in weber (Wb) in honour of the German physicist Wilhelm Eduard Weber. In the SI system, the unit of magnetic flux is weber (Wb) and one weber equals 10^8 lines. The weber is a very large unit; thus, in most practical situations, the microweber (μWb) is used.

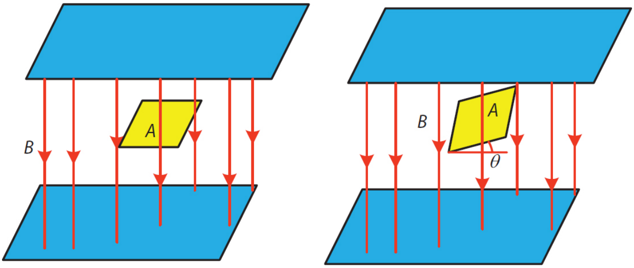


Fig. 2.7 Flux lines with a cross-sectional area.

According to Fig. 2.7, the magnetic flux can be written as,

$$\phi = B \bullet A \tag{2.9}$$

When the object is placed in an angle θ as shown in Fig. 2.7, then the flux is expressed as,

$$\phi = BA\cos\theta \tag{2.10}$$

Magnetic flux density is defined as the amount of magnetic flux in a unit area that is perpendicular to the direction of the magnetic flux. Alternatively, the magnetic flux density is defined as the flux per unit area and it is represented by the letter *B*. The magnetic field lines with a cross-sectional area are shown in Fig. 2.8.

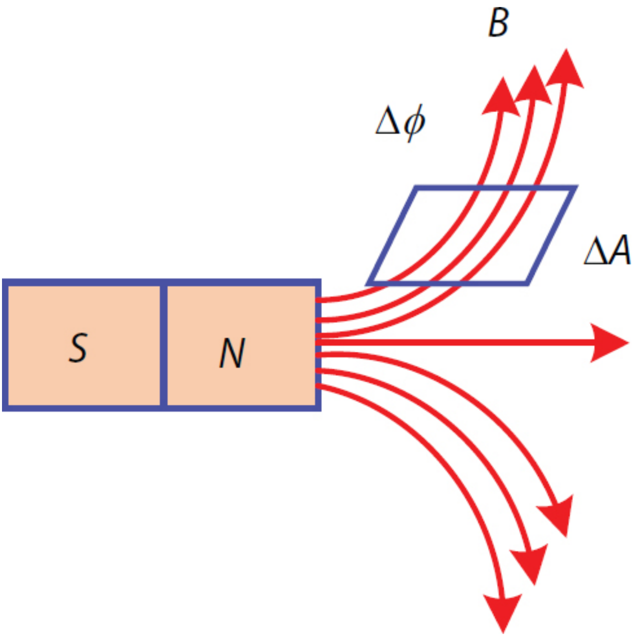


Fig. 2.8 Magnetic flux with a cross-sectional area.

Mathematically, the magnetic flux density is represented as,

$$B = \frac{\phi}{A} \tag{2.11}$$

where,

B is the flux density in Wb/m^2 or Tesla,

ϕ is the magnetic flux in Wb,

A is the cross-sectional area in m^2.

Example 2.1

The area of a bar magnet is 6 m^2. Calculate the magnetic flux density if the flux is 1.25 Wb.

Solution

The value of the magnetic flux is calculated as,

$$B = \frac{\phi}{A} = \frac{1.5}{6} = 0.25\,\mathrm{Wb/m^2\,(T)} \tag{2.12}$$

Example 2.2

An object with an area of 10 m^2 is placed in a magnetic field at an angle of 45°. Find the magnetic flux density if the flux is 1.5 Wb.

Solution

The value of the magnetic flux density is calculated as,

$$B = \frac{\phi}{A\cos\theta} = \frac{1.5}{10\cos 45^\circ} = 0.21\,\mathrm{Wb/m^2\,(T)} \tag{2.13}$$

Practice Problem 2.1

The area of a bar magnet is 16 m^2. Calculate the magnetic flux if the magnetic flux density is 1. 5 Wb/m^2.

Practice Problem 2.2

An object with an area of 4 m^2 is placed in a magnetic field at an angle of 65°. Calculate the magnetic flux density if the flux is 0.95 Wb.

2.5 Magnetomotive Force

In an electric circuit, an electromotive force is required to start the current flowing in the electric current. Similarly, a magnetomotive force which is abbreviated as MMF is required to set up the flux in a magnetic circuit. The external force that is required to set up the magnetic flux lines in the magnetic circuit is known as the magnetomotive force. The N number of turns are wound around a rectangular core and the current I flows through the coils when it is connected to the source as shown in Fig. 2.9.

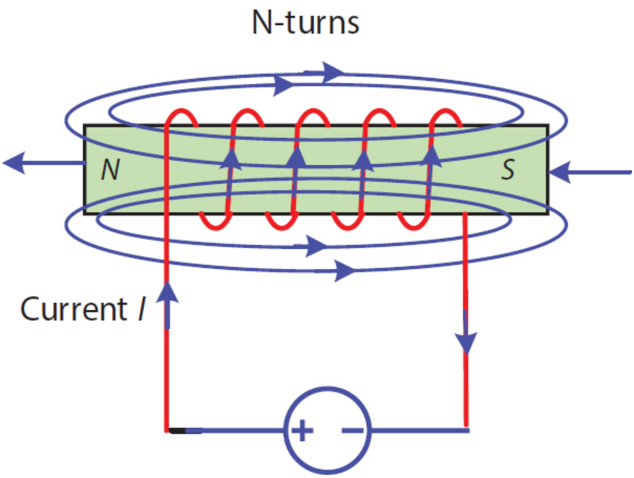


Fig. 2.9 Rectangular core wound with coils.

Alternatively, the magnetomotive force is defined as the product of the number of turns wound around the core and the current that flows through the turns of the coil. The magnetomotive force is represented by the Greek letter I. In the SI system, the unit of magnetomotive force is Ampere-turns (At) and in the CGS system, the unit is G (Gilburt). Mathematically, it is directly proportional to the number of turns of the coil and the current in the coil. These are written as,

$$(2.14)\Im \infty N$$

$$(2.15)\Im \infty I$$

$$(2.16)\Im = NI$$

where,

N is the number of turns in the coil,

I is the current in A,

$\Im$ is the magnetomotive force in At.

2.6 Magnetizing Force

The magnetizing force is often known as magnetic field intensity.

The magnetic field intensity is different from than magnetic flux density. The magnetic field intensity is defined as the magnetomotive force per unit length. Consider N number of turns of the coil is partly wound around the core as shown in Fig. 2.10. Magnetizing force is represented by the capital letter H and its unit is expressed as At/m. Mathematically, it can be represented as,

$$H = \frac{\Im}{l} \tag{2.17}$$

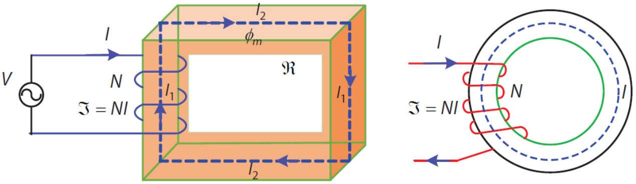


Fig. 2.10 Rectangular and toroidal cores wound with coils.

Substituting equation (2.16) into equation (2.17) yields,

$$H = \frac{NI}{l} \tag{2.18}$$

The magnetic flux density and magnetizing force are also related by the following equation,

$$B = \mu H \tag{2.19}$$

where,

l is the mean length of the magnetic core in m,

μ_0 is the permeability with a value of 4π x 10^{-7} H/m,

NI is the mmf in ampere-turns.

Example 2.3

A cast iron toroid has an inside diameter of 2 cm and an outside diameter of 5 cm. The core is wound by the coil of 150 turns and carries a current of 2A. The value of the flux is 0.012 Wb and it has a circular cross-section as shown in Fig. 2.11. Find the flux density, magnetomotive force, and magnetic field intensity.

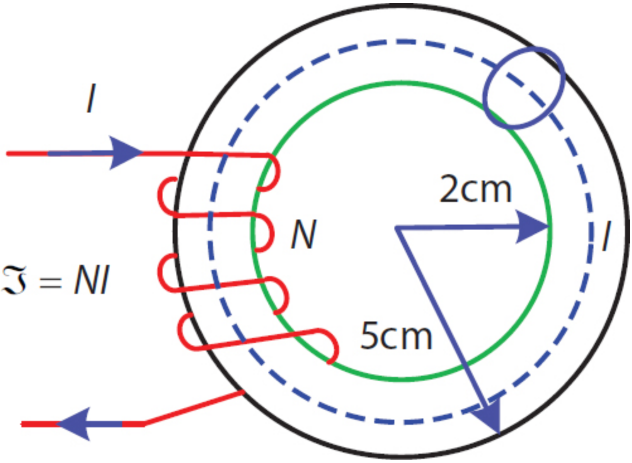


Fig. 2.11 Circuit for example 2.3.

Solution

The value of the mean distance is determined as,

$$d = 2 + \frac{5-2}{2} = 3.5\text{cm} \tag{2.20}$$

The mean length of the core is calculated as,

$$l = \pi d = \pi \times 3.5 = 11\text{cm} \tag{2.21}$$

The cross-sectional area of the core is circular and has a diameter of 3 cm (5 cm-2 cm). The value of the area is calculated as,

$$A = \frac{\pi d^2}{4} = \frac{\pi \times \left(\frac{3}{100}\right)^2}{4} = 7.06 \times 10^{-4}\,\text{m}^2 \tag{2.22}$$

The value of the flux density is calculated as,

$$B = \frac{\phi}{A} = \frac{0.012}{7.06 \times 10^{-4}} = 17\text{Wb/m}^2 \tag{2.23}$$

The value of the magnetomotive force is calculated as,

$$\Im = NI = 150 \times 2 = 300\ \text{At} \tag{2.24}$$

The value of the magnetic field intensity is calculated as,

$$H = \frac{NI}{l} = \frac{300}{0.11} = 2727.27\ \text{At/m} \tag{2.25}$$

Practice Problem 2.3

The mean length of the toroidal coil is 2.15 cm. The toroidal coil is wound by 120 turns. Find the value of the current if the magnetic field intensity is 180 At/m.

2.7 Permeability and Relative Permeability

The magnetic flux density of any material is directly proportional to the magnetic field intensity and mathematically, it is expressed as,

$$B \propto H \tag{2.26}$$

$$B = \mu H \tag{2.27}$$

$$\mu = \mu_0 \mu_r = \frac{B}{H} \tag{2.28}$$

where,

μ is the permeability of the medium,

μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m),

μ_r is the relative permeability of the medium.

The permeability of the material is the ability to form and sustain the magnetic fields in a material. It is defined as the ratio of the magnetic flux density to the magnetic field intensity. Its unit is Henry per meter (H/m).

The relative permeability of any material is defined as the ratio of the permeability of the material to the permeability of free space. Mathematically, the relative permeability is expressed as,

(2.29)
$$\mu_r = \frac{\mu}{\mu_0}$$

Example 2.4

The relative permeability of the rectangular core is 1625. Calculate the magnetic field intensity, permeability and magnetic flux density. The core is wound by 5 turns and carries a current of 2A as shown in Fig. 2.12.

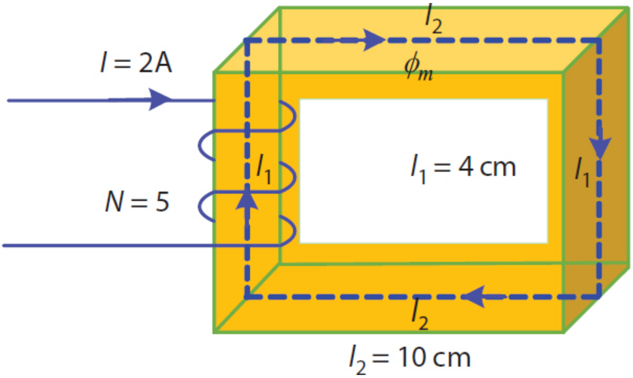


Fig. 2.12 Circuit for example 2.4.

Solution

The mean length of the core is calculated as,

(2.30)
$$l = 2(10 + 4) = 28\text{cm}$$

The value of the magnetic field intensity is calculated as,

(2.31)
$$H = \frac{NI}{l} = \frac{5 \times 2}{0.28} = 35.71\text{At/m}$$

The value of the permeability is calculated as,

(2.32)
$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1625 = 2.04 \times 10^{-3}$$

The value of the magnetic flux density is calculated as,

$$B = \mu H = 2.04 \times 10^{-3} \times 35.71 = 0.073\,\mathrm{Wb/m^2} \tag{2.33}$$

Practice Problem 2.4

A rectangular core with the dimensions is shown in Fig. 2.13. The relative permeability of the rectangular core is 1800 and it is wound by 6 turns and carries a current of 1.2 A. Calculate the magnetic field intensity, permeability and magnetic flux density.

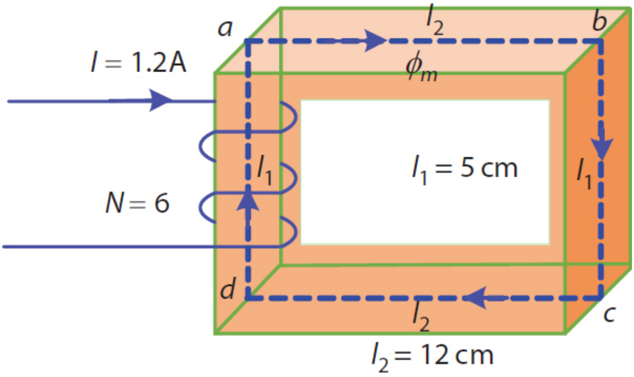


Fig. 2.13 Circuit for practice problem 2.4.

2.8 Reluctance

In an electric circuit, resistance opposes the flow of electric current. Similarly, reluctance opposes setting up flux in a magnetic circuit. In May 1888, British scientist Oliver Heaviside invented the term reluctance. In Ohm's law, the current is directly proportional to the voltage and it is expressed as,

$$V \propto I \tag{2.34}$$

$$V = IR \tag{2.35}$$

In a magnetic circuit, the magnetomotive force is directly proportional to the flux and it is written as,

$$\Im \propto \phi \tag{2.36}$$

$$\Im = \phi \Re \tag{2.37}$$

$$\Re = \frac{\Im}{\phi} \tag{2.38}$$

From equation (2.38), reluctance is defined as the ratio of magnetomotive force to the flux of a magnetic circuit. Substituting equations (2.11) and (2.16) into equation (2.38) yields,

$$\Re = \frac{NI}{BA} \tag{2.39}$$

Substituting equation (2.27) into equation (2.39) yields,

$$\Re = \frac{NI}{\mu HA} \tag{2.40}$$

Substituting equation (2.18) into equation (2.40) yields,

$$\Re = \frac{NI}{\mu \frac{NI}{l} A} \tag{2.41}$$

$$\Re = \frac{l}{\mu A} \tag{2.42}$$

In general, the expression of reluctance is written as,

$$\Re = \frac{\Im}{\phi} = \frac{l}{\mu A} \tag{2.43}$$

where,

$\Re$ is the reluctance of the magnetic circuit in At/Wb,

l is the average length of the magnetic path in meter (m),

A is the cross-sectional area of the magnetic circuit in m^2.

2.9 Permeance

Permeance is also important in a magnetic circuit. The permeance is defined as the ratio of the magnetic flux to the magnetomotive force of a magnetic circuit is known as permeance. Alternatively, it can be defined as the reciprocal of reluctance. It is denoted by the letter P and its unit is Wb/At. The permeance is directly proportional to the magnetic flux and inversely proportional to the magnetomotive force. Mathematically, it can be expressed as,

$$P = \frac{\phi}{\Im} \tag{2.44}$$

Substituting equations (2.11) and (2.16) into equation (2.40) yields,

$$P = \frac{BA}{NI} \tag{2.45}$$

Substituting equations (2.27) into equation (2.45) yields,

$$P = \frac{\mu HA}{NI} \tag{2.46}$$

Substituting equation (2.18) into equation (2.46) yields,

$$P = \frac{\mu \frac{NI}{l} A}{NI} \tag{2.47}$$

$$P = \frac{\mu A}{l} \tag{2.48}$$

$$P = \frac{1}{\frac{l}{\mu A}} \tag{2.49}$$

Substituting equation (2.42) into equation (2.49) yields,

$$P = \frac{1}{\mathfrak{R}} \tag{2.50}$$

In general, the permeance is expressed as,

$$P = \frac{\phi}{\mathfrak{I}} = \frac{1}{\mathfrak{R}} \tag{2.51}$$

Example 2.5

A core is wound with 10 turns and carries a current of 1.5 A as shown in Fig. 2.14. Determine the magnetomotive force, reluctance and permeance of the circuit if the flux is 0.95 Wb.

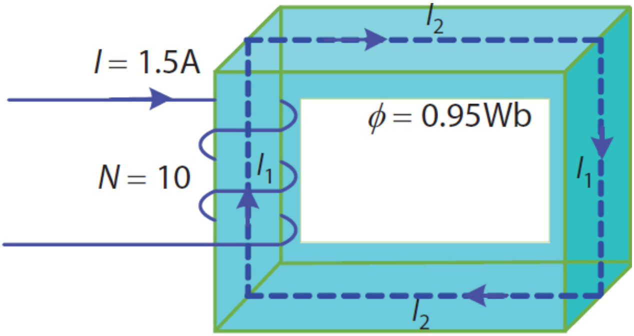


Fig. 2.14 Circuit for example 2.5.

Solution

The magnetomotive force is calculated as,

$$\mathfrak{I} = NI = 10 \times 1.5 = 15\,\text{At} \tag{2.52}$$

The value of reluctance is calculated as,

$$\mathfrak{R} = \frac{\mathfrak{I}}{\phi} = \frac{15}{0.95} = 15.79\,\text{At/Wb} \tag{2.53}$$

Practice Problem 2.5

A toroidal coil carries a current of 1.5 A and the magnetomotive is found to be 20 At as shown in Fig. 2.15. Calculate the number of turns of the coil and the reluctance of the circuit if the flux is 1.2 Wb.

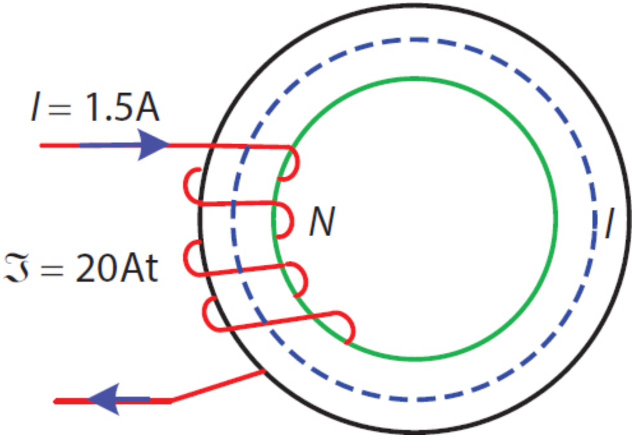


Fig. 2.15 Circuit for practice problem 2.5.

2.10 Fleming’s Rules

In the 19th century, Professor Sir John Ambrose Fleming, an English electrical engineer and physicist. invented two rules related to the direction of motion of the electric motor and the direction of the current in the generator. According to his name, these rules are known as **Fleming’s left-hand rule** and **Fleming’s right-hand rule**.

Fleming’s right-hand rule is used in the generator to identify the direction of the current. According to Faraday’s laws of electromagnetic induction, when a conductor moves in a magnetic field it induces a current. Fleming’s right-hand rule is used to identify the direction of the induced current. The statement of **Fleming’s right-hand rule** is “Hold the thumb finger, forefinger and middle finger of the right hand in such a way that they are perpendicular to each other; the thumb indicates the direction of motion and the forefinger indicates the direction of the magnetic field, then the middle finger indicates the direction of induced voltage or direction of induced current,” as shown in Fig.

2.16.

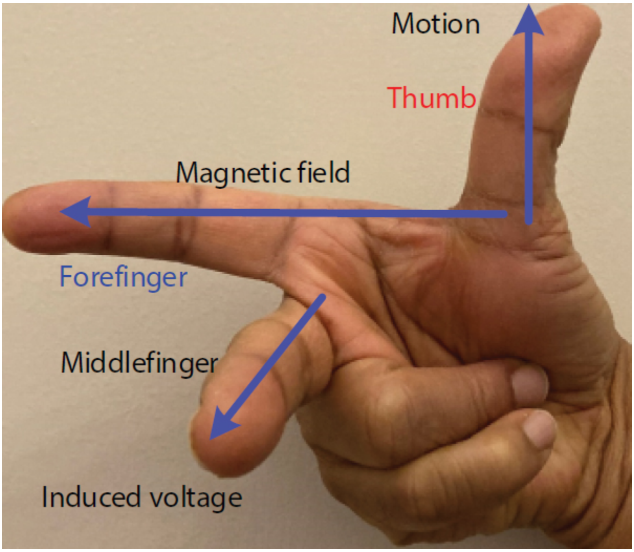


Fig. 2.16 Fleming's right-hand rule.

When a current-carrying conductor is placed in a magnetic field it experiences a force that is perpendicular to the magnetic field and the direction of the current. The statement of Fleming's left-hand rule is "Hold the thumb finger, forefinger and middle finger of the left hand in such a way that they are perpendicular to each other; the middle finger indicates the direction of the current and the forefinger indicates the direction of the magnetic field, then the thumb finger indicates the direction of motion or movement," as shown in Fig. 2.17.

2.11 Analogy of Magnetic and Electric Circuits

In the circuit the MMF creates the magnetic flux in the path against its reluctance; this is known as the magnetic circuit, whereas in the circuit EMF creates the current in the path against the resistance is known as an electric circuit. Magnetic and electric circuits have similarities and the corresponding analogies of these circuits are mentioned in Table 2.1.

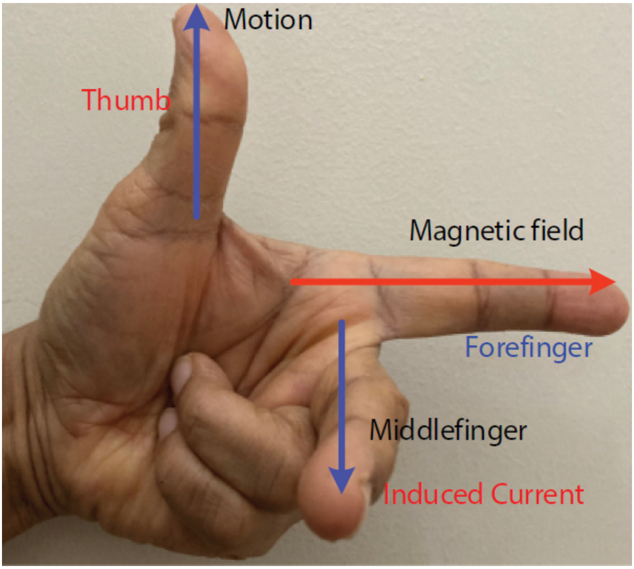


Fig. 2.17 Fleming's left-hand rule.

Table 2.1 Analogies of magnetic and electric circuits.

	Magnetic circuit	**Electric circuit**
1	The magnetic circuit follows the magnetic flux	The electric circuit follows the electric current
2	The flux of the magnetic circuit is defined as the MMF per reluctance, $\phi = \mathfrak{I}/\mathfrak{R}$	The current is defined as the voltage per resistance, $I = V/R$
3	The flux density is defined as flux per cross-sectional area, $B = \phi/A$	The current density is defined as the current per cross-sectional area, $J = I/A$
4	The reluctance is defined as $\mathfrak{R} = l/\mu A$	The resistance is defined as $R = \rho l/A$
5	The MMF drop is expressed as $\mathfrak{I} = \phi \mathfrak{R}$	The voltage drop is expressed as $V = IR$

6	Magnetic circuits store energy in their fields	Electric circuit releases energy as heat
7	Permeance is expressed as P = 1/ $\mathfrak{R}$	Conductance is written as G = 1/R
8	The unit of magnetic flux is weber (Wb)	The unit of the current is ampere (A)

2.12 Ampere’s Circuital Law

French physicist André-Mare Ampère (1775-1836) experimented to formulate the famous law of electromagnetism. According to his name, it is known as Ampere’s law or Ampere’s circuital law. Ampere’s law is an important relationship between the magnetic fields and the electric current that allows calculating the magnetic field produced by a current flowing through a wire in any shape. Let us consider an arbitrary path of a wire that carries a current I that cuts the infinitesimal length dl that contains the magnetic field B as shown in Fig. 2.18. This magnetic field makes an angle θ and it is constant along the path.

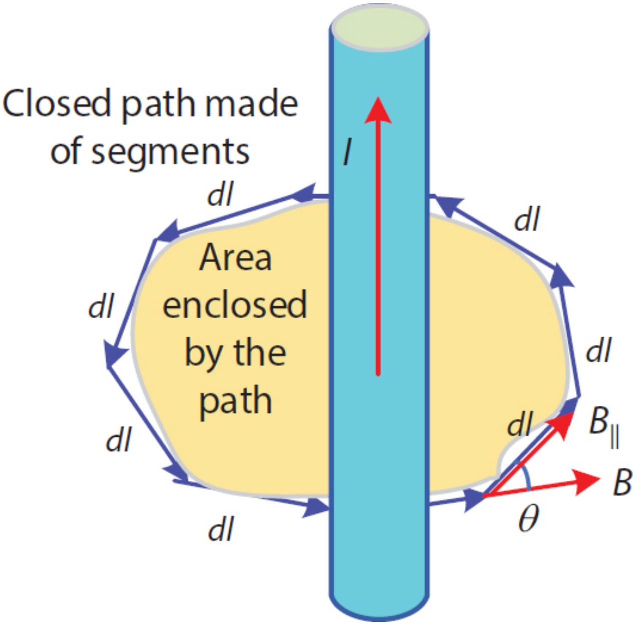

Closed path made of segments
I
dl
dl
Area enclosed by the path
dl
dl
dl
dl
dl
dl
B_||
B
θ

Fig. 2.18 A wire with a closed path.

Ampere's law states that the line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the currents flowing through the area enclosed by the path.

$$\oint B \bullet dl = \sum B_{\|}\, dl = \mu I_{enclosed} \tag{2.54}$$

where,

B_{II} is the parallel component of the magnetic component,

dl is the infinitesimal length of the closed path,

$I_{cnclosed}$ is the current enclosed by the path.

2.13 Magnetic Flux Density of a Long Straight Wire

Let us consider a long straight conductor that carries a current I as shown in Fig. 2.19. However, this conductor can extend in either direction and the conductor cuts into the infinitesimal lengths *dl*. Again consider that the magnetic field at point *A* is a distance *r* from the center of a conductor is *B*.

Then the magnetic field is calculated as,

$$\oint B \bullet dl = \int_0^{2\pi r} B\cos\theta dl = B\cos\theta \left| l \right|_0^{2\pi r} = B\cos\theta(2\pi r) \tag{2.55}$$

If the field forms a circle then cosθ = 1, equation (2.55) becomes,

$$\oint B \bullet dl = \int_0^{2\pi r} B\cos\theta dl = B\cos\theta \left| l \right|_0^{2\pi r} = B(2\pi r) \tag{2.56}$$

Finally, applying Ampere’s law,

$$\oint B \bullet dl = B(2\pi r) = \mu I_{enclosed} \tag{2.57}$$

$$B(2\pi r) = \mu I_{enclosed} = \mu I \tag{2.58}$$

$$B = \frac{\mu I}{2\pi r} \tag{2.59}$$

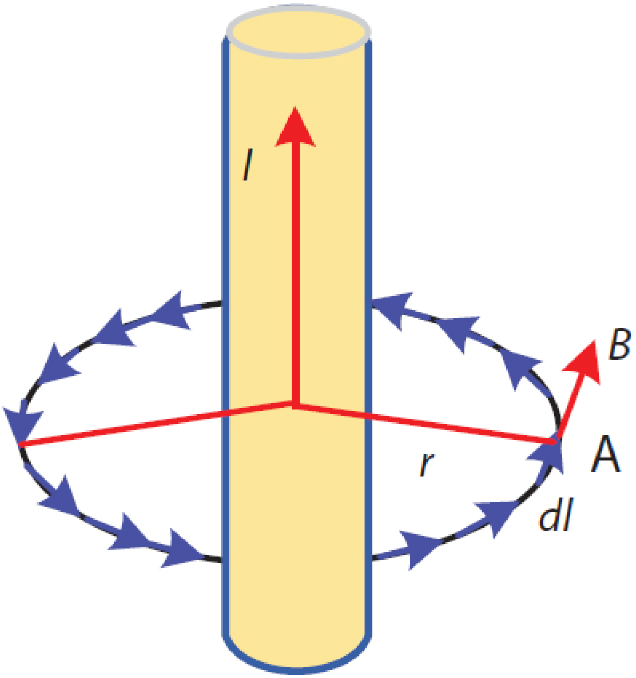


Fig. 2.19 A wire with a radial distance *r*.

Substituting equation (2.27) into equation (2.59) yields,

$$\mu H = \frac{\mu I}{2\pi r} \tag{2.60}$$

$$H = \frac{I}{2\pi r} \tag{2.61}$$

For N number of turns, the magnetic field and magnetic flux density are expressed as,

$$B = \frac{\mu N I}{2\pi r} \tag{2.62}$$

$$H = \frac{N I}{2\pi r} \tag{2.63}$$

$$I = \frac{H \times 2\pi r}{N} = 6.28\frac{Hr}{N} \tag{2.64}$$

where,

B is the magnetic flux density in Wb/m^2,

H is the magnetic field intensity in At/m,

I is the current in the coil in A,

r is the radial distance in m,

N is the number of turns,

Example 2.6

A long straight conductor carries a current of 8 A and the conductor is surrounded by air. The conductor is wound by 12 turns. Calculate the magnetic flux density at a distance of 3 cm from the center of the conductor.

Solution

The value of the magnetic flux density is calculated as,

$$B = \mu H = \frac{\mu_0 \mu_r NI}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1 \times 120 \times 8}{2\pi \times 0.03} = 6.4 \times 10^{-3}\,\mathrm{T} \tag{2.65}$$

Practice Problem 2.6

A long straight wire is wound by 150 turns and carries a current of 10 A. Determine the magnetic field intensity and flux density at a distance 0.6 m from the center of the wire if the conductor is surrounded by air.

2.14 Toroidal Coil

Let us consider the inner and outer diameters of the toroidal coil are a and b. The distance from the center to the mean length is r. The toroidal coil is wound by N number of turns and carries a current of I ampere as shown in Fig. 2.20. The mean length of the coil is,

$$l = 2\pi r \tag{2.66}$$

Apply Ampere’s law to the coil and the equation becomes,

$$Bl = N\mu I \tag{2.67}$$

Substituting equation (2.66) into equation (2.67) yields,

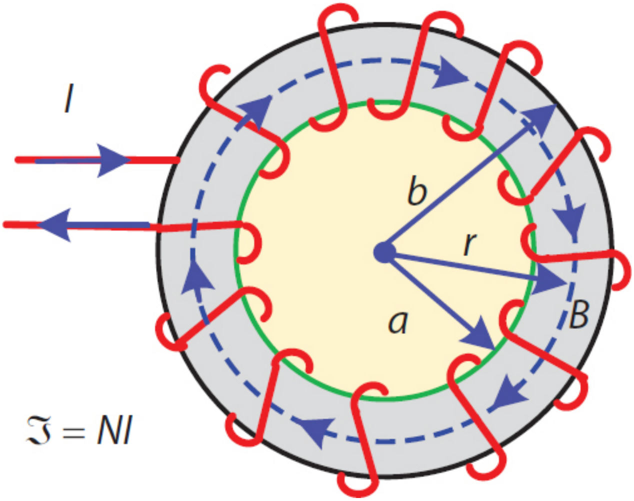


Fig. 2.20 Toroid with N turns.

$$B(2\pi r) = N\mu I \tag{2.68}$$

$$B = \frac{N\mu I}{2\pi r} \tag{2.69}$$

Substituting equation (2.27) into equation (2.69) yields,

$$\mu H = \frac{N\mu I}{2\pi r} \tag{2.70}$$

$$H = \frac{NI}{2\pi r} \tag{2.71}$$

Example 2.7

A wooden toroid is wound by 220 turns and has an inner diameter of 6 cm and an outer diameter of 12 cm as shown in Fig. 2.21. The toroid has a circular cross-sectional area of $\pi 1.5^2$ cm^2 and the current 3 A flows in the coil. Calculate the magnetic field strength, flux density and flux.

Solution

The mean circumference length is calculated as,

$$l = 2\pi r = 2\pi\left(6 + \frac{12-6}{2}\right) = 0.57\,\mathrm{m} \tag{2.72}$$

The value of the magnetic field intensity is determined as,

$$H = \frac{NI}{l} = \frac{220 \times 3}{0.57} = 1157.89\,\mathrm{At/m} \tag{2.73}$$

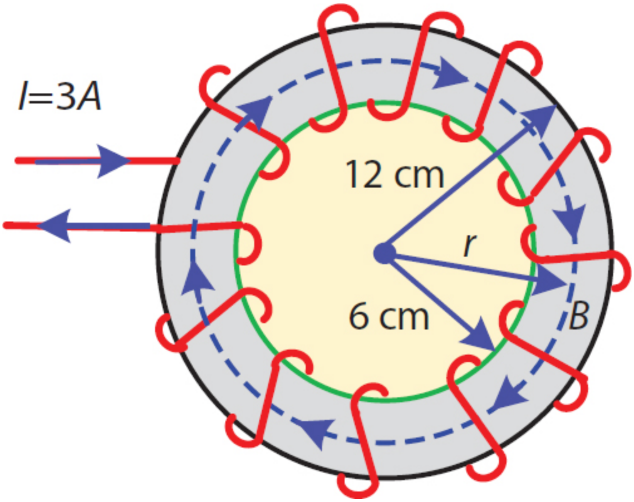


Fig. 2.21 Toroid coil for example 2.7.

The value of the flux density is calculated as,

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 1157.89 = 1.45\,\mathrm{mWb/m^2} \tag{2.74}$$

The value of the flux is calculated as,

$$\phi = BA = 1.45\times10^{-3}\times\pi\times1.5^2\times10^{-4} = 1.02\,\mu\text{Wb} \tag{2.75}$$

Practice Problem 2.7

A toroid is wound by 325 turns and has an inner diameter of 8 cm and an outer diameter of 14 cm as shown in Fig. 2.22. The toroid has a circular cross-sectional area and 1.5 A flows in the coil. Find the magnetic field strength, flux density and flux.

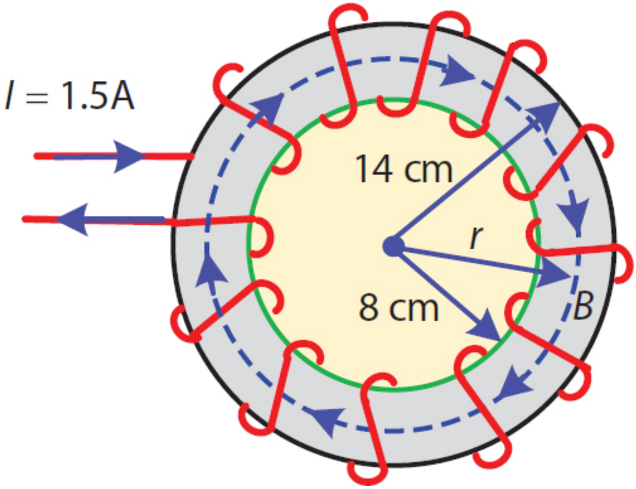


Fig. 2.22 Toroid coil for practice problem 2.7.

2.15 Series Magnetic Circuit

A magnetic circuit with different materials but carrying the same magnetic flux is called a series magnetic circuit. Fig. 2.23 shows a magnetic circuit that consists of three materials. The mean lengths, cross-sectional areas and relative permeabilities are l_1, l_2, l_3, A_1, A_2, A_3, μ_{r1}, μ_{r2} and μ_{r1}, respectively. The core of the first material is wound by N turns that carry a current I ampere. The equivalent circuit is shown in Fig. 2.24. Apply the magnetic potential drop around the circuit and the equation is,

$$NI = H_1 l_1 + H_2 l_2 + H_3 l_3 \tag{2.76}$$

Substituting equation (2.27) into equation (2.76) yields,

$$NI = \frac{B_1}{\mu_1} l_1 + \frac{B_2}{\mu_2} l_2 + \frac{B_3}{\mu_3} l_3 \tag{2.77}$$

$$NI = B_1 A_1 \frac{l_1}{\mu_1 A_1} + B_2 A_2 \frac{l_2}{\mu_2 A_2} + B_3 A_3 \frac{l_3}{\mu_3 A_3} \tag{2.78}$$

Substituting equation (2.11) into equation (2.78) yields,

$$\mathfrak{I} = \phi \mathfrak{R}_1 + \phi \mathfrak{R}_2 + \phi \mathfrak{R}_3 = \phi (\mathfrak{R}_1 + \mathfrak{R}_2 + \mathfrak{R}_3) \tag{2.79}$$

$$\mathfrak{I} = \phi \mathfrak{R} \tag{2.80}$$

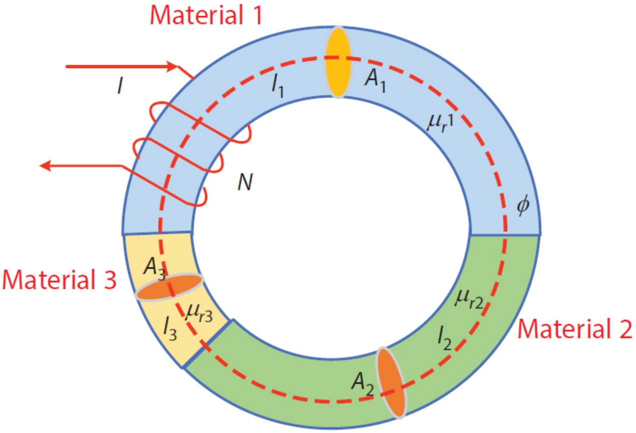

Material 1
I
l_1
A_1
μ_r1
N
ϕ
Material 3
A_3
μ_{r3}
l_3
μ_{r2}
l_2
Material 2
A_2

Fig. 2.23 Magnetic circuit with different materials.

where the total reluctance is written as,

$$\mathfrak{R} = (\mathfrak{R}_1 + \mathfrak{R}_2 + \mathfrak{R}_3) \tag{2.81}$$

The total flux is written as,

$$\phi = B_1 A_1 = B_2 A_2 = B_3 A_3 \tag{2.82}$$

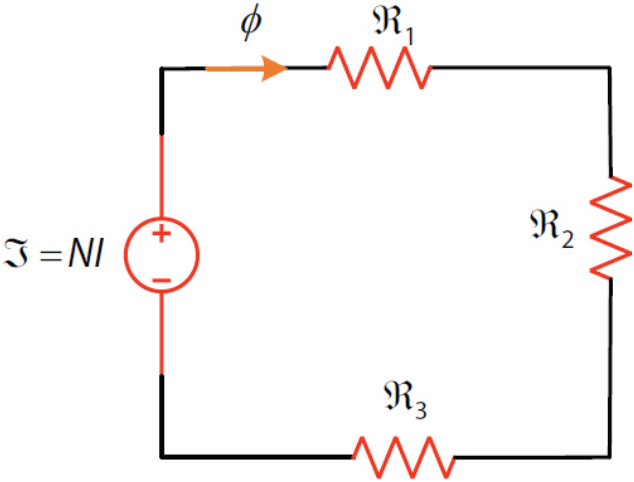


Fig. 2.24 Equivalent series magnetic circuit.

Example 2.8

The mean length of the iron ring is 70 cm and it has an air gap of 5 cm as shown in Fig. 2.25. The iron ring is wound by 200 turns and carries a current of 2.5 A. Find the flux density if the relative permeability of iron is 400.

Solution

The value of the total magnetomotive force is written as,

$$\Im = NI = H_i l_i + H_g l_g \tag{2.83}$$

$$\Im = NI = \frac{B}{\mu_0 \mu_{ri}} l_i + \frac{B}{\mu_0 \mu_{rg}} l_g \tag{2.84}$$

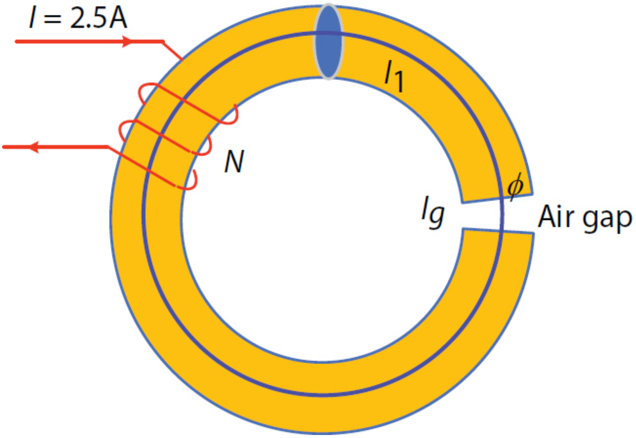


Fig. 2.25 Magnetic circuit for example 2.8.

Substituting the relevant values in equation (2.84) yields,

$$200 \times 2.5 = \frac{B}{4\pi \times 10^{-7}}\left[\frac{0.7}{400} + \frac{0.05}{1}\right] \tag{2.85}$$

$$B = 0.012\,\mathrm{Wb/m^2} \tag{2.86}$$

Example 2.9

The mean length of the first material is 85 cm and the second material is 75 cm as shown in Fig. 2.26. The areas of the first and the second materials are 9 cm^2 and 4 cm^2. The iron ring is wound by 120 turns and carries a current of 3 A. Calculate the reluctances and magnetic flux if $\mu_{r1} = 600$ and $\mu_{r2} = 700$.

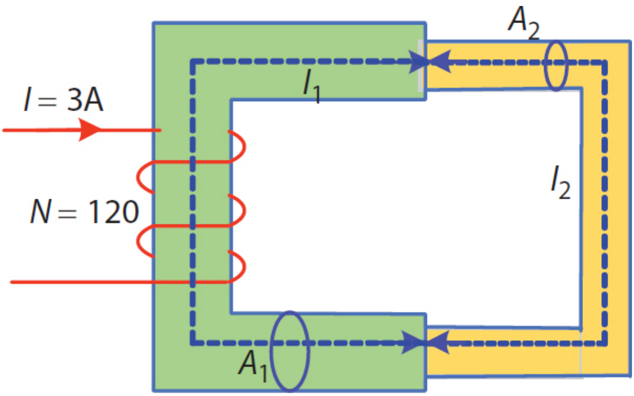


Fig. 2.26 Magnetic circuit for example 2.9.

Solution

The value of the reluctance of the first material is calculated as,

$$\Re_1 = \frac{l_1}{\mu_0 \mu_r A_1} = \frac{0.85}{4\pi \times 10^{-7} \times 600 \times 9 \times 10^{-4}} = 1252608.35\,\text{At/Wb} \tag{2.87}$$

The value of the reluctance of the second material is calculated as,

$$\Re_2 = \frac{l_2}{\mu_0 \mu_r A_2} = \frac{0.75}{4\pi \times 10^{-7} \times 700 \times 4 \times 10^{-4}} = 2131539.42\,\text{At/Wb} \tag{2.88}$$

The value of the magnetic flux is calculated as,

$$\phi = \frac{\Im}{\Re_1 + \Re_1} = \frac{3 \times 120}{1252608.35 + 2131529.42} = 0.11\,\text{mWb} \tag{2.89}$$

Practice Problem 2.8

The mean length of the iron core is 95 cm and it has an air gap of 6 cm as shown in Fig. 2.27. The iron ring is wound by 200 turns and carries a current of 1.5 A. Calculate the flux density if the relative permeability of iron is 540.

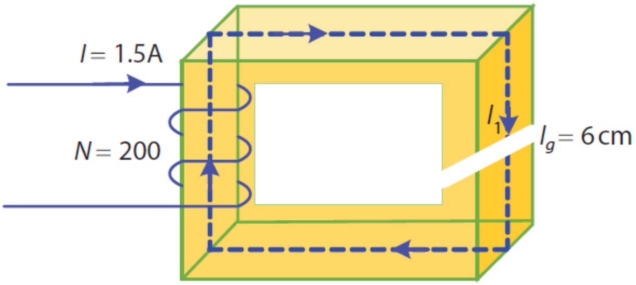


Fig. 2.27 Magnetic circuit for practice problem 2.8.

Practice Problem 2.9

The mean length of the first material is 105 cm and the second material is 95 cm as shown in Fig. 2.28. The areas of the first and the second materials are 16 cm^2 and 9 cm^2. The iron ring is wound by 220 turns and carries a current of 3.5 A. Calculate the reluctances and magnetic flux if $\mu_{r1} = 1400$ and $\mu_{r2} = 1700$.

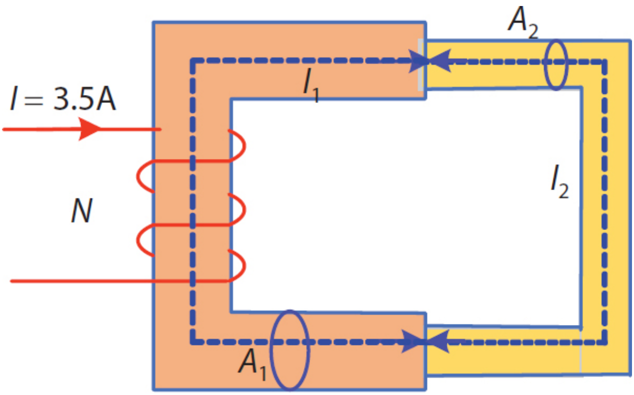


Fig. 2.28 Magnetic circuit for practice problem 2.9.

2.16 Parallel Magnetic Circuit

A magnetic circuit that has more than one path for the flux to divide into or when the main flux is divided into two or more branches in the magnetic path is called a parallel magnetic circuit. A magnetomotive force is connected in the left limb of the circuit as shown in Fig. 2.29. This MMF creates the flux ϕ and it is divided into ϕ_1 and ϕ_2 to the middle and right limbs whose lengths are l_1 and l_2, respectively.

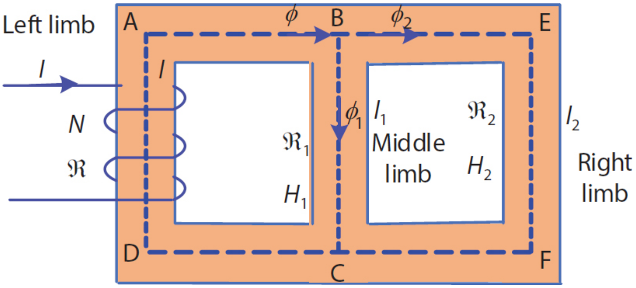

Left limb
A
ϕ
B
ϕ_2
E
I
I
N
$\Re$
$\Re_1$
H_1
ϕ_1
l_1
Middle limb
$\Re_2$
H_2
l_2
Right limb
D
C
F

Fig. 2.29 Parallel magnetic circuit.

Consider the magnetic field intensity of the left, middle and right limbs are H, H_1 and H_2, respectively. The equivalent magnetic circuit is shown in Fig. 2.30.

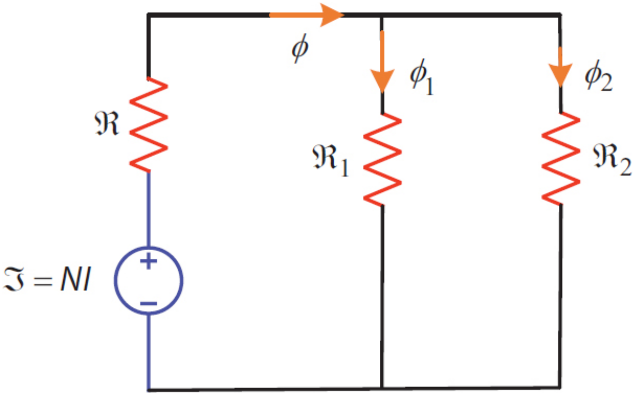

ϕ
ϕ_1
ϕ_2
$\Re$
$\Re_1$
$\Re_2$
$\Im = NI$
+
−

Fig. 2.30 Equivalent circuit of parallel magnetic.

From Fig. 2.29, the magnetic potential drop around the equivalent circuit is written as,

$$\Im - \Re = \Im_1 = \Im_2 \tag{2.90}$$

Substituting equation (2.17) into equation (2.90) yields,

$$\Im - Hl = H_1 l_1 = H_2 l_2 \tag{2.91}$$

Based on the KCL in an electrical circuit, the total flux is written as,

$$\phi = \phi_1 + \phi_2 \tag{2.92}$$

Substituting equation (2.38) into equation (2.90) yields,

$$\mathfrak{I} - \mathfrak{R}\phi = \mathfrak{R}_1\phi_1 = \mathfrak{R}_2\phi_2 \tag{2.93}$$

$$\mathfrak{I} = \mathfrak{R}\phi + \mathfrak{R}_1\phi_1 = \mathfrak{R}\phi + \mathfrak{R}_2\phi_2 \tag{2.94}$$

The total reluctance of R_1 and R_2 is written as,

$$\mathfrak{R} = \frac{\mathfrak{R}_1\mathfrak{R}_2}{\mathfrak{R}_1 + \mathfrak{R}_2} \tag{2.95}$$

Example 2.10

Fig. 2.31 shows a magnetic circuit with a cross-sectional area is 9 cm^2. The flux in the right limb is 4 Wb and the left limb is wound by 1500 turns. Find the value of the current if the relative permeability is 1000.

Solution

The magnetic potential drop in both the middle and right limbs are the same and it is written as,

$$\phi_1\mathfrak{R}_1 = \phi_2\mathfrak{R}_2 \tag{2.96}$$

$$\phi_1\frac{l_1}{\mu A} = \phi_2\frac{l_2}{\mu A} \tag{2.97}$$

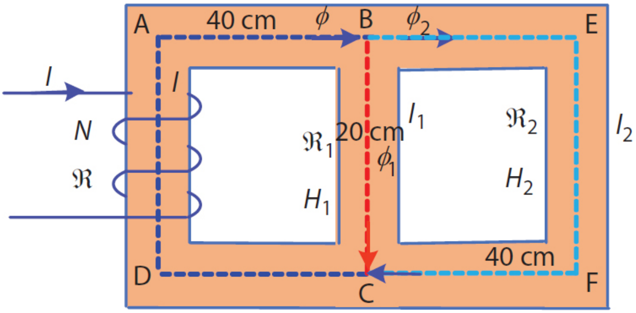


Fig. 2.31 Magnetic circuit for example 2.10.

$$\phi_1 \frac{20}{\mu A} = 4\frac{40}{\mu A} \tag{2.98}$$

$$\phi_1 = 8\,\text{Wb} \tag{2.99}$$

The value of the total flux is calculated as,

$$\phi = \phi_1 + \phi_2 = 8 + 4 = 12\,\text{Wb} \tag{2.100}$$

The value of the flux density in the left limb is calculated as,

$$B_1 = \frac{\phi}{A} = \frac{12}{9\times10^{-4}} = 13300\,\text{Wb/m}^2 \tag{2.101}$$

The value of the magnetic field intensity is calculated as,

$$H_1 = \frac{B}{\mu_0\mu_r} = \frac{13300}{4\pi\times10^{-7}\times1000} = 10583803.72\,\text{At/m} \tag{2.102}$$

The value of the magnetomotive force is calculated as,

$$\Im_1 = H_1 l_1 = 10583803.72\times0.40 = 4233521.49\,\text{At} \tag{2.103}$$

The value of the magnetic flux density in the right limb is calculated as,

$$B_2 = \frac{\phi_2}{A} = \frac{4}{9\times10^{-4}} = 4444.44\,\text{Wb/m}^2 \tag{2.104}$$

The value of the magnetic field intensity is determined as,

$$H_2 = \frac{B_2}{\mu_0\mu_r} = \frac{4444.44}{4\pi\times10^{-7}\times1000} = 3536772.98\,\text{At/m} \tag{2.105}$$

The value of the magnetomotive force is calculated as,

$$\Im_2 = H_2 l_2 = 3536772.98\times0.4 = 1414709.19\,\text{At} \tag{2.106}$$

The value of the total magnetomotive force is calculated as,

$$\Im_t = NI = 4233521.49 + 1414709.19 = 5648230.68\,\text{At} \tag{2.107}$$

The value of the current is determined as,

$$I = \frac{5648230.68}{1500} = 3765.49\,\mathrm{A} \tag{2.108}$$

Practice Problem 2.10

Fig. 2.32 shows a magnetic circuit with a cross-sectional area is 16 m^2. The flux in the right limb is 3 Wb and the left limb is wound by N turns. Find the value of the turns if the 15 A current flows in the coil. Consider relative permeability is 800.

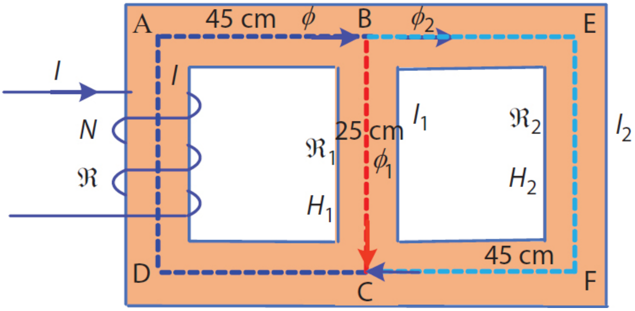


Fig. 2.32 Magnetic circuit for practice problem 2.10.

2.17 Air Gap

The air gap is very important in electrical machines. The air gap is considered between the stator and rotor. The gap between two parts of the magnetic bar is called the air gap and this air gap is a non-magnetic part. This non-magnetic part is connected in series with the magnetic circuit so that the substantial magnetic field or flux can flow in the circuit. In addition to that, the air gap is filled with a non-magnetic material such as wood, plastic, gas, water and also air. The reluctance of an air gap increases. Therefore, at the edge, the flux that comes out from the magnetic part bends outward, reducing the magnetic flux density. This phenomenon is known as the fringing effect as shown in Fig. 2.33. However, in the middle of the air gap, the flux passes directly. The fringing effect increases as the air gap increases or vice versa. For a small air gap, the fringing effect is neglected.

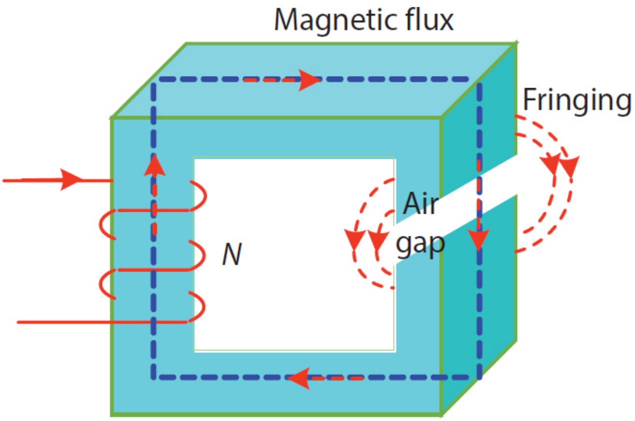


Fig. 2.33 Magnetic circuit with fringing.

2.18 Analysis of Magnetic Circuit with Air Gap

A magnetic circuit with an air gap is shown in Fig. 2.34. Consider the mean length, permeability, and cross-sectional area of the core are l_c, μ and A_c, respectively. However, these parameters for the air gap are l_{ag}, μ and A_{ag}, respectively. The air gap part is connected in series with the core part.

Therefore, the same magnetic flux will pass through it. The equivalent circuit can be considered a series magnetic circuit as shown in Fig. 2.35. The magnetic flux density for the core is written as,

$$B_c = \frac{\phi}{A_c} \tag{2.109}$$

The magnetic flux density for the air gap is written as,

$$B_{ag} = \frac{\phi}{A_{ag}} \tag{2.110}$$

In the series magnetic circuit, the total magnetomotive force is written as,

$$\Im_t = \Im_c + \Im_{ag} \tag{2.111}$$

Substituting equation (2.17) into equation (2.111) yields,

$$\Im_t = H_c l_c + H_{ag} l_{ag} \tag{2.112}$$

Substituting equation (2.27) into equation (2.112) yields,

$$\Im_t = \frac{B_c}{\mu} l_c + \frac{B_{ag}}{\mu_0} l_{ag} \tag{2.113}$$

Substituting equation (2.11) into equation (2.113) yields,

$$\Im_t = \frac{\phi}{\mu A_c} l_c + \frac{\phi}{\mu_0 A_{ag}} l_{ag} \tag{2.114}$$

$$\Im_t = \phi\left(\frac{l_c}{\mu A_c} + \frac{l_{ag}}{\mu_0 A_{ag}}\right) \tag{2.115}$$

Substituting equation (2.42) into equation (2.115) yields,

$$\Im_t = \phi(\Re_c + \Re_{ag}) \tag{2.116}$$

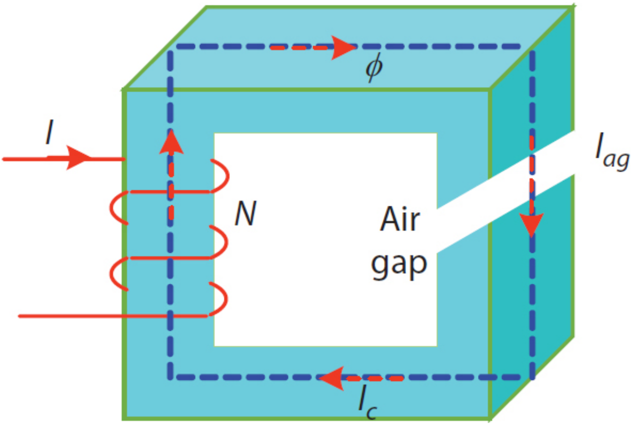


Fig. 2.34 Magnetic circuit with an air gap.

$$\Im_t = \phi \Re_t \tag{2.117}$$

where the total reluctance is expressed as,

$$\Re_t = \Re_c + \Re_{ag} \tag{2.118}$$

where the reluctance is written as,

$$\Re_c = \frac{l_c}{\mu A_c} \tag{2.119}$$

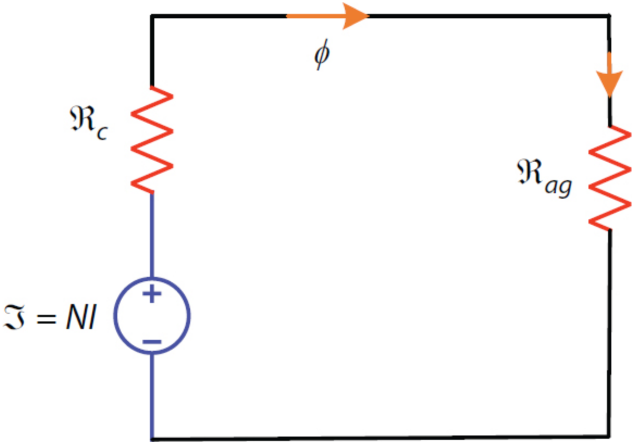


Fig. 2.35 Equivalent circuit of Fig. 2.33.

$$\Re_{ag} = \frac{l_{ag}}{\mu_0 A_{ag}} \tag{2.120}$$

Example 2.11

The mean length of an iron ring is 55 cm and the cross-sectional area is 4 cm^2. The air gap of the core is 0.56 mm as shown in Fig. 2.36. The iron ring core is wound by 600 turns and carries a current of 3 A that produces 1.5 mWb flux. Calculate the reluctance and the relative permeability of the iron ring.

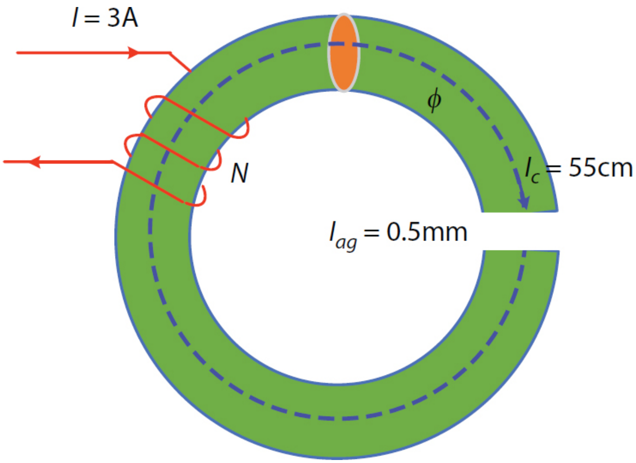


Fig. 2.36 Circuit for example 2.11.

Solution

The magnetic flux density is calculated as,

$$B = \frac{\phi}{A} = \frac{1.5\times10^{-3}}{4\times10^{-4}} = 3.75\,\text{Wb/m}^2 \tag{2.121}$$

The value of total magnetomotive force is determined as,

$$\Im = NI = 600\times3 = 1800\,\text{At} \tag{2.122}$$

The value of magnetizing force for the air gap is calculated as,

$$H_{ag} = \frac{B}{\mu_0\mu_r} = \frac{3.75}{4\pi\times10^{-7}\times1} = 2984155.18\,\text{At/m} \tag{2.123}$$

The value of magnetomotive force for the air gap is calculated as,

$$\Im_{ag} = H_{ag}l_{ag} = 2984155.18\times0.56\times10^{-3} = 1671.13\,\text{At} \tag{2.124}$$

The magnetomotive force for an iron ring is calculated as,

$$\Im_i = 1800 - 1671.13 = 128.87\,\text{At} \tag{2.125}$$

The value of magnetic field intensity for an iron ring is determined as,

$$H_i = \frac{\Im_i}{l_c} = \frac{128.87}{0.55} = 234.31\,\text{At/m} \tag{2.126}$$

The value of the relative permeability for the iron ring is calculated as,

$$\mu_r = \frac{B}{\mu_0 H_i} = \frac{3.75}{4\pi \times 10^{-7} \times 234.31} = 12735.93 \tag{2.127}$$

Example 2.12

The cross-sectional area of the iron core is 4 cm^2 and it is wound by 600 turns as shown in Fig. 2.37. The core is made of iron and the relative permeability of the core is 4000. The total flux of the circuit is 3.5 mWb and the relative of the iron core is 4000. Calculate the total reluctance.

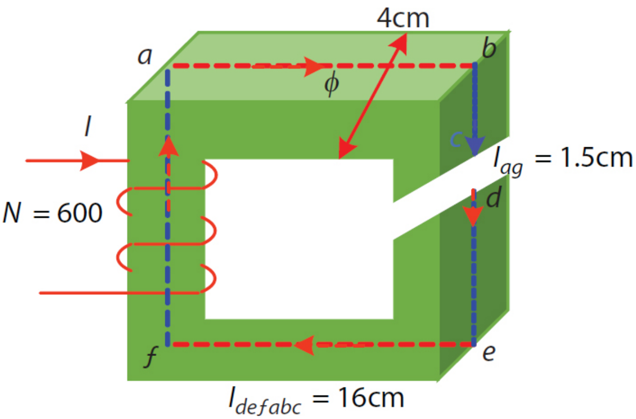


Fig. 2.37 Circuit for example 2.12.

Solution

For air gap, the reluctance is calculated as,

$$\Re_{ag} = \frac{l}{\mu A} = \frac{0.015}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 2.98 \times 10^{7}\ \text{At/Wb} \tag{2.128}$$

For the iron, the mean length is calculated as,

$$l_i = 16 - 1.5 = 14.5\,\text{cm} \tag{2.129}$$

The permeability is calculated as,

$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 4000 = 0.0050 \tag{2.130}$$

The value of the reluctance of the iron core is calculated as,

$$\Re_i = \frac{l}{\mu A} = \frac{0.145}{0.0050 \times 4 \times 10^{-4}} = 72500\,\text{At/Wb} \tag{2.131}$$

The value of the total reluctance is determined as,

$$\Re_t = \Re_{ag} + \Re_i = 2.98 \times 10^7 + 72500 = 29872500\,\text{At/Wb} \tag{2.132}$$

The value of the current can be determined by using the following formula,

$$\phi = \frac{\Im}{\Re_t} = \frac{NI}{\Re_t} \tag{2.133}$$

$$3.5 \times 10^{-3} = \frac{600 \times I}{29872500} \tag{2.134}$$

$$I = 174.26\,\text{A} \tag{2.135}$$

Practice Problem 2.11

The mean length of an iron ring is 75 cm and the cross-sectional area is 0.9 m^2. The air gap of the core is 2 cm as shown in Fig. 2.38. The iron ring core is wound by 1600 turns and carries a current of 3.5 A that produces 2.5 mWb flux. Calculate the reluctance and the relative permeability of the iron ring.

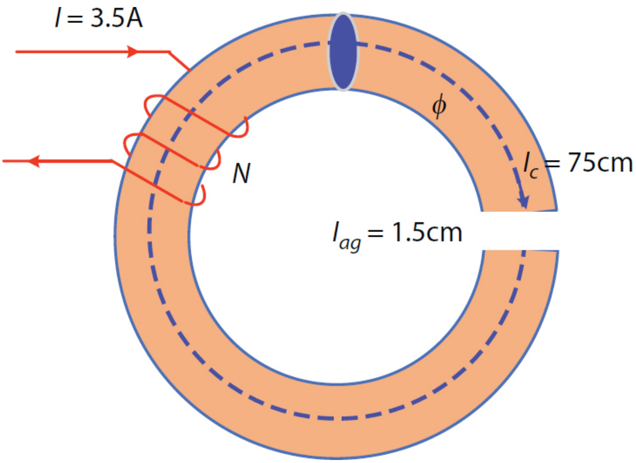


Fig. 2.38 Circuit for practice problem 2.11.

Practice Problem 2.12

The cross-sectional area of the iron core is 9 cm^2 and it is wound by 1600 turns as shown in similar Fig. 2.39. The core is made of iron and the relative permeability of the core is 4000. The total flux of the circuit is 2.5 mWb and the relative of the iron core is 4000. Calculate the total reluctance and current.

2.19 Electromagnetic Force on a Conductor

A conductor is a substance, system or material that conducts current. The force of the magnetic field is observed around the permanent magnet or moving charge particles in a conductor. In 1820, scientist Hans Christian Oersted was the first to discover that a current-carrying conductor influenced the orientation of a nearby compass needle. In 1825, an English scientist, Michael Faraday, discovered that a current-carrying conductor can move when placed in a magnetic field. He also discovered the electric generator and the electric motor. Therefore, the knowledge of electromagnetic force is important to understand the working principle of the motor, generator and other electrical devices. When a current-carrying conductor is placed in a magnetic field, it experiences a force. This force is known as electromagnetic force or Lorentz force. This force increases with increasing current flowing in the conductor as shown in Fig. 2.40. This force also depends on the orientation of the conductor. From equation (2.3), it is written as,

$$F = q\,vB\sin\phi = q\frac{l}{t}B\sin\phi \tag{2.136}$$

$$F = \frac{q}{t}Bl\sin\phi \tag{2.137}$$

$$F = BlI\sin\phi \tag{2.138}$$

where,

B is the magnetic flux density in Tesla,

l is the length of the conductor in m,

ϕ is the angle between B and I,

I is the current flowing through the conductor.

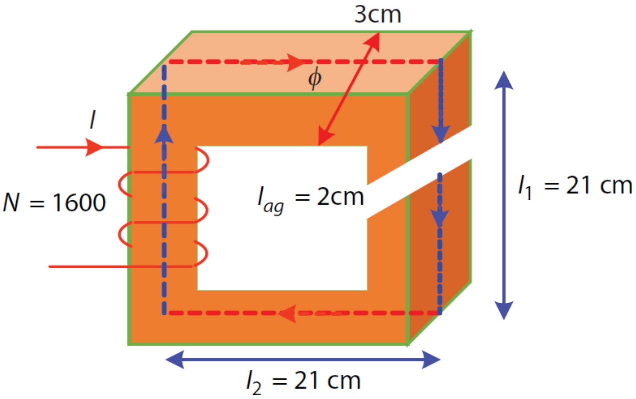


Fig. 2.39 Circuit for practice problem 2.11.

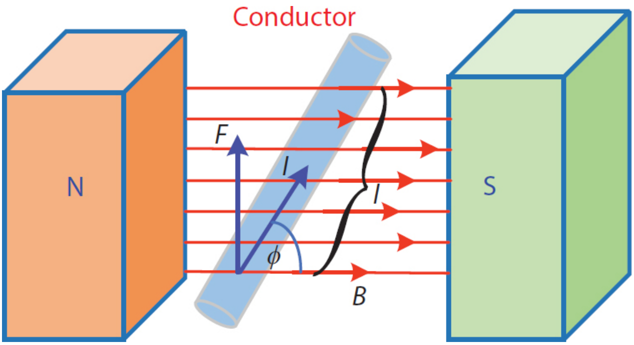


Fig. 2.40 Magnetic field with a current-carrying conductor.

The force will be maximum if the conductor is perpendicular to the magnetic field. The electromagnetic force also depends on the direction of the current direction and the magnetic field direction.

Alternative approach: A horseshoe permanent magnet is shown in Fig. 2.41. Two slider bars S1 and S2 are connected with a source. A current-carrying conductor is attached with two sliders and it is free to move over the sliders as shown in Fig. 2.41. The DC voltage source is connected across the two sliders and the current flows in slider S_1 to the conductor and then returns to the source through slider S_2 which forms a closed loop. This current creates the magnetic field that interacts with the south pole of the magnet which generates the force. This force moves the conductor toward the DC source. The length of the conductor is l and the angle between the magnetic field and the current is θ. The magnetic force will be larger if the current in the conductor increases and vice versa. Therefore, the magnetic force is directly proportional to the current, length of the conductor, external magnetic field and sin of the angle between the external magnetic field B and the current. Mathematically, it is expressed as,

$$(2.139)\, F \infty I$$

$$(2.140)\, F \infty l$$

$$(2.141)\, F \infty B$$

$$(2.142)\, F \infty \sin\theta$$

Combining equations (2.139) to (2.142) yields,

$$(2.143)\, F \infty BlI \sin\theta$$

$$(2.144)\, F = k\, BlI \sin\theta$$

If $k = 1$, then equation (2.144) is written as,

$$(2.145)\, F = BlI \sin\theta$$

In vector form, equation (2.145) is written as,

$$(2.146)\, F = I\, B \times l$$

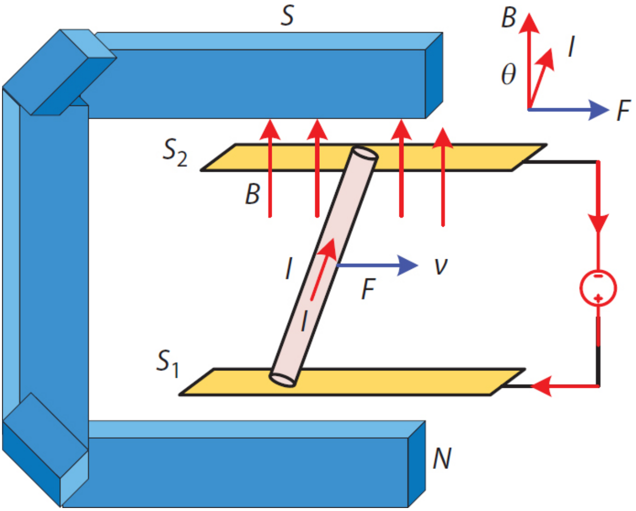
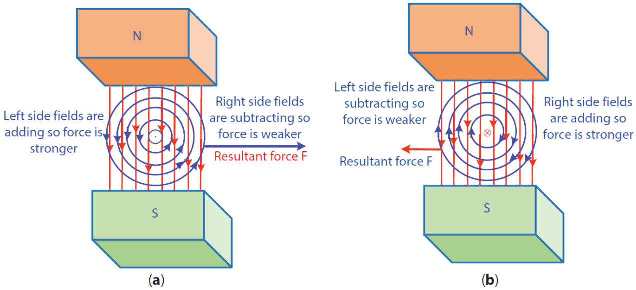


Fig. 2.41 Magnetic a horseshoe magnet with sliders and a current-carrying conductor.

A current is coming out from the plane as shown in Fig. 2.41. The magnetic field lines are coming out from the north pole and entering the south pole. The conductor with a dot indicates the current coming out from the plane. According to Fleming’s right-hand rule, the thumb indicates the direction of the current and the other fingers indicate the direction. The magnetic fields due to the current-carrying conductor move in an anticlockwise direction as shown in Fig. 2.42(a). As a result, the magnetic fields are additive on the left-hand side of the conductor. Therefore, the resultant force is stronger, whereas the magnetic fields are subtractive in the right-hand side of the conductor which in turn is a weaker force. Therefore, the conductor will move from left to right in the magnetic field. Similarly, the conductor will move from right to left in the magnetic field as shown in Fig. 2.42(b).


N
Left side fields are adding so force is stronger
Right side fields are subtracting so force is weaker
Resultant force F
S
N
Left side fields are subtracting so force is weaker
Right side fields are adding so force is stronger
Resultant force F
S

(a)

(b)

Fig. 2.42 Magnetic force direction identification.

Example 2.13

A conductor is placed in a magnetic and carries a current of 3 A. The active length of the conductor is 450 cm and the magnetic flux density is 1.5 T. Determine the force if the conductor is placed in a magnetic field with 45° and 90°.

Solution

The value of the force at the angle of 45° is calculated as,

$$F = BlI\sin\theta = 1.5\times4.5\times3\times\sin45^\circ = 14.32\,\mathrm{N} \tag{2.147}$$

The value of the force at the angle of 90° is calculated as,

$$F = BlI\sin\theta = 1.5\times4.5\times3\times\sin90^\circ = 20.25\,\mathrm{N} \tag{2.148}$$

Practice Problem 2.13

A conductor is placed in a magnetic and carries a current of 3 A. The active length of the conductor is 450 cm and the magnetic flux density is 1.5 T. Calculate the force if the conductor is placed in a magnetic field with 30° and 85°.

2.20 Force Between Two Parallel Conductors

Consider two conductors C_1 and C_2 whose lengths l_1 and l_2 are separated by a distance d. These conductors carry currents I_1 and I_2 as shown in Fig. 2.43. The current I_1 flows in the conductor C_1 which creates the magnetic field B_1 as shown in Fig. 2.43(a). The force exerted by conductor C_2 due to magnetic field B_1 is F_{21}. Based on equation (2.146), the force exerted by conductor C_2 due to the magnetic field in conductor C_1 is expressed as,

$$F_{21} = I_2 B_1 l_2 \tag{2.149}$$

Substituting the value of B_1 in equation (2.149) yields,

$$F_{21} = I_2 \frac{\mu_0 I_1}{2\pi d} l_2 \tag{2.150}$$

The current I_2 flows in the conductor C_2 which creates the magnetic field B_2. The force exerted by conductor C_1 due to magnetic field B_2 is F_{12}.

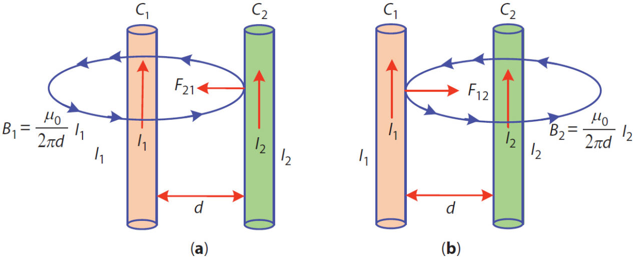


Fig. 2.43 Magnetic force of parallel conductors.

Based on equation (2.146), the force exerted by conductor C_1 due to the magnetic field in conductor C_2 is expressed as,

$$F_{12} = I_2 B_2 l_1 \tag{2.151}$$

Substituting the value of B_1 in equation (2.151) yields,

$$F_{12} = I_1 \frac{\mu_0 I_2}{2\pi d} l_1 \tag{2.152}$$

In general, the force is expressed as,

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} l \tag{2.153}$$

Example 2.14

Two conductors A and B are placed 4 m apart in parallel. Conductor A carries a current of 10 A toward the south and the conductor carries a current of 15 A toward the south. Calculate the force if the conductors' length is 6 m.

Solution

The value of the force is calculated as,

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} l = \frac{4\pi \times 10^{-7} \times 10 \times 15}{2\pi \times 4} \times 6 = 3 \times 10^{-5}\,\mathrm{N} \tag{2.154}$$

Practice Problem 2.14

Two conductors A and B are placed 5 m apart in parallel. Conductor A carries a current of 15 A toward the south and the conductor carries a current of 20 A toward the north. Calculate the force exerted by conductor B due to conductor A if the conductor's length is 5 m.

2.21 Faraday's Laws

In 1831, English scientist Michael Faraday discovered two laws related to the voltage induced in a magnetic field. Based on his name, it is known as Faraday's laws of electromagnetic induction. The statement of the first law is "Whenever a current-carrying conductor is placed in a magnetic field voltage is induced". Faraday's second law is related to the induced voltage. The second law is "The magnitude of the induced voltage equals the rate of change of flux". Mathematically, for N number of turns of the coil, it is written as,

$$e = -N\frac{d\phi}{dt} \quad (2.155)$$

where,

N is the number of turns,

ϕ is the flux in Wb,

e is the induced voltage in V.

Example 2.15

The number of turns of the coil is 150 and the magnetic flux changes from 15 mWb to 35 mWb in 1.2 s. Calculate the induced voltage.

Solution

The value of the induced voltage is calculated as,

$$e = -N\frac{d\phi}{dt} = 150 \times \frac{35-15}{1.2} \times 10^{-3} = 2.5\,\mathrm{V} \quad (2.156)$$

Practice Problem 2.15

The magnetic flux changes from 10 mWb to 25 mWb in 1.1 s. Calculate the number of turns if the induced voltage is 5 V.

2.22 Magnetic Materials and B-H Curve

All materials have different atomic structures. When these materials are placed in a magnetic field they react differently. In atoms, electrons exist in a pair and rotate in different directions that cancel each other out as shown in Fig. 2.44. As a result, no net magnetic field exists. However, some materials have unpaired electrons that generate a high magnetic field. This high magnetic field has a higher reaction to the external magnetic field. Magnetic materials are used in different types of electrical machines. Based on the applications, magnetic materials are classified as diamagnetic, ferromagnetic, and paramagnetic materials.

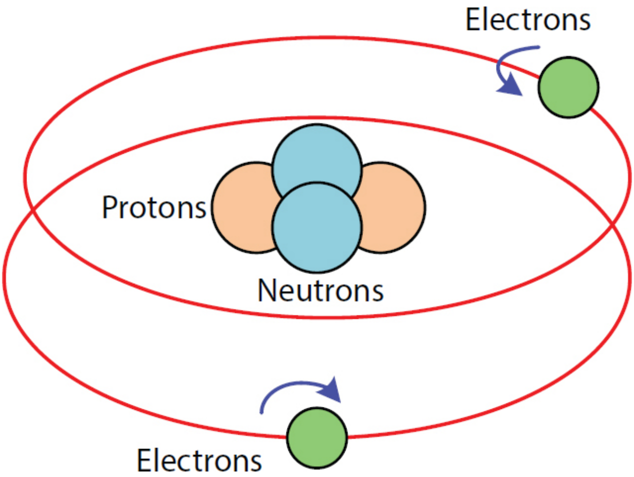


Fig. 2.44 Materials atomic path.

In a diamagnetic material, electrons are paired and they do not generate their magnetic field which means net spin is zero as shown in Fig. 2.45. The magnetic domains of this material are aligned or placed in a such a way that they repel the external magnetic field, which means repelling any poles of the magnet. It has strong repelling behaviour of the magnetic poles depending on the material. Diamagnetic materials have a weak and negative susceptibility or sensitivity to the magnetic field. It is repelled slightly by a magnetic field and does not retain this property after the external field has been removed. In brief, the materials that have less permeability than the free space are called diamagnetic materials. These types of materials are copper, gold and silver.

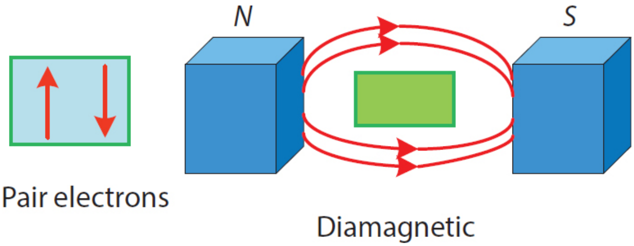


Fig. 2.45 Diamagnetic material.

Paramagnetic materials have a small and positive susceptibility to the magnetic field. Electrons are placed unpair in an orbital and have a net spin as shown in Fig. 2.46. The magnetic fields of these materials align with the external magnetic fields. It is attracted slightly by a magnetic field and does not retain this property after the external field is removed. In brief, the materials that have permeability slightly greater than the free space are called paramagnetic materials. These types of materials are magnesium, molybdenum, lithium, etc.

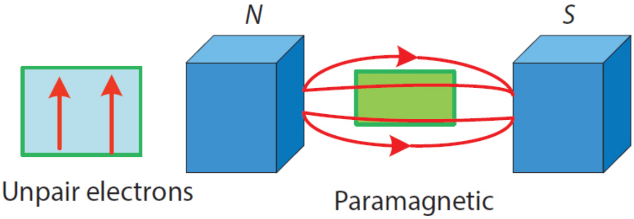


Fig. 2.46 Paramagnetic material.

Ferromagnetic materials are strongly attracted by both magnetic poles. Ferromagnetic materials align their magnetic fields in the direction of the external magnetic fields and create the region as shown in Fig. 2.47. These materials have high and positive susceptibility to a magnetic field. They have a strong attraction to the magnetic field and retain this property after the external field has been removed. In short, the materials having permeability a thousand times greater than the free space are called ferromagnetic materials. These types of materials are iron, nickel, steel, etc.

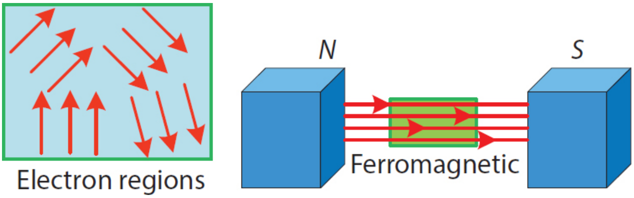


Fig. 2.47 Ferromagnetic material.

B-H curve: The value of the permeability for air and other non-magnetic materials is usually constant with the variation of magnetic field intensity. However, the permeability of ferromagnetic materials varies with magnetic field intensity. The magnetic field intensity also varies directly with the current. The curve between the magnetic flux density and magnetic field intensity is known as the B-H curve and it is very important to analyze the magnetic materials. The relationship between B and H is shown in Fig. 2.48.

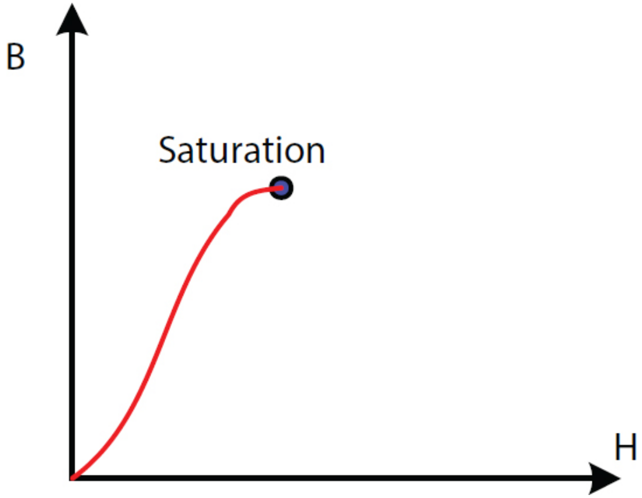


Fig. 2.48 BH curve.

2.23 Inductance and Mutual Inductance

The value of pure inductance is important to achieve performance in electrical machines. The inductance is defined as the property of a coil that opposes any change of current flowing through it. When a coil is connected across the source, current will flow through it. As a result, the flux will be associated with this current. Therefore, the inductance is defined as the flux linkage (ψ) per unit current. Mathematically, it can be expressed as,

$$L = \frac{\psi}{i} \tag{2.157}$$

The SI unit of inductance is Henry (H) and the symbols of air core, ferrite core, iron core and variable core inductors are shown in Fig. 2.49.

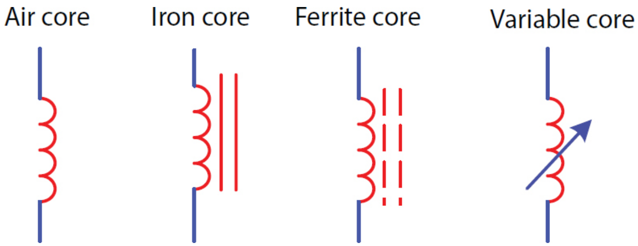


Fig. 2.49 Inductor symbols.

Consider a magnetic circuit with N number of turns as shown in Fig. 2.50. The length of the core is l and the cross-sectional area of the core is A. When the voltage source is connected to the coil, the current i flows in the coil which creates the magnetic flux ϕ. Mathematically, it is expressed as,

$$\phi \propto i \tag{2.158}$$

$$\phi = L i \tag{2.159}$$

where L is the inductance and it is expressed as,

$$L = \frac{\phi}{i} \tag{2.160}$$

For N turns of the coil, it is expressed as,

$$L = \frac{N\phi}{i} \tag{2.161}$$

According to Faraday’s laws of electromagnetic induction, the voltage induced across the coil for a single turn of the coil is expressed as,

$$e = N\frac{d\phi}{dt} \tag{2.162}$$

Substituting equation (2.160) into equation (2.162) and for a single turn of coil yields,

$$e = \frac{d(Li)}{dt} = L\frac{di}{dt} \quad (2.163)$$

Equation (2.162) can be re-arranged as,

$$e = N\frac{d\phi}{di}\frac{di}{dt} \quad (2.164)$$

$$e = L\frac{di}{dt} \quad (2.165)$$

where the expression of inductance is also written as,

$$L = N\frac{d\phi}{di} \quad (2.166)$$

However, the flux is defined as,

$$\phi = \frac{\Im}{\Re} \quad (2.167)$$

Substituting equation (2.42) into equation (2.167) yields,

$$\phi = \frac{Ni}{\dfrac{l}{\mu A}} = \frac{N\mu Ai}{l} \quad (2.168)$$

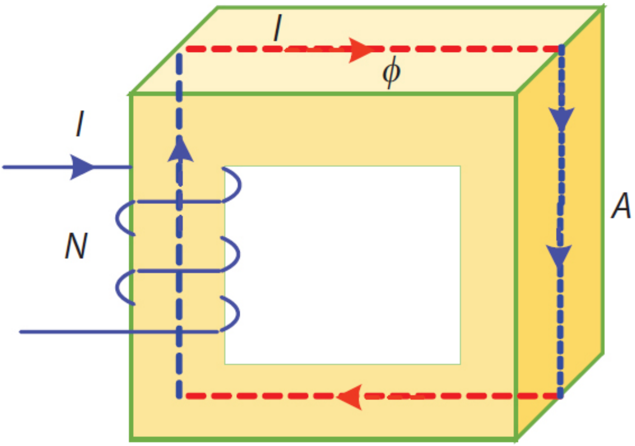


Fig. 2.50 Simple magnetic circuit.

Differentiating equation (2.168) with respect to *i* yields,

$$\frac{d\phi}{di} = \frac{N\mu A}{l} \tag{2.169}$$

Substituting equation (2.169) into equation (2.166) yields,

$$L = \frac{N^2\mu A}{l} \tag{2.170}$$

Again, consider that the magnetic circuit contains two coils. The self-inductances of the first and the second coils are represented as L_{11} (L_1) and L_{11} (L_1), respectively. The magnetic flux caused by the current in the first coil links with the second coil as denoted by ϕ_{21}. As a result, voltage is induced in the second coil and this phenomenon is known as mutual inductance. Consider a magnetic circuit as shown in Fig. 2.51, where the first coil is energized and the second coil is left open.

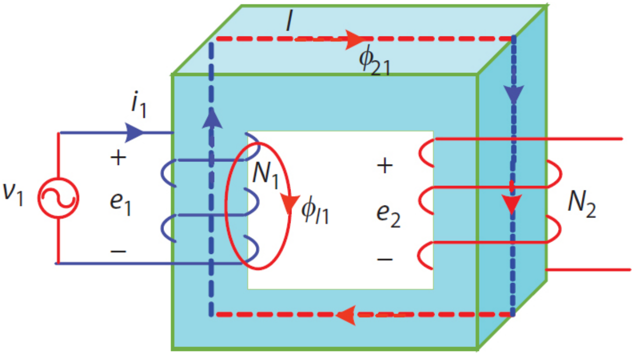


Fig. 2.51 First coil is energized and the second coil is left open.

The total flux in the first coil is written as,

$$\phi_1 = \phi_{l1} + \phi_{21} \tag{2.171}$$

The voltage induced in the first coil is expressed as,

$$e_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} \tag{2.172}$$

$$e_1 = L_1 \frac{di_1}{dt} \tag{2.173}$$

where the self-inductance is expressed as,

$$L_1 = N_1 \frac{d\phi_1}{di_1} \tag{2.174}$$

The voltage induced in the second coil due to the flux ϕ_{21} is written as,

$$e_2 = N_2 \frac{d\phi_{21}}{dt} = N_2 \frac{d\phi_{21}}{di_1}\frac{di_1}{dt} \tag{2.175}$$

$$e_2 = M_{21} \frac{di_1}{dt} \tag{2.176}$$

where the mutual inductance of the second coil due to the first coil is written as,

$$M_{21} = N_2 \frac{d\phi_{21}}{di_1} \tag{2.177}$$

The second magnetic circuit is shown in Fig. 2.52, where the second coil is energized and the first coil is left open. The total flux in the second coil is expressed as,

$$\phi_2 = \phi_{l2} + \phi_{12} \tag{2.178}$$

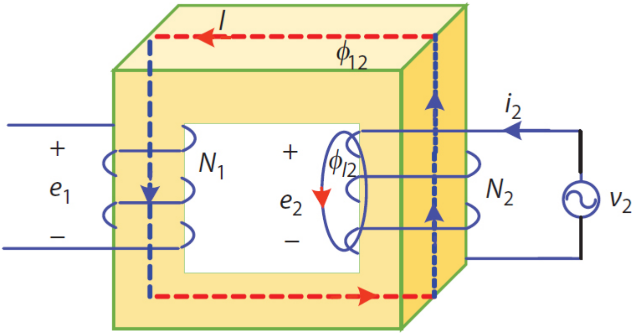


Fig. 2.52 Coil 2 is energized and coil 1 is left open.

The voltage induced in the first coil is written as,

$$e_1 = N_1 \frac{d\phi_{12}}{dt} = N_1 \frac{d\phi_{12}}{di_2} \frac{di_2}{dt} \tag{2.179}$$

$$e_1 = M_{12} \frac{di_2}{dt} \tag{2.180}$$

where the mutual inductance of the first coil due to the second coil is written as,

$$M_{12} = N_1 \frac{d\phi_{12}}{di_2} \tag{2.181}$$

The voltage induced in the second coil is written as,

$$e_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} \tag{2.182}$$

$$e_2 = L_2 \frac{di_2}{dt} \tag{2.183}$$

where the self-inductance of the second coil is expressed as,

$$L_2 = N_2 \frac{d\phi_2}{di_2} \tag{2.184}$$

The term coefficient of coupling is defined as the ratio of mutual flux to the total flux produced by the respective coil. For the first coil, the coefficient of coupling is expressed as,

$$k = \frac{\phi_{21}}{\phi_{l1} + \phi_{21}} = \frac{\phi_{21}}{\phi_1} \tag{2.185}$$

$$\phi_{21} = k\phi_1 \tag{2.186}$$

Similarly, for the second coil, the coefficient of coupling is written as,

$$k = \frac{\phi_{12}}{\phi_{l2} + \phi_{12}} = \frac{\phi_{12}}{\phi_2} \tag{2.187}$$

$$\phi_{12} = k\phi_2 \tag{2.188}$$

Multiplying the equations (2.177) and (2.181) yields,

$$M_{12} M_{21} = N_1 \frac{d\phi_{12}}{di_2} N_2 \frac{d\phi_{21}}{di_1} \tag{2.189}$$

Substituting equations (2.186) and (2.188) yields,

$$M_{12} M_{21} = N_1 k \frac{d\phi_2}{di_2} k N_2 \frac{d\phi_1}{di_1} \tag{2.190}$$

Substituting equations (2.174) and (2.184) yields,

$$M_{12}M_{21} = kL_1 \times kL_2 \tag{2.191}$$

Considering $M_{12} = M_{21} = M$, equation (2.191) is revised as,

$$M \times M = kL_1 \times kL_2 \tag{2.192}$$

$$k = \frac{M}{\sqrt{L_1 L_2}} \tag{2.193}$$

The inductors are said to be well coupled if the value k is higher than 0.5. The inductors are said to be poorly coupled if the value k is less than 0.5. The flux in the first coil reaches completely to the second coil when $k = 1$.

Example 2.16

The length of an air-core coil is 15 cm and the cross-sectional area is 9 cm^2. The coil is wound by 166 turns. Calculate the inductance of the coil.

Solution

The value of the inductance can be calculated as,

$$L = \frac{N^2 \mu A}{l} = \frac{166^2 \times 1 \times 4\pi \times 10^{-7} \times 9 \times 10^{-4}}{0.15} = 207.78\,\mu\text{H} \tag{2.194}$$

Practice Problem 2.16

The mean length of the air core is 125 mm and the circular diameter is 8 mm. The core is wound by 125 turns. Determine the inductance of the core.

2.24 Hysteresis Curve and Losses

In electrical machines, the calculation of hysteresis loss is very important for design purposes. However, hysteresis phenomena occur in ferromagnetic materials, in which a lag occurs between the application and the removal of the field and its subsequent effect. The hysteresis curve is obtained by plotting the magnetic flux density on the x-axis and the magnetic field intensity on the y-axis. The expression of magnetic field intensity ($H = NI/l$, $B = \mu H$) is required to explain the hysteresis curve.

The H increases as the current increases in the coil from the zero position. After a certain time, if the current in the coil increases, the *H* does not increase anymore. That means the material becomes saturated as shown in Fig. 2.53. After that, if the current decreases, the *H* decreases but not in the same path. At $H = 0$, the *B* is not zero which means the magnetic core sustains some flux density and the value of this magnetic flux density (*om*) is known as the retentivity of the material.

Again, if we increase the magnetic field intensity in the reverse direction then the magnetic flux density will be zero at the point on the x-axis. This value (*on*) of the magnetic field is required to demagnetize the material and this amount of magnetic field intensity is known as a coercive force. If we magnetize the core in the same direction, the magnetic flux density will be developed in the opposite direction and it will increase slowly with increasing the value of magnetic field intensity. The value of the magnetic flux density will be reduced if the value of the magnetic field intensity is reduced and met at point *p* on the y-axis. Again, increases the current in the forward direction and the curve will meet with the saturation point. This complete curve is known as the hysteresis curve.

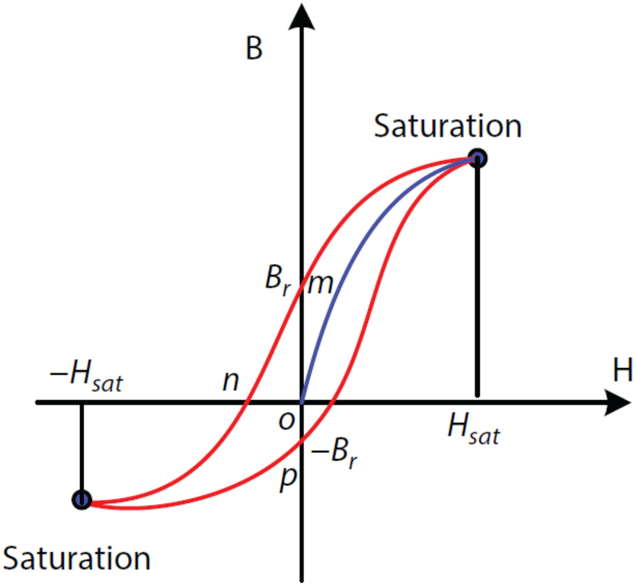


Fig. 2.53 Hysteresis curve.

The magnetic material loses energy during the formation of the hysteresis curve. This loss of energy is known as hysteresis loss. However, hysteresis loss is defined as the amount of electric energy required to overcome the retentivity of the iron during magnetization. The hysteresis loss can be expressed as,

$$P_h = k_h f B_m{}^x Vol \tag{2.195}$$

In general, the hysteresis loss can be expressed as,

$$P_h = k_h V_{vol} B_m{}^n f \tag{2.196}$$

where,

k_h is the hysteresis coefficient,

x is the Steinmetz coefficient and it is equal to 2.6,

f is the frequency, Vol is the volume of the iron,

B_m is the maximum flux density.

P_h is the hysteresis loss.

The flux set up in the core of the magnetic material cut the conductor around it as well as cut the core. As a result, voltage is induced in the core. Due to this voltage, current flows through the body of the core as it forms a closed circuit. This current is known as eddy current and the power loss due to this current is known as the eddy current loss. The power loss (P_{eddy}) is expressed as,

$$P_{eddy} = k_e \delta^2 f^2 B_m{}^2 Vol \tag{2.197}$$

where,

B_m is the maximum flux density,

f is the frequency,

Vol is the volume of the iron,

δ is the thickness of the iron,

Vol is the volume of the iron,

k_e is the constant.

Example 2.17

The hysteresis coefficient of the core material of a power transformer is 100 J/m^3. The volume of the core is 0.004 m^3 and the maximum value of the flux density is 1.15T. Determine the hysteresis loss if the supply frequency is 60Hz.

Solution

The hysteresis loss can be determined as,

$$P_h = k_h V_{cvol} B_m{}^n f = 100 \times 0.004 \times 1.5^{1.6} \times 60 = 45.92\,\mathrm{W} \tag{2.198}$$

Practice Problem 2.17

The hysteresis coefficient of the core material of a distribution transformer is 130 J/m^3. The maximum flux density is 1.04 T and the supply frequency is 60 Hz. Find the volume of the core, if the hysteresis loss is 60 W.

Exercise Problems

2.1 The cross-sectional area of a bar magnet is 8 m^2. Calculate the magnetic flux density if the flux in the magnet is 0.05 Wb.

2.2 The flux in the bar magnet is obtained as 0.04 Wb. and the magnetic flux density is 0.03T. Find the area of the magnet.

2.3 The mean circumference length and area of a wooden ring are 155 mm and 0.04 m^2. The wooden ring is wound by 400 turns and carries a current of 2 A. Calculate the (i) magnetic field intensity, magnetic flux density and total flux.

2.4 The diameter of a mild steel ring is found to be 135 cm and the cross-sectional area is 200 cm^2. Find the reluctance of the ring if the relative permeability of the mild steel is 1320 and magnetomotive force to produce a flux of 250 µWb.

2.5 A coil carries a current of 3 A that produces a magnetomotive force of 45 At. Determine the number of turns of the coil and the reluctance of the circuit if the flux is 0.3 Wb.

2.6 A coil with 60 turns carries a current of 2.5 A and the total flux produced by the coil is 0.06 Wb. Find the magnetomotive force and reluctance of the circuit.

2.7 The magnetomotive force of a coil is found to be 156 At when carries a current of 4 A. Calculate the number of turns and reluctance if the flux of the coil is 0.4 Wb.

2.8 The reluctance of an iron ring is 5000 At/Wb and the flux is found to be 4.5 Wb. If the ring is wound by 400 turns, find the current in the iron ring.

2.9 A iron ring carries a current of 2 A which establishes the flux of 0.5 mWb. Find the number of turns if the total reluctance of the iron ring is 0.56 Mega At/Wb.

2.10 A air-core toroid whose inner and outer diameters are given in Fig. E2.1. Find the reluctance of the toroid by assuming a circular cross-section.

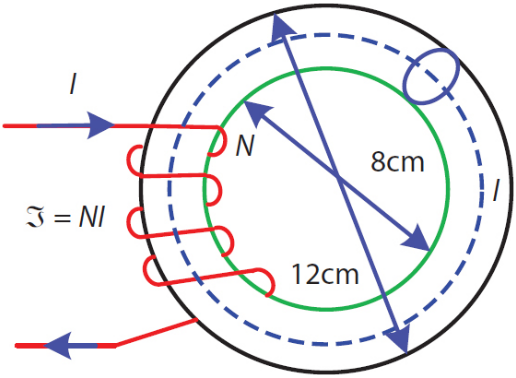


Fig. E2.1 Circuit for problem 2.10.

2.11 The dimensions of a cast iron toroid are shown in Fig. E2.2 The core carries a current of 3 A and it is wound by 400 turns that produce the flux of 0.04 Wb. Calculate the flux density, magnetomotive force and magnetizing intensity. Use a circular cross-section.

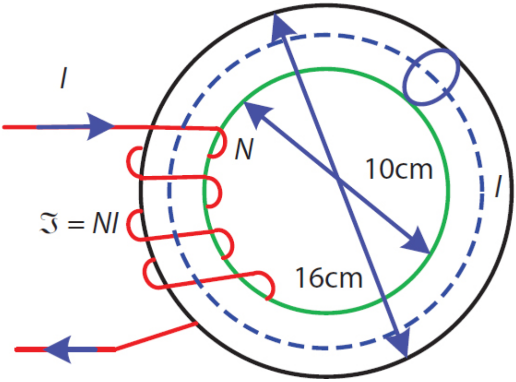


Fig. E2.2 Circuit for problem 2.11.

2.12 A toroidal coil is wound by 100 turns and its mean length is found to be 40 cm. Calculate the current flow through the coil if the magnetic field intensity is 185 At/m.

2.13 The inner and outer diameters of a toroidal core are 6 cm and 8 cm, respectively. The core is wound by 200 turns and carries a current of $i = 4$ sin314t A. Find the total flux of the core. Assume that the relative permeability is 3500.

2.14 The inner and outer diameters of a toroidal core are 6 cm and 9 cm, respectively. The coil is wound by 250 turns and carries a current of $i = 2.5 \sin 314t$ A. This current generates a voltage of $v = 12 \cos 314t$ V. Find the total flux as a function of time and relative permeability of the core.

2.15 Fig. E2.3 represents the rectangular core whose dimensions for length, width and height are 0.42 m, 0.04 m and 0.24 m, respectively. Calculate the field intensity, flux density and flux of the magnetic circuit. Assume the relative permeability of the magnetic material is 800. Also, assume that it has a square cross-section.

2.16 The dimensions of the magnetic circuit are shown in Fig. E2.4. The relative permeability of the magnetic material is 520. Calculate the field intensity and total flux of the magnetic circuit. Assume a square cross-sectional area.

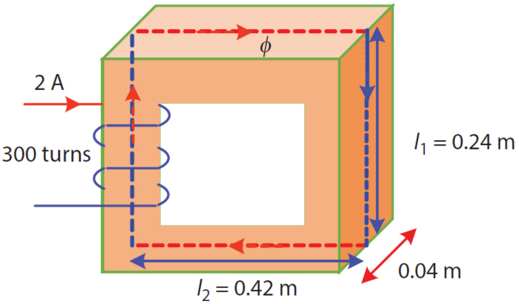


Fig. E2.3 Circuit for problem 2.15.

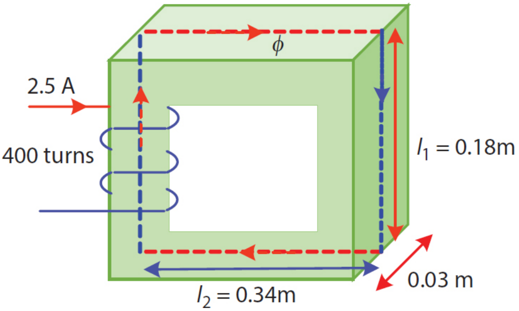


Fig. E2.4 Circuit for problem 2.16.

2.17 The dimensions of the magnetic circuit are shown in Fig. E2.5. The cross-sectional area of the core is 4 cm^2 and the relative permeability is 3500. Find the flux of the limbs if the total flux is 8 mWb.

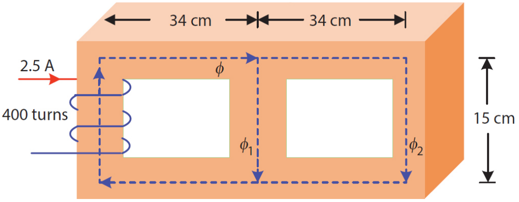

34 cm
34 cm
2.5 A
400 turns
ϕ
ϕ_1
ϕ_2
15 cm

Fig. E2.5 Circuit for problem 2.17.

2.18 The flux in the center limb is 6 mWb and N number of turns is wound on the left limb which is shown in Fig. E2.6. Calculate the number of turns if 2 A current flows in the coil. Assume that the relative permeability of the structure is 2500 and the cross-sectional area of the magnetic structure is 4 cm^2.

2.19 The iron core has a cross-section of 3 cm by 3 cm and its relative permeability is 2150. The core is wound by 400 turns as shown in Fig. E2.7. Determine the total reluctance of the circuit and the current flowing through the coil if the total flux of the circuit is 5 mWb.

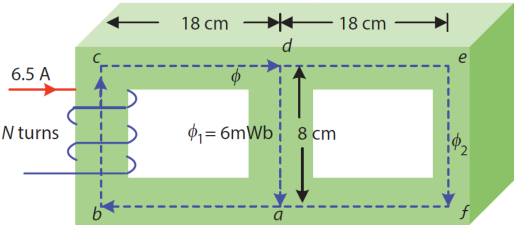

18 cm
18 cm
c
d
e
6.5 A
ϕ
N turns
ϕ_1= 6mWb
8 cm
ϕ_2
b
a
f

Fig. E2.6 Circuit for problem 2.18.

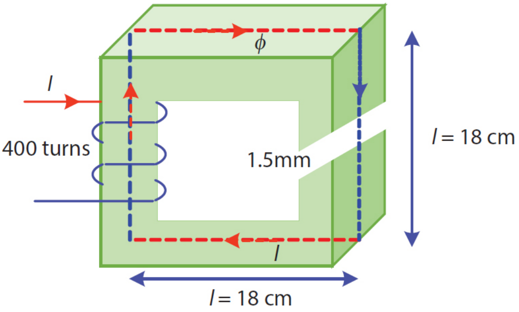

ϕ
I
400 turns
1.5mm
l = 18 cm
l
l = 18 cm

Fig. E2.7 Circuit for problem 2.19.

2.20. A magnetic bar is made of a steel whose cross section is 3 cm by 3 cm. The left of the limb is would with 600 turns and carries a current *I*. Find the current I that produces the flux 3 mWb in the right limb as shown in the Fig. E2.8. Assume the relative permeability is 2100.

2.21 A cylindrical plastic core is 155cm long and the diameter is 0.6cm. The plastic core is wound by 160 turns. Determine the inductance of the core if the permeability is 2290.

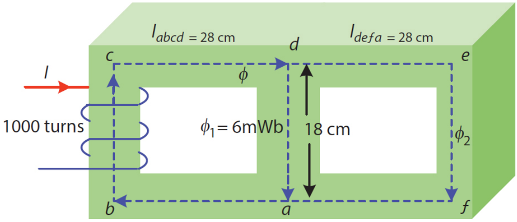


Fig. E2.8 Circuit for problem 2.20.

2.22 The length of the cylindrical plastic core and diameter is 110 cm and 0.35 cm. The plastic core is wound by 250 turns. Calculate the inductance of the core if the permeability is 2290.

2.23 The number of turns of the coil is 300 and the magnetic flux is 1.5 mWb. This flux (1.5 mWb) reverses in 2.5 ms. Calculate the induced voltage.

3
Transformer

3.1 Introduction

Transformers in different ratings are used in the power station, transmission, and distribution networks. The transformer is a static device that transfers electrical voltage from one circuit to another without changing frequency. It converts electrical to the electrical voltage from one magnitude to another magnitude. Usually, the three-phase transformer is used in the power station for the voltage for transmission. According to the current rating, the transformer is classified as single-phase and three-phase. But based on the construction, the transformer is also classified as a core-type and shell-type. The coil that surrounds the core of the transformer is known as a core-type transformer as shown in Fig. 3.1. It does not provide a path for zero sequence current which creates heat in the tank and it is used for a high voltage and high rating power transformer.

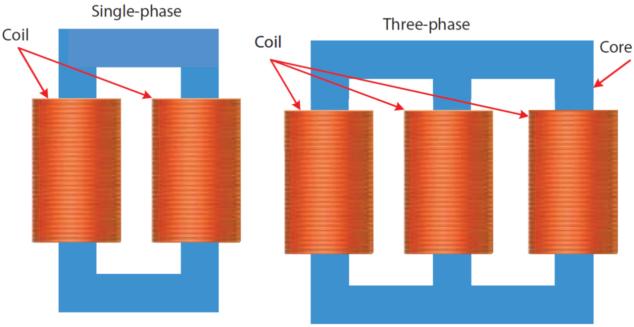


Fig. 3.1 Schematic of a core-type transformer.

The core that surrounds the coil is known as a shell-type transformer. The single-phase and three-phase shell-type transformers are shown in Fig. 3.2. A shell-type transformer provides a path for zero sequence current. Therefore, it does not create any heat in the tank and it is used for low voltage and low power rating transformers. Based on the service, the transformer is classified as a power transformer and a distribution transformer. The power transformer is above 500 kVA and its rating, usually represented by MVA, varies from 10 MVA to 50 MVA. The primary of a power transformer is star-connected and the secondary is delta-connected. A power transformer is used to step the voltage from 11 kV to 120 kV or more. This higher magnitude of voltage is then transmitted to the grid substation by three-phase transmission lines. Then a distribution transformer is used to step down the voltage either from the 11 kV or 33 kV lines. The rating of a distribution transformer is up to 500 kVA. The primary of a distribution transformer is delta-connected and the secondary is star-connected.

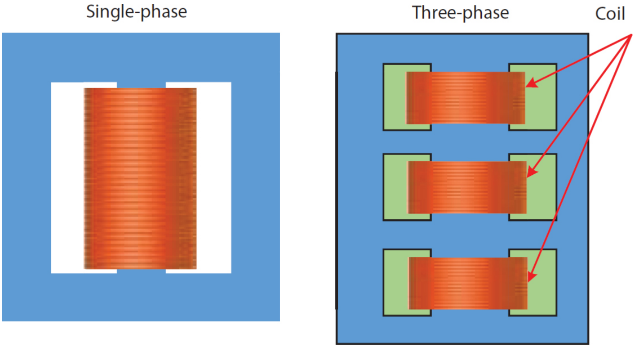


Fig. 3.2 Schematic of a shell-type transformer.

A transformer plays an important role in power transmission and distribution networks as shown in Fig. 3.3. Therefore, students and engineers need to know the details of the transformer.

There are some important points to know about the transformer. The transformer has (i) no moving parts, (ii) no electrical connection between the primary and secondary windings, (iii) windings are magnetically coupled, (iv) rugged and durable in construction, (v) efficiency is very high, i.e., more than 95%, and (vi) frequency is unchanged.

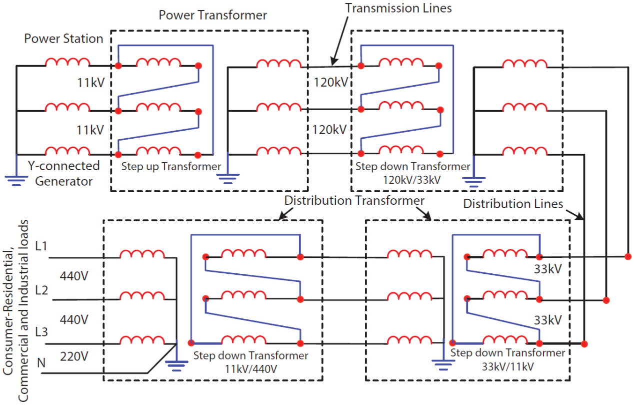

Power Transformer
Transmission Lines
Power Station
11kV
11kV
120kV
120kV
Y-connected
Generator
Step up Transformer
Step down Transformer
120kV/33kV
Distribution Transformer
Distribution Lines
Consumer-Residential,
Commercial and Industrial loads
L1
L2
L3
N
440V
440V
220V
33kV
33kV
Step down Transformer
11kV/440V
Step down Transformer
33kV/11kV

Fig. 3.3 Schematic of power and distribution transformers.

3.2 Working Principle of Transformer

A schematic of a single-phase transformer with two types of windings is shown in Fig. 3.4. These windings are called primary and secondary windings or coils. The winding that is connected to the alternating voltage source is known as the primary winding. The winding that delivers power to the load or is connected to the load is known as the secondary winding. The parameters of the primary and secondary winding are represented by the suffix p or 1 and s or 2, respectively.

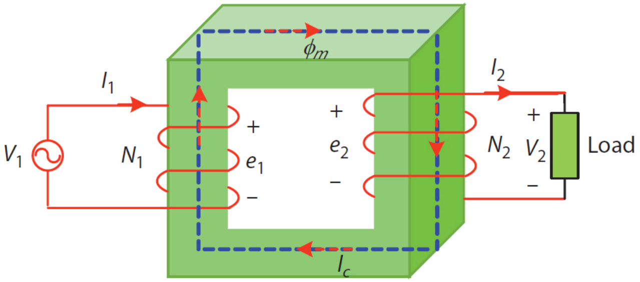

ϕ_m
I_1
V_1
N_1
+
e_1
−
+
e_2
−
I_2
N_2
+
V_2
−
Load
I_c

Fig. 3.4 Schematic of a single-phase transformer.

A sinusoidal current flows in the primary winding when connected to an alternating voltage source. This current establishes a flux ϕ and about 95% of this flux moves from the primary to the secondary through the low reluctance path of the magnetic core.

This flux is linked by both windings, and a small percentage of this flux links to the primary winding.

According to Faraday's laws of electromagnetic induction, a voltage is induced across the primary and secondary windings. These induced voltages are represented by e_1 and e_2, respectively.

Due to this induced voltage, a current I_2 flows through the load if it is connected with the secondary winding. Hence, the primary voltage is transferred to the secondary winding by induction without a change in frequency.

3.3 Transformer Flux

A sinusoidal voltage source is connected in the primary of the transformer as shown in Fig. 3.5. As a closed circuit in the primary side, the current will flow in the primary winding that creates a magnetic flux. About 95% of this flux moves from primary to secondary windings and links both the windings; it is called the mutual flux. The maximum value of this flux is represented by ϕ_m. The flux that links only the primary winding and completes the magnetic path through the surrounding air is known as primary leakage flux and it is represented by ϕl_1. The secondary of the transformer is connected to the load and the current will to the load. In this case, the flux links only the secondary winding and completes the magnetic path through the surrounding air; this is known as the secondary leakage flux and it is represented by ϕl_2. These leakage fluxes also created two different types of losses known as hysteresis loss and eddy current loss. These losses have already been discussed in the magnetic circuit in Chapter 2. The transformer with the mutual and leakage fluxes is shown in Fig. 3.5.

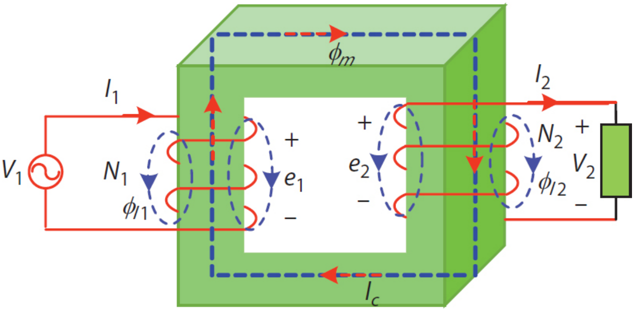


Fig. 3.5 Mutual and leakage fluxes.

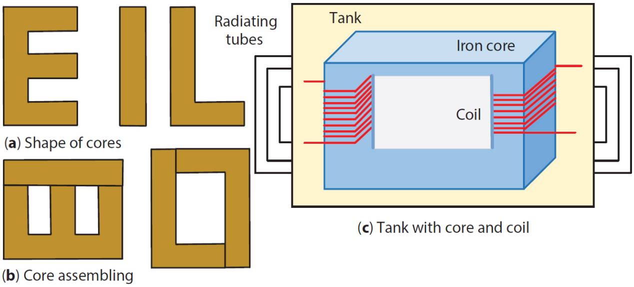
3.4 Construction of a Transformer

The construction of the transformer depends on the voltage and current ratings. The transformer is constructed by the magnetic core, coils, insulating materials, tank and cooling system. Brief descriptions of these parts are given below.

Magnetic core: The core is made of laminated sheet steel with an approximately 0.35 mm thickness for 50 Hz. The cores are constructed by the L-type, I-type and E-type laminated sheet steel as shown in Fig. 3.6(a). If the core is made of non-magnetic material, the transformer is known as an air-core transformer. When the core is made of ferromagnetic material, the transformer is known as an iron-core transformer. According to construction, the transformer is divided into core-type and shell-type. The core-type construction is normally used in the high rating transformer whereas the shell-type is used in the low rating transformer.

Coils: The coils are made of bare copper wire with a suitable number of turns. The coils are insulated from each other by insulating materials. Depending on the number of turns, the coils or windings are categorized either by the high voltage (HV) side or by the low voltage (LV) side.

Insulating materials: Insulation is very important for both primary and secondary windings of a transformer. Enamel is a common insulating material which is used for inter-turn insulation. Other insulating materials are insulating papers, cotton taps, winding leads, high-voltage bushing and low-voltage bushing; these are not shown in Fig. 3.6. Transformer oil Pyranol is normally used inside the transformer for insulation under energized conditions.



(a) Shape of cores

(b) Core assembling

(c) Tank with core and coil

Fig. 3.6 Parts of a transformer.

Tank and cooling system: The tank is used to hold the coils, core and transformer oil as shown in Fig. 3.6(c). Tank is also used for safety and protection against any kind of outside damage. Under loaded conditions, more heat is generated inside the transformer. As a main cooling, transformer oil is used inside the transformer. In addition, a natural air-cooling system is also used to reduce the heat inside the transformer.

3.5 Ideal Transformer

The concept of an ideal transformer is important. A transformer that does not supply any energy to the load is known as an ideal transformer. It means that the secondary winding is not connected to the load. In an ideal transformer, the core is highly permeable and requires less magnetomotive force to set up the flux, leakage flux is zero, primary and secondary windings resistance is negligible, and there are no losses due to resistance, hysteresis and eddy current.

A single-phase transformer without a load is shown in Fig. 3.7(a). When the primary winding is connected to the alternating voltage source, a small primary current (I_1 or I_o) will flow in this coil. This current is equivalent to the magnetizing current as can be seen in the phasor diagram. The magnetizing current (I_m) lags behind the supply voltage, V_1 by 90º and creates the flux, ϕ. Due to this flux, an emf will induce in the primary (E_1) and secondary (E_2). These induced emfs lag the flux ϕm by 90º and lag behind the supply voltage by 180º as shown in Fig. 3.7(b).

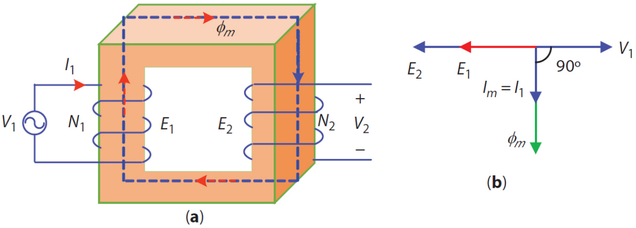

ϕ_m
I_1
V_1
N_1
E_1
E_2
N_2
V_2
+
−
(a)
E_2
E_1
V_1
90º
$I_m = I_1$
ϕ_m
(b)

Fig. 3.7 Ideal transformer with phasor diagram.

3.6 E.M.F. Equation of Transformer

The primary winding draws a current when it is connected to an alternating voltage source. This current creates the sinusoidal flux as shown in Fig. 3.8. This primary sinusoidal current produces a sinusoidal flux φ that can be expressed as,

$$\phi = \phi_m \sin \omega t \tag{3.1}$$

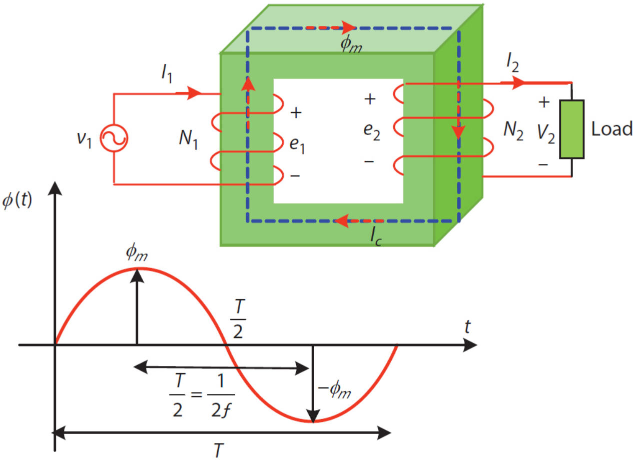

ϕ_m
I_1
I_2
v_1
N_1
+
e_1
−
+
e_2
−
N_2
+
V_2
−
Load
I_c
$\phi(t)$
ϕ_m
$\frac{T}{2}$
t
$\frac{T}{2} = \frac{1}{2f}$
$-\phi_m$
T

Fig. 3.8 Transformer with a sinusoidal flux waveform.

The change in flux for the period $T/2$ is expressed as,

$$d\phi = \phi_m - (-\phi_m) = 2\phi_m \tag{3.2}$$

The induced emf is expressed as,

$$e_1 = N_1 \frac{d\phi}{dt} = N_1 \frac{2\phi_m}{T/2} = N_1 \frac{2\phi_m}{\frac{1}{2f}} = 4\phi_m N_1 f \tag{3.3}$$

The average value of the induced emf is written as,

$$E_{avg} = 4\phi_m N_1 f \tag{3.4}$$

For a sinusoidal voltage, the form factor is defined as the ratio of rms voltage to the average voltage and it is expressed as,

$$\text{Form Factor} = \frac{\text{RMS value of votage } E_1}{\text{Average value of votage } E_{avg}} \tag{3.5}$$

Substituting form factor 1.11 for a sinusoidal voltage and equation (3.4) into equation (3.5) yields,

$$1.11 = \frac{E_1}{4N_1\phi_m f} \tag{3.6}$$

$$E_1 = 4.44N_1\phi_m f \tag{3.7}$$

EMF is also derived in an alternative way. Instantaneous emf induced in the primary winding is,

$$e_1 = -N_1 \frac{d\phi}{dt} \tag{3.8}$$

Similarly, instantaneous emf induced in the secondary winding is,

$$e_2 = -N_2 \frac{d\phi}{dt} \tag{3.9}$$

Substituting equation (3.1) into equation (3.8) yields,

$$e_1 = -N_1 \frac{d}{dt}(\phi_m \sin \omega t) \tag{3.10}$$

$$e_1 = -N_1 \omega \phi_m \cos \omega t \tag{3.11}$$

$$e_1 = N_1 \omega \phi_m \sin(\omega t - 90^\circ) \tag{3.12}$$

The maximum value of e_1 is written as,

$$E_{m1} = N_1 \omega \phi_m \tag{3.13}$$

The rms value of the primary emf is expressed as,

$$E_1 = \frac{E_{m1}}{\sqrt{2}} \tag{3.14}$$

Substituting equation (3.13) into equation (3.14) yields,

$$E_1 = \frac{N_1 2\pi f \phi_m}{\sqrt{2}} \tag{3.15}$$

$$E_1 = 4.44 f \phi_m N_1 \tag{3.16}$$

Similarly, the expression of the secondary emf is,

$$E_2 = 4.44 f \phi_m N_2 \tag{3.17}$$

The primary and secondary voltages can be determined from equations (3.16) and (3.17) if other parameters of a transformer are known.

3.7 Turns Ratio of Transformer

Turns ratio plays an important role in the analysis and drawing of an equivalent circuit of a transformer. The turns ratio is also used to identify whether the transformer is step-up or step-down. According to Faraday’s laws of electromagnetic induction, the induced emf in the primary(e_1) and the secondary (e_2) windings are written as,

$$e_1 = -N_1 \frac{d\phi}{dt} \tag{3.18}$$

$$e_2 = -N_2 \frac{d\phi}{dt} \tag{3.19}$$

Dividing equation (3.18) by equation (3.19) yields,

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \tag{3.20}$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = a \tag{3.21}$$

Similarly, dividing equation (3.16) by (3.17) yields,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \tag{3.22}$$

Here, a is the turn ratio of a transformer. If $N_1 > N_2$ and $E_1 > E_2$, then $a > 1$, the transformer is said to be a step-down transformer, whereas for $N_2 > N_1$ and $E_2 > E_1$, then $a < 1$, the transformer is said to be a step-up transformer. If $N_2 = N_1$ and $E_2 = E_1$, then $a = 1$, the transformer is said to be an isolation transformer. However, the isolation transformer is not used practically. In an ideal transformer, the secondary coil does not deliver power to the load. Therefore, the losses are negligible and considered zero. In this case, the input power of the transformer is equal to its output power and it is written as,

$$V_1 I_1 = V_2 I_2 \tag{3.23}$$

Equation (3.23) can be rearranged as,

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = a \tag{3.24}$$

The ratio of primary current to secondary current is,

$$\frac{I_1}{I_2} = \frac{1}{a} \tag{3.25}$$

From equation (3.25), it is seen that the current ratio of a single-phase transformer is inversely proportional to the voltage ratio.

Again, the magnetomotive force produced by the primary current will be equal to the magnetomotive force produced by the secondary current and it can be expressed as,

$$\Im = \Im_1 - \Im_2 = 0 \tag{3.26}$$

Substituting magnetomotive force I = *NI* in equation (3.26) yields,

$$N_1 I_1 = N_2 I_2 \tag{3.27}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{3.28}$$

From equation (3.28), it is concluded that the ratio of primary to secondary current is inversely proportional to the turn ratio of the transformer.

The input and output power of an ideal transformer is written as,

$$P_{in} = V_1 I_1 \cos\phi_1 \tag{3.29}$$

$$P_{out} = V_2 I_2 \cos\phi_2 \tag{3.30}$$

For an ideal condition, the angle φ_1 is equal to the angle φ_2 and the output power can be re-arranged as,

$$P_{out} = \frac{V_1}{a} a I_1 \cos\phi_1 \tag{3.31}$$

$$P_{out} = V_1 I_1 \cos\phi_1 \tag{3.32}$$

Substituting equation (3.29) into equation (3.32) yields,

$$P_{out} = P_{in} \tag{3.33}$$

From equation (3.33), it is seen that the input and the output power are the same for an ideal transformer. Similarly, the input and output reactive powers are the same and it can be written as,

$$Q_{out} = V_2 I_2 \sin\phi_2 = V_1 I_1 \sin\phi_1 = Q_{in} \tag{3.34}$$

From equations (3.33) and (3.34), it is seen that the input and output power and reactive power can be calculated if other parameters are given.

Example 3.1

A 22 kVA, 2200 V/220 V single-phase transformer has the number of turns 50 in the secondary coil. Determine the number of primary turns, primary full load current, and secondary full load current. Neglect all kinds of losses in the transformer.

Solution

The primary and secondary voltages are given as 2200V and 220V. Therefore, the value of the turns ratio is calculated as,

$$a = \frac{V_1}{V_2} = \frac{2200}{220} = 10 \tag{3.35}$$

The value of the primary turns can be determined as,

$$N_1 = aN_2 = 10 \times 50 = 500 \tag{3.36}$$

The value of the primary full load current is calculated as,

$$I_1 = \frac{22 \times 10^3}{2200} = 10\,\text{A} \tag{3.37}$$

The value of the secondary full load current is calculated as,

$$I_2 = \frac{22 \times 10^3}{220} = 100\,\text{A} \tag{3.38}$$

Example 3.2

The number of primary and secondary turns of a 25 kVA single-phase transformer are 200 and 400, respectively. The transformer is connected to a 220 V, 50 Hz source. Find the turns ratio and mutual flux in the core.

Solution

The primary and the secondary turns are given by 200 and 400, respectively. Therefore, the turns ratio is calculated as,

$$a = \frac{N_1}{N_2} = \frac{200}{400} = 0.5 \tag{3.39}$$

The value of the mutual flux is calculated as,

$$\phi_m = \frac{V_1}{4.44 f N_1} = \frac{220}{4.44 \times 50 \times 200} = 4.95\,\text{mWb} \tag{3.40}$$

Practice Problem 3.1

The primary voltage of an iron core single-phase transformer is 220 V with 60 Hz. The number of primary and secondary turns of the transformer are 200 and 50, respectively. Determine the voltage of the secondary coil.

Practice Problem 3.2

The number of primary turns of a 40 kVA, 2200 V/220 V, 50 Hz single-phase transformer is 200. Calculate the turns ratio, the mutual flux in the core, and full load primary and secondary currents.

3.8 Analysis of No-Load Phasor Diagram

The small magnitude of the primary current flows in the transformer coil when it is connected to the source. This current creates iron loss (hysteresis loss and eddy current loss) and a small amount of copper loss ($I_0^2 R_1$). However, the input power with a small input current I_0 is expressed as,

$$P = V_1 I_0 \cos\theta_0 \quad (3.41)$$

The primary current (I_0) is the combination of I_w and I_m. The current I_w is in phase with supply voltage V_1, creating the iron loss and small copper loss in the primary. Therefore, the current I_w is known as iron loss or the active or working component of the primary current. Whereas, the current I_m is in quadrature of supply voltage and sustains the magnetic flux in the core. This current is known as a magnetizing component of the primary current. The phasor diagram of the components of the current is shown in Fig. 3.9.

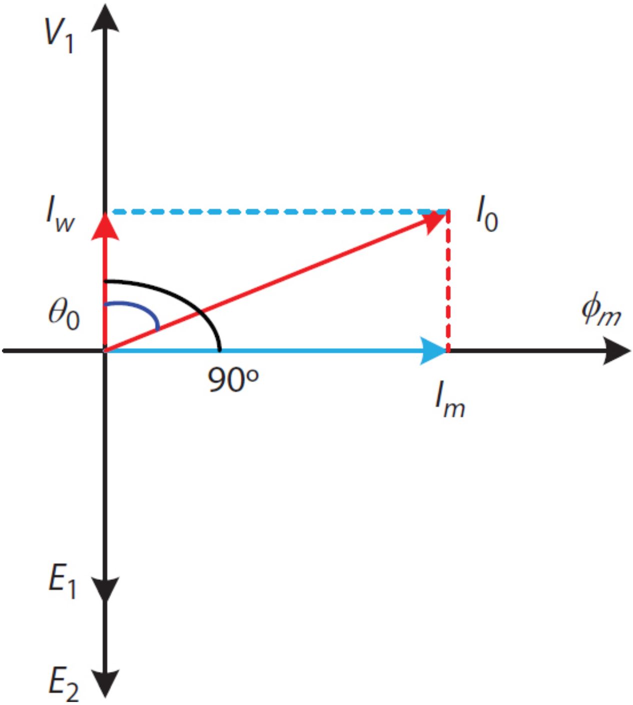


Fig. 3.9 Phasor diagram at no load.

From Fig. 3.8, the following equations are written as,

$$I_w = I_0 \cos\theta_0 \quad (3.42)$$

$$I_m = I_0 \sin\theta_0 \quad (3.43)$$

Squared equations (3.42) and (3.43) and adding yields,

$$(3.44)\quad I_0^2 = I_w^2 + I_m^2$$

$$(3.45)\quad I_0 = \sqrt{I_w^2 + I_m^2}$$

From equation (3.45), it is seen that the no-load current can be calculated if core loss and a magnetizing component of the currents are given.

Example 3.3

The no-load current of an 1100 V/120 V single-phase transformer is 0.06 A. The transformer consumes power of 60 W when it is connected to an 1100 V source. Determine the power factor at no-load current, iron loss and magnetizing components of the current.

Solution

The value of the power factor at no-load current is calculated as,

$$(3.46)\quad 60 = 1100 \times 0.06 \times \cos\theta_0$$

$$(3.47)\quad \cos\theta_0 = 0.90$$

The value of the iron loss component of the current is calculated as,

$$(3.48)\quad I_w = I_0 \cos\theta_0 = 0.06 \times 0.90 = 0.054\,\text{A}$$

The value of the magnetizing component of the current is calculated as,

$$(3.49)\quad I_m = I_0 \sin\theta_0 = 0.06 \times \sin(\cos^{-1} 0.90) = 0.026\,\text{A}$$

Practice Problem 3.3

The no-load current of an 1100 V/120 V, 10 kVA, single-phase transformer is 1.2 A and the iron loss component of the current is 0.7A. The high-voltage side is kept open and the low-voltage side is connected to the source. Determine the magnetizing component of the current, power factor and input power.

3.9 Analysis of Load Phasor Diagram

The small magnitude of the primary current flows in the transformer coil when is connected to the source. This current creates the flux in the core that generates the voltage across the primary and the secondary winding. Due to this voltage, secondary current I_2 will flow in the load as shown in Fig. 3.10(a).

The current I_1 lags the supply voltage V_1 and the resistive drop I_1R_1 is in phase with the current I_1 as shown in Fig. 3.10(b). The input voltage from Fig. 3.10(b) is expressed as,

$$V_1 = V_1^{'} + I_1R_1 + jI_1X_1 \tag{3.50}$$

Similarly, the secondary voltage is expressed as,

$$V_2 = E_2 - I_2R_2 - jI_2X_2 \tag{3.51}$$

3.10 Rules for Referring Impedance

The primary and secondary parameters of a transformer need to be shifted when developing an equivalent circuit of a transformer. These parameters are the resistance, reactance, impedance, current and voltage. A simple circuit of a single-phase transformer with a load impedance is shown in Fig. 3.11.

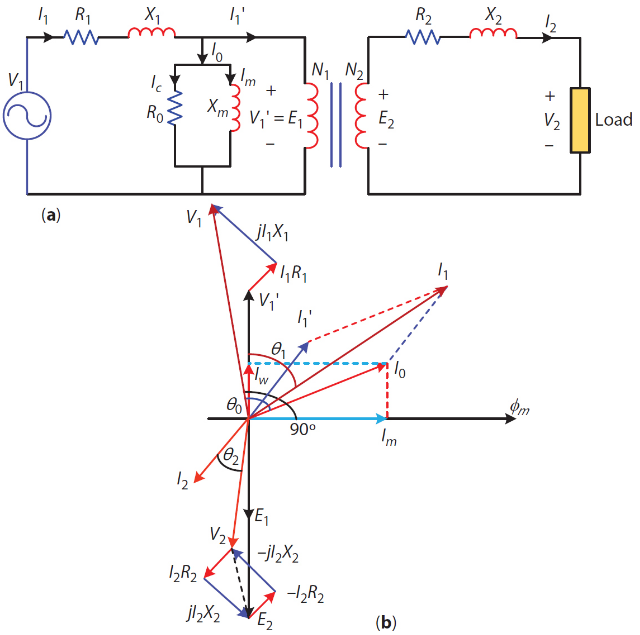

I_1
R_1
X_1
I_1'
I_0
I_c
I_m
R_0
X_m
V_1
$V_1' = E_1$
N_1
N_2
R_2
X_2
I_2
E_2
V_2
Load
(a)
V_1
jI_1X_1
I_1R_1
V_1'
I_1
I_1'
θ_1
I_w
I_0
θ_0
$90°$
ϕ_m
I_m
θ_2
I_2
E_1
V_2
$-jI_2X_2$
I_2R_2
$-I_2R_2$
jI_2X_2
E_2
(b)

Fig. 3.10 Phasor diagram at no load.

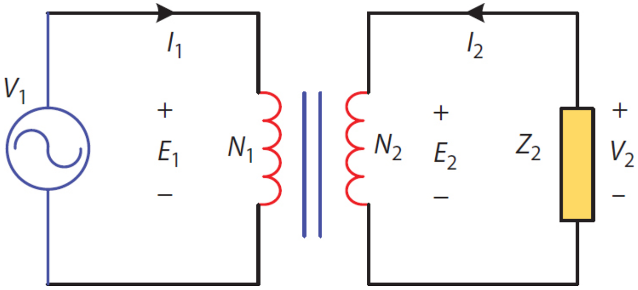

I_1
I_2
V_1
E_1
N_1
N_2
E_2
Z_2
V_2

Fig. 3.11 Transformer with a load impedance.

The ratio of primary voltage to secondary voltage is written as,

$$\frac{V_1}{V_2} = a \qquad (3.52)$$

The ratio of primary current to secondary current is expressed as,

$$\frac{I_1}{I_2} = \frac{1}{a} \tag{3.53}$$

Dividing equation (3.52) by equation (3.53) yields,

$$\frac{\frac{V_1}{V_2}}{\frac{I_1}{I_2}} = \frac{a}{\frac{1}{a}} \tag{3.54}$$

Rearranging equation (3.54) yields,

$$\frac{\frac{V_1}{I_1}}{\frac{V_2}{I_2}} = a^2 \tag{3.55}$$

Substituting $Z = V/I$ in equation (3.55) yields,

$$\frac{Z_1}{Z_2} = a^2 \tag{3.56}$$

Alternative approach: The impedances in the primary and secondary windings are expressed as,

$$Z_1 = \frac{V_1}{I_1} \tag{3.57}$$

$$Z_2 = \frac{V_2}{I_2} \tag{3.58}$$

Dividing equation (3.57) by equation (3.58) yields,

$$\frac{Z_1}{Z_2} = \frac{\frac{V_1}{I_1}}{\frac{V_2}{I_2}} \quad (3.59)$$

$$\frac{Z_1}{Z_2} = \frac{V_1}{V_2} \times \frac{I_2}{I_1} \quad (3.60)$$

Substituting equations (3.52) and (3.53) in equation (3.60) yields,

$$\frac{Z_1}{Z_2} = a \times a \quad (3.61)$$

$$\frac{Z_1}{Z_2} = a^2 \quad (3.62)$$

From equation (3.62), it can be concluded that the impedance ratio is equal to the square of the turns ratio. The important points for transferring parameters are (i) R_1 in the primary becomes R_1/a^2 when referred to the secondary, (ii) R_2 in the secondary becomes a^2R_2 when referred to the primary, (iii) X_1 in the primary becomes X_1/a^2 when referred to the secondary, and (iv) X_2 in the secondary becomes a^2X_2 when referred to the primary.

Example 3.4

The turn ratio of a single-phase transformer is 1:5. The secondary coil is connected to a resistive load of 80 Ω and the primary is connected to a 120V source. Calculate the resistance referred to as primary, primary current and the voltage drop across 80 Ω resistance.

Solution

The value of the turns ratio is given as,

$$a = \frac{N_1}{N_2} = \frac{1}{5} \quad (3.63)$$

The value of the resistance referred to as the primary is calculated as,

$$R_1 = R_2' = a^2 R_2 = \left(\frac{1}{5}\right)^2 \times 150 = 6\Omega \quad (3.64)$$

The value of the primary current is calculated as,

$$I_1 = \frac{V_1}{R_1} = \frac{120}{6} = 20\,\text{A} \quad (3.65)$$

The value of the current in the secondary winding is calculated as,

$$I_2 = I_1 a = 20 \times \frac{1}{5} = 4\,\text{A} \quad (3.66)$$

The voltage drop across 80 Ω resistance is calculated as,

$$V_2 = I_2 R_2 = 4 \times 80 = 320\,\text{V} \quad (3.67)$$

Practice Problem 3.4

The turn ratio of a single-phase transformer is 1:2. The secondary coil is connected to a resistive load of 10 Ω and the primary is connected to a 220V source. Calculate the resistance referred to as primary, primary current and the voltage drop across 10 Ω resistance.

3.11 Equivalent Circuit of a Transformer

The windings of a transformer are not connected electrically. However, the windings are magnetically coupled with each other. In this configuration and condition, it is tedious to do an appropriate analysis. Therefore, for easy computation and visualization, the practical transformer needs to be converted into an equivalent circuit by maintaining the same properties as the main transformer. In the equivalent circuit, the related parameters need to be transferred either from the primary to the secondary or vice versa. A two-winding ideal transformer is shown in Fig. 3.12.

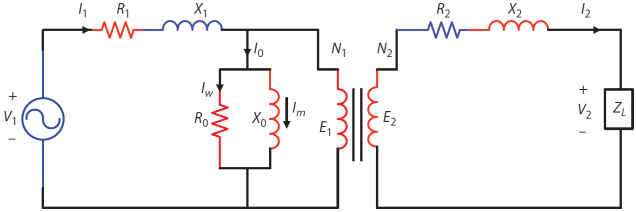


Fig. 3.12 Two windings transformer.

3.12 Exact Equivalent Circuit

Transformer parameters such as current, voltage, resistance and inductance can be transferred either from the primary winding to the secondary winding or vice versa. An exact equivalent circuit refers to the primary where all the parameters are transferred from the secondary winding to the primary winding as shown in Fig. 3.13. Let the secondary resistance R_2 be R_2' when transferred to the primary side. The copper loss $I_2^2 R_2$ due to the secondary resistance will be equal to the copper loss $I_1^2 R_2'$ due to resistance R_2' in the primary. These copper losses are expressed as,

$$I_1^2 R_2' = I_2^2 R_2 \tag{3.68}$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = a^2 R_2 \tag{3.69}$$

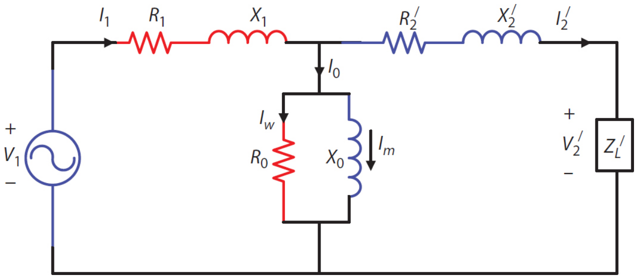


Fig. 3.13 Exact equivalent circuit is referred to as the primary.

Similarly, for reactance, load impedance, current and voltage in the primary side are expressed as,

$$X_2^{/} = a^2 X_2 \tag{3.70}$$

$$Z_L^{/} = a^2 Z_L \tag{3.71}$$

$$I_2^{/} = \frac{I_2}{a} \tag{3.72}$$

$$V_2^{/} = aV_2 \tag{3.73}$$

The exact equivalent circuit refers to the secondary where all the parameters are transferred from the primary to the secondary as shown in Fig. 3.14. Let the secondary resistance R_1 be $R_1^{/}$ when transferred to the secondary side. The copper loss $I_1^2R_1$ due to the primary resistance R_1 will be equal to the copper loss $I_2^2R_1^{/}$ due to resistance $R_1^{/}$ in the secondary. These copper losses are expressed as,

$$I_1^2 R_1 = I_2^2 R_1^{/} \tag{3.74}$$

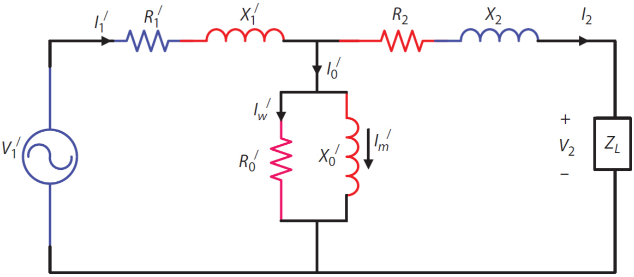


Fig. 3.14 Exact equivalent circuit is referred to as the secondary.

$$R_1^{/} = \left(\frac{I_1}{I_2}\right)^2 R_1 = \frac{R_1}{a^2} \tag{3.75}$$

Similarly, the reactance, current and voltages are expressed as,

$$X_1' = \frac{X_1}{a^2} \quad (3.76)$$

$$I_1' = aI_1 \quad (3.77)$$

$$V_1' = \frac{V_1}{a} \quad (3.78)$$

$$I_w' = aI_w \quad (3.79)$$

$$I_m' = aI_m \quad (3.80)$$

$$I_0' = aI_0 \quad (3.81)$$

3.13 Approximate Equivalent Circuit

The rated primary and secondary currents along with other parameters are included in the nameplate of the transformer. However, the no-load current is very small compared to the rated primary current. Therefore, there is a negligible voltage drop due to primary resistance R_1 and reactance X_1. In this condition, it can be assumed that the voltage drop across the no-load circuit is the same as the applied voltage without counting any significant error. The approximate equivalent circuit can be drawn by shifting the no-load circuit across the supply voltage, V_1. An approximate equivalent circuit is referred to as the primary shown in Fig. 3.15. The total or equivalent resistance, reactance and impedance are referred to as the primary is written as,

$$R_{01} = R_1 + R_2' = R_1 + a^2 R_2 \quad (3.82)$$

$$X_{01} = X_1 + X_2' = X_1 + a^2 X_2 \quad (3.83)$$

$$Z_{01} = R_{01} + jX_{01} \quad (3.84)$$

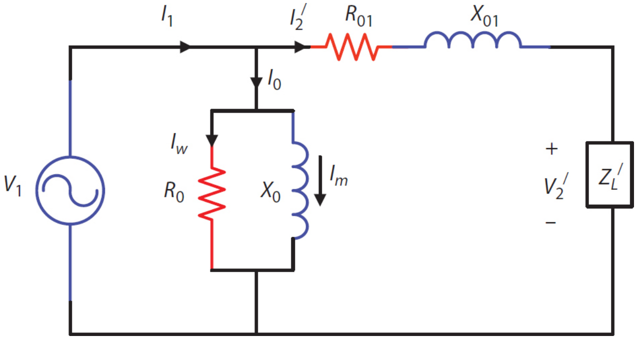

I_1
I_2'
R_{01}
X_{01}
I_0
I_w
R_0
X_0
I_m
V_1
+
V_2'
−
Z_L'

Fig. 3.15 Approximate equivalent circuit referred to primary.

The no-load circuit resistance and reactance are expressed as,

$$R_0 = \frac{V_1}{I_w} \tag{3.85}$$

$$X_0 = \frac{V_1}{I_m} \tag{3.86}$$

An approximate equivalent circuit is referred to as the secondary shown in Fig. 3.16.

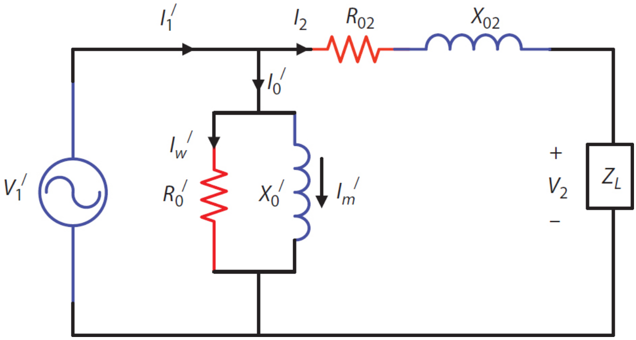

I_1'
I_2
R_{02}
X_{02}
I_0'
I_w'
R_0'
X_0'
I_m'
V_1'
+
V_2
−
Z_L

Fig. 3.16 Approximate equivalent circuit referred to secondary.

The equivalent resistance, reactance and impedance referred to as the secondary is expressed as,

$$R_{02} = R_2 + R_1' = R_2 + \frac{R_1}{a^2} \tag{3.87}$$

$$X_{02} = X_2 + X_1' = X_2 + \frac{X_1}{a^2} \tag{3.88}$$

$$Z_{02} = R_{02} + jX_{02} \tag{3.89}$$

The no-load circuit resistance and reactance referred to as the secondary is written as,

$$R_0' = \frac{V_1'}{I_w'} \tag{3.90}$$

$$X_0' = \frac{V_1'}{I_m'} \tag{3.91}$$

Example 3.5

A 5 kVA, 200 V/40 V single-phase transformer has the primary resistance and reactance of 3Ω and 12Ω, respectively. On the secondary side, these values are 0.3Ω and 0.1Ω, respectively. Determine the equivalent impedance referred to the primary and the secondary windings of the transformer.

Solution

The value of the turns ratio is calculated as,

$$a = \frac{V_1}{V_2} = \frac{200}{40} = 5 \tag{3.92}$$

The value of the equivalent resistance referred to as the primary is determined as,

$$R_{01} = R_1 + a^2 R_2 = 3 + 25 \times 0.3 = 10.5\Omega \tag{3.93}$$

The value of the equivalent reactance referred to as the primary is determined as,

$$X_{01} = X_1 + a^2 X_2 = 12 + 25 \times 0.1 = 14.5\Omega \tag{3.94}$$

The value of the equivalent impedance referred to as the primary is determined as,

$$Z_{01} = \sqrt{10.5^2 + 14.5^2} = 17.9\Omega \tag{3.95}$$

The value of the equivalent resistance referred to as the secondary is determined as,

$$R_{02} = R_2 + \frac{R_1}{a^2} = 0.3 + \frac{3}{25} = 0.42\Omega \tag{3.96}$$

The value of the equivalent reactance referred to as the secondary is determined as,

$$X_{02} = X_2 + \frac{X_1}{a^2} = 0.1 + \frac{12}{25} = 0.58\Omega \tag{3.97}$$

The value of the equivalent impedance referred to as the secondary is determined as,

$$Z_{02} = \sqrt{0.42^2 + 0.58^2} = 0.72\Omega \tag{3.98}$$

Practice Problem 3.5

A 15 kVA, 220 V/80 V single-phase transformer has the primary resistance and reactance of 4 Ω and 14Ω, respectively. The resistance and reactance at the secondary side are 0.5Ω and 0.9Ω, respectively. Calculate the equivalent impedance referred to as the primary and the secondary.

3.14 Polarity of a Transformer

The relative directions of induced voltages between the high-voltage and low-voltage terminals are known as the polarity of a transformer. The polarity of a transformer is very important to constructing a three-phase transformer bank, the parallel connection of the transformer, the connection of the current transformer (CT) and the potential transformer (PT) power with the metering device. Two polarities, namely additive and subtractive, are used to identify the polarity in the transformer as shown in Fig. 3.17 and Fig. 3.18, respectively.

For safety issues, a lower voltage needs to be applied to the primary side, i.e., high voltage terminals. Suppose, a voltage of 110V is applied to the primary side of a transformer.

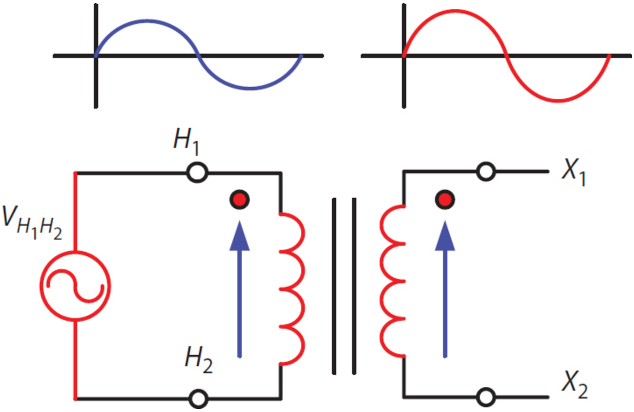


Fig. 3.17 Additive polarity of a transformer.

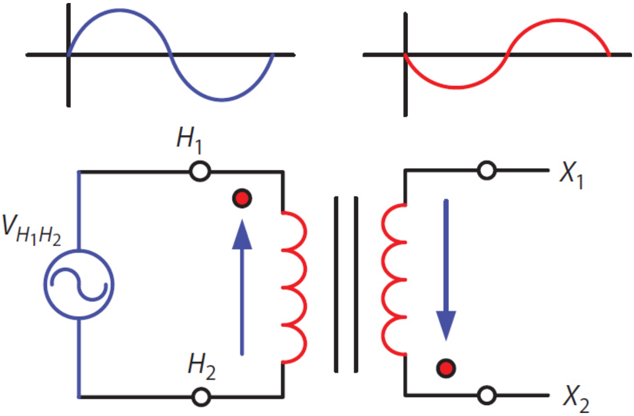


Fig. 3.18 Subtractive polarity of a transformer.

In this case, a voltage of 55 V (110/2) will appear at the secondary terminals. If the meter reads out the voltage of 165 V (110 + 55), then the transformer is said to be in additive polarity. This additive connection is shown in Fig. 3.19.

If the voltmeter reads the voltage of 55 V (110-55), then the transformer is said to be in subtractive polarity as shown in Fig. 3.20.

3.15 Three-Phase Transformer

A three-phase power transformer is used at the power generating station to step up the voltage from 11 kV to 120 kV, whereas in the power distribution substation, the three-phase voltage is again used to step down the voltage to 11 kV using a three-phase distribution transformer. A three-phase wye-delta transformer is shown in Fig. 3.21.

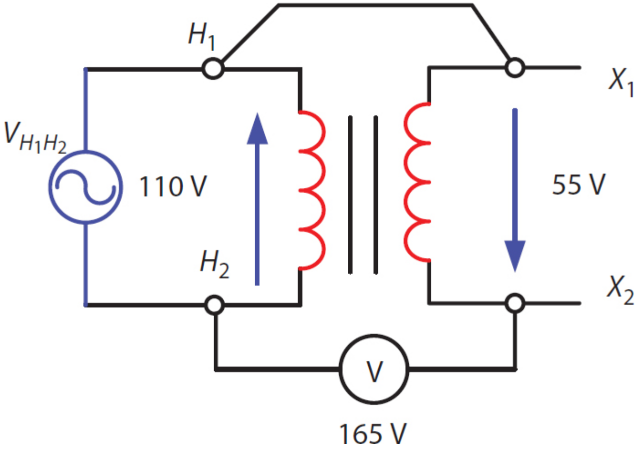


Fig. 3.19 Testing of additive polarity.

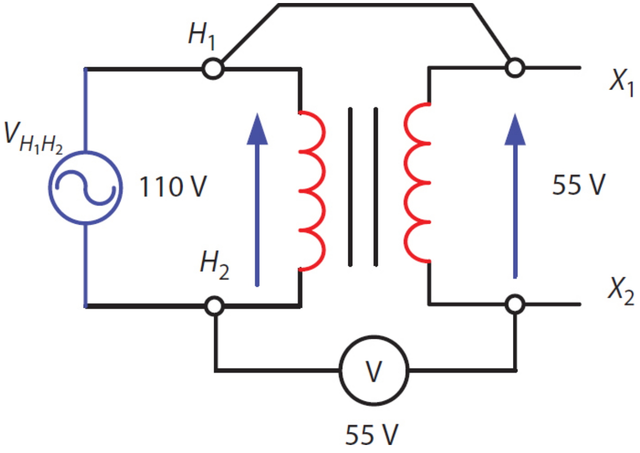

H_1
X_1
$V_{H_1H_2}$
110 V
55 V
H_2
X_2
V
55 V

Fig. 3.20 Testing of subtractive polarity.

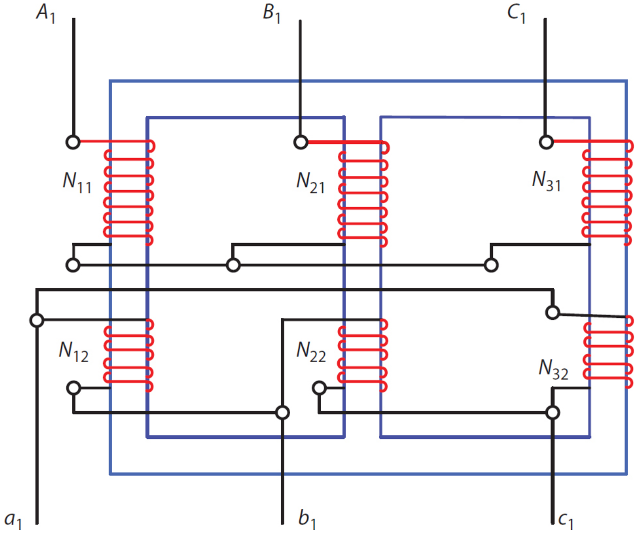

A_1
B_1
C_1
N_{11}
N_{21}
N_{31}
N_{12}
N_{22}
N_{32}
a_1
b_1
c_1

Fig. 3.21 Three windings on a common core and wye-delta connection.

Therefore, a three-phase transformer can be made either by three windings wound on a common core or by three single-phase transformers connected in a three-phase bank as shown in Fig. 3.22. The first approach is a cheaper one that results in a transformer with a smaller size and less weight. The main disadvantage of the first approach is that if one phase becomes defective, then the whole transformer needs to be replaced, whereas in the second approach, if one of the transformers becomes defective, then the system can be given power by an open delta at a reduced capacity. In this case, the defective transformer is normally replaced by a new one.

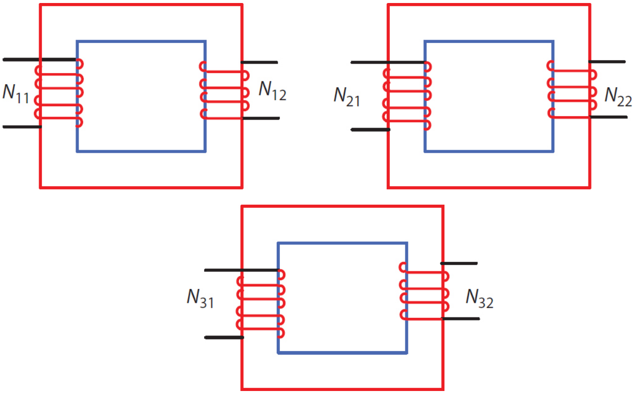


Fig. 3.22 Three single-phase transformers.

3.16 Transformer Vector Group

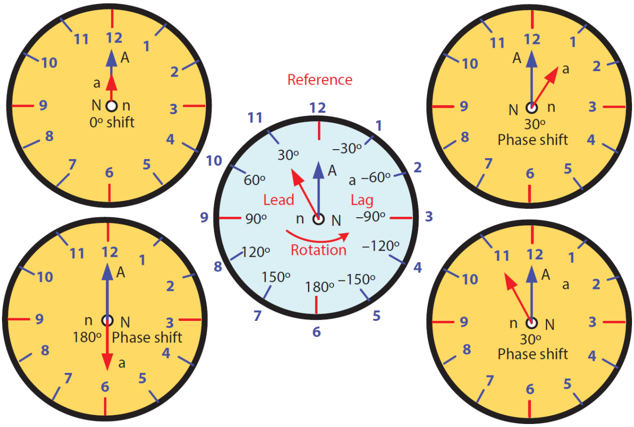
In the power station, transmission and distribution networks three-phase transformers with different ratings and connections are used. The primary (HV) and secondary windings (LV) of a three-phase transformer are connected to either delta-delta or star-star or delta-star or star-delta configuration pair.

The transformer vector group is labelled with capital letters and small letters plus numbers from 1 to 12 in a typical clock-like diagram. The capital letters (D, Y, Z, N) indicate the primary winding and the small letters (d, y, z, n) represent the secondary winding.

The digits 0, 1, 11, etc., are used to represent the phase displacement between the HV (High Voltage) and LV (Low Voltage) windings. Consider HV winding as a reference and set it to 12 o'clock as shown in Fig. 3.23. The digit zero represents the 0° phase shift between the HV and LV. The digit 1 represents the 30° phase shift (Lag) and the digit 11 represents the +30° phase shift (Lead) between the HV and LV windings as shown in Fig. 3.23.

There are four vector groups used in the three-phase transformer connection. These vector groups are (i) Group I: 0 o'clock, zero phase displacement (Yy0, Dd0, Dz0), (ii) Group II: 6 o'clock, 180^0 phase displacement (Yy6, Dd6, Dz6), (iii) Group III: 1 o'clock, -30^0 lag phase displacement (Dy1, Yd1, Yz1), and (iv) Group IV: 11 o'clock, 30^0 lead phase displacement (Dy11, Yd11, Yz11).

Here, Y represents the wye connection, D represents the delta connection and z represents the zigzag connection. The connection diagrams for different combinations are shown in Figs. 3.24 to 3.35.



12
1
2
3
4
5
6
7
8
9
10
11
A
a
N
n
0° shift
Reference
30°
−30°
60°
a −60°
Lead
Lag
90°
−90°
120°
−120°
Rotation
150°
180°
−150°
30° Phase shift
180° Phase shift
30° Phase shift

Fig. 3.23 Representation of vector groups with the clock.

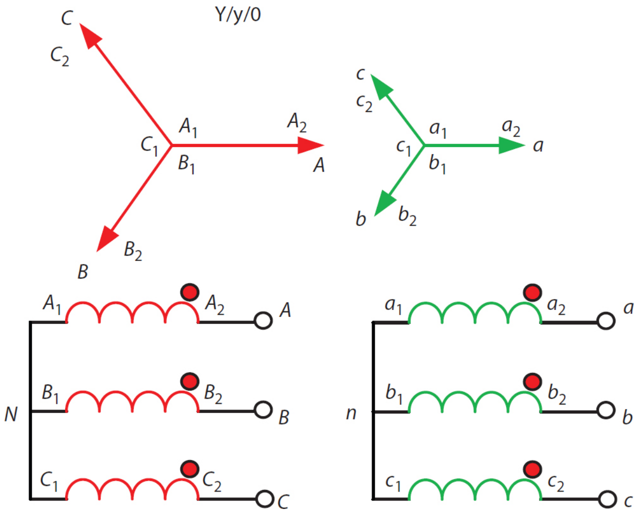

Y/y/0
C
C_2
A_1
A_2
C_1
B_1
A
B_2
B
c
c_2
a_1
a_2
a
c_1
b_1
b
b_2
A_1
A_2
A
N
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
n
b_1
b_2
b
c_1
c_2
c

Fig. 3.24 Connection of Y-y-0.

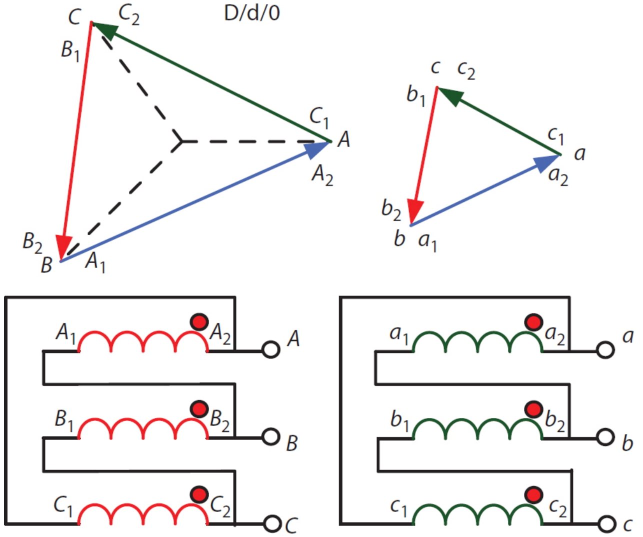

D/d/0
C
C_2
B_1
C_1
A
A_2
B_2
B
A_1
c
c_2
b_1
c_1
a
a_2
b_2
b
a_1
A_1
A_2
A
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
b_1
b_2
b
c_1
c_2
c

Fig. 3.25 Connection of D-d-0.

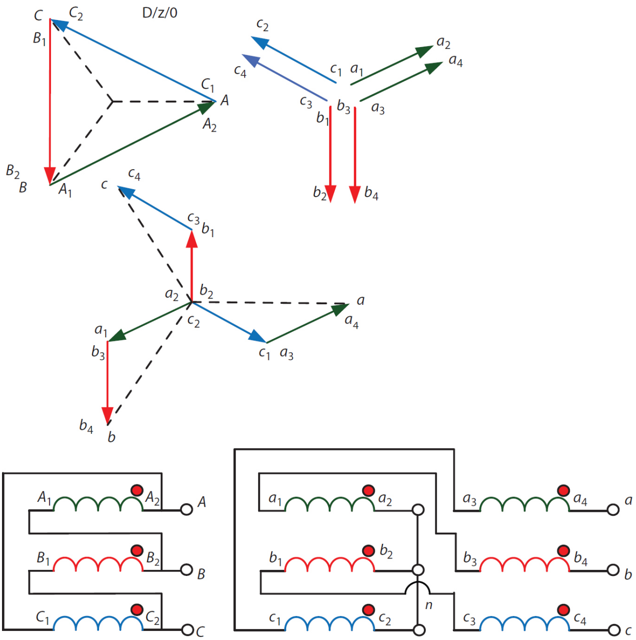

D/z/0
C
C_2
B_1
C_1
A
A_2
B_2
B
A_1
c_2
a_2
c_1
a_1
a_4
c_4
c_3
b_3
a_3
b_1
b_2
b_4
c
c_4
c_3
b_1
a_2
b_2
a
c_2
a_4
a_1
b_3
c_1
a_3
b_4
b
A_1
A_2
A
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a_3
a_4
a
b_1
b_2
b_3
b_4
b
n
c_1
c_2
c_3
c_4
c

Fig. 3.26 Connection of D-z-0.

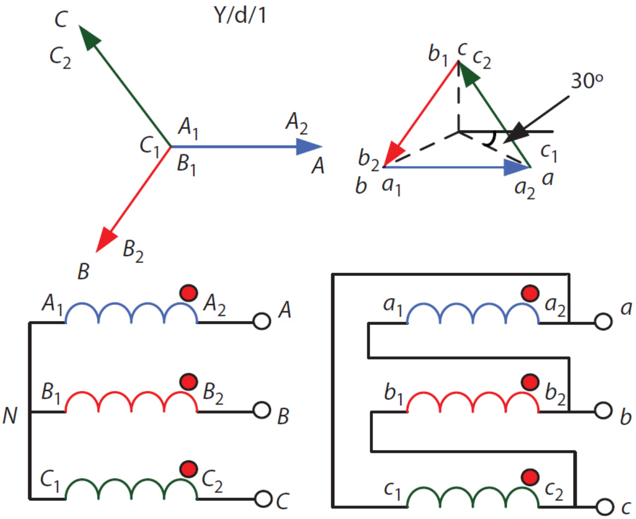

Y/d/1
C
C_2
A_1
A_2
C_1
B_1
A
B_2
B
b_1
c
c_2
30°
b_2
c_1
b
a_1
a_2
a
A_1
A_2
A
N
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
b_1
b_2
b
c_1
c_2
c

Fig. 3.27 Connection of Y-d-1.

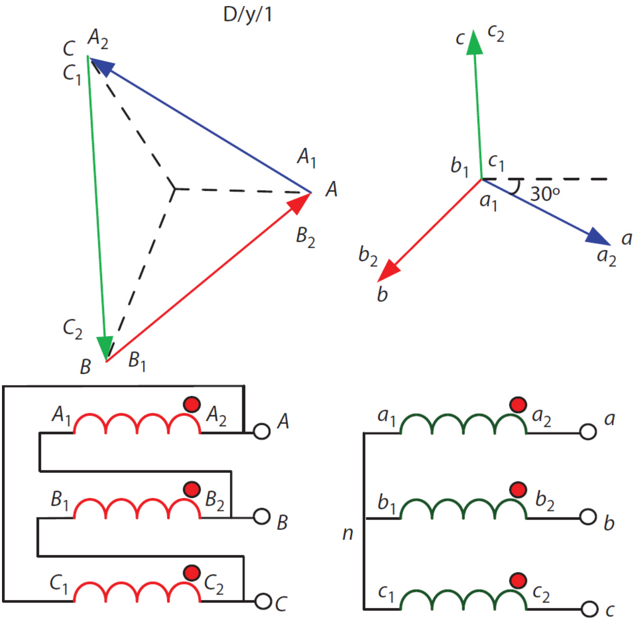

D/y/1
C
C_1
A_2
A_1
A
B_2
C_2
B
B_1
c
c_2
b_1
c_1
a_1
30º
a
a_2
b_2
b
A_1
A_2
A
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
b_1
b_2
b
n
c_1
c_2
c

Fig. 3.28 Connection of D-y-1.

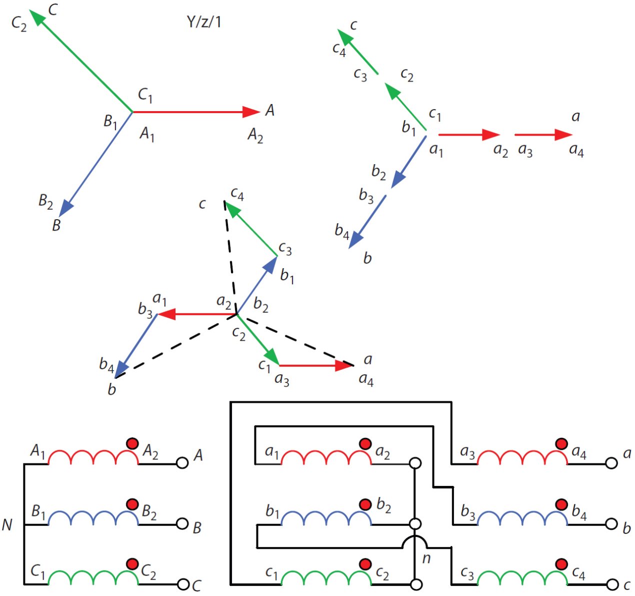

Y/z/1
C_2
C
C_1
B_1
A_1
A
A_2
B_2
B
c
c_4
c_3
c_2
c_1
b_1
a_1
a_2
a_3
a
a_4
b_2
b_3
b_4
b
c
c_4
c_3
b_1
b_3
a_1
a_2
b_2
c_2
b_4
b
c_1
a_3
a
a_4
A_1
A_2
A
N
B_1
B_2
B
C_1
C_2
C
a_1
a_2
b_1
b_2
n
c_1
c_2
a_3
a_4
a
b_3
b_4
b
c_3
c_4
c

Fig. 3.29 Connection of Y-z-1.

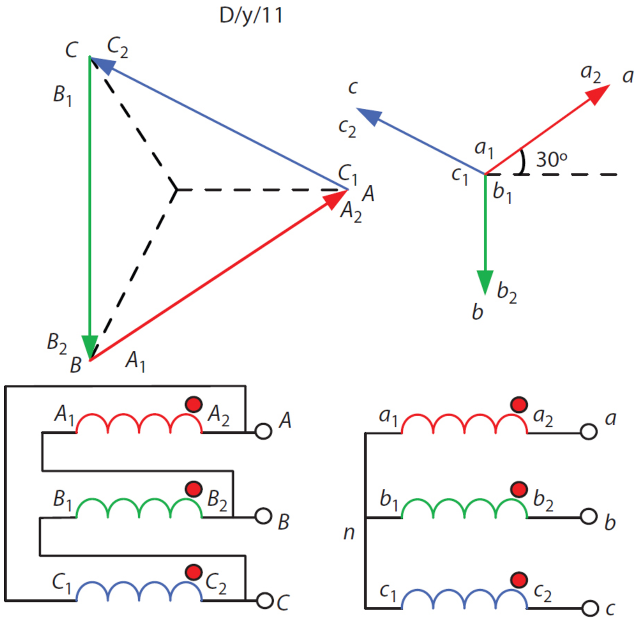

D/y/11
C
C_2
B_1
c
c_2
a_2
a
a_1
30º
C_1
A
A_2
c_1
b_1
b_2
b
B_2
B
A_1
A_1
A_2
A
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
b_1
b_2
b
n
c_1
c_2
c

Fig. 3.30 Connection of D-y-11.

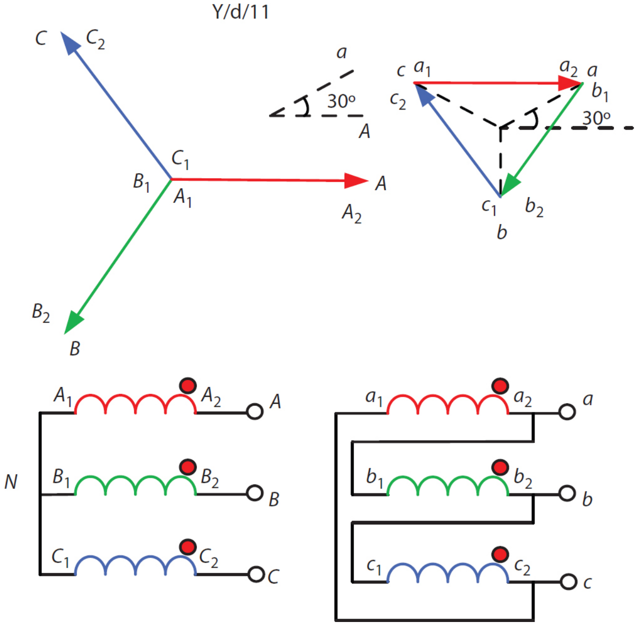

Y/d/11
C
C_2
a
30º
A
c
a_1
a_2
a
c_2
b_1
30º
C_1
B_1
A_1
A
A_2
c_1
b_2
b
B_2
B
A_1
A_2
A
N
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
b_1
b_2
b
c_1
c_2
c

Fig. 3.31 Connection of Y-d-11.

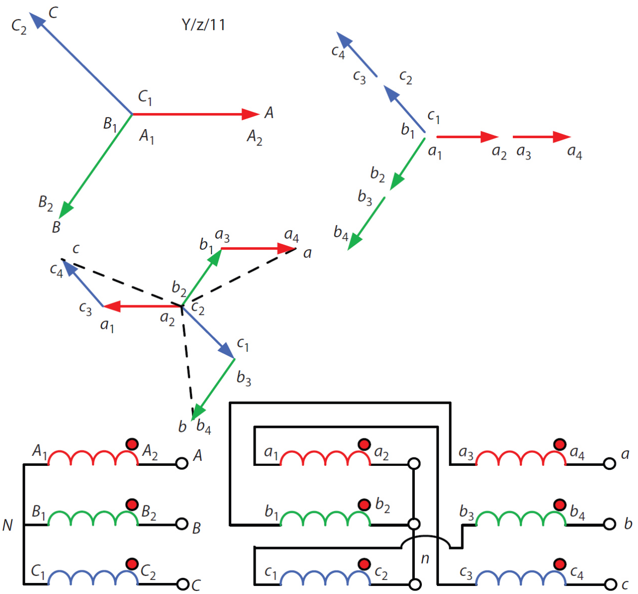

Y/z/11
C
C2
C1
B1
A1
A
A2
B2
B
c4
c3
c2
c1
b1
a1
a2
a3
a4
b2
b3
b4
c
a
b
N
n

Fig. 3.32 Connection of Y-z-11.

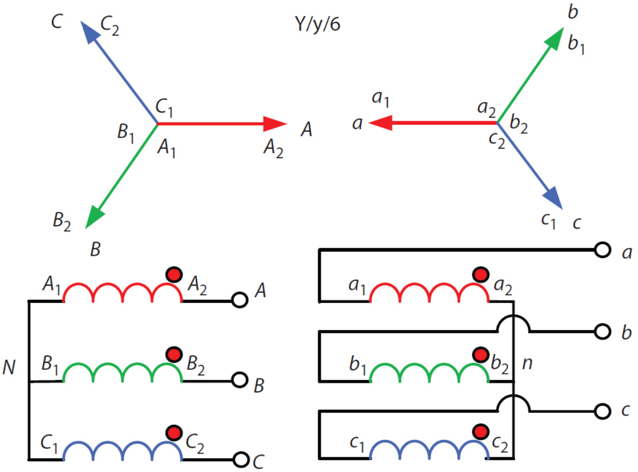

Y/y/6
C
C2
C1
B1
A1
A
A2
B2
B
b
b1
a1
a
a2
b2
c2
c1
c
N
n

Fig. 3.33 Connection of Y-y-6.

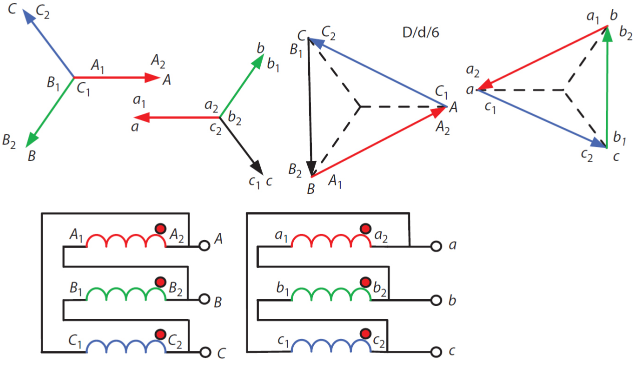

C
C_2
B_1
C_1
A_1
A_2
A
B_2
B
a_1
a
a_2
c_2
b_2
b
b_1
c_1 c
D/d/6
C
C_2
B_1
C_1
A
A_2
B_2
B
A_1
a_1
b
b_2
a_2
a
c_1
b_1
c_2
c
A_1
A_2
A
B_1
B_2
B
C_1
C_2
C
a_1
a_2
a
b_1
b_2
b
c_1
c_2
c

Fig. 3.34 Connection of D-d-6.

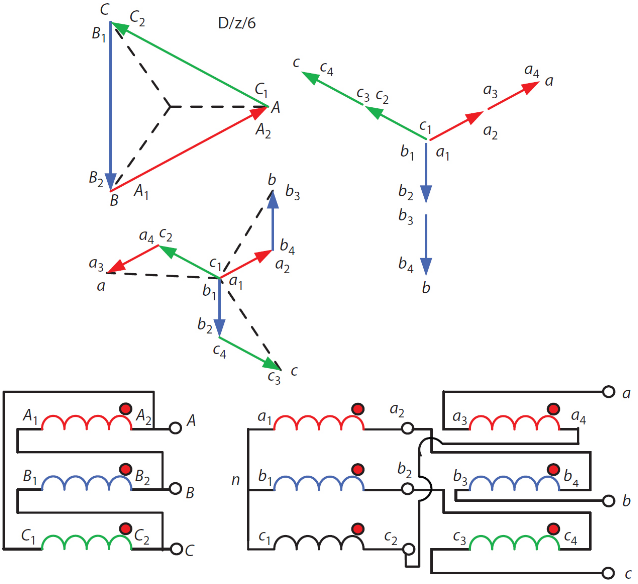

D/z/6
C
C_2
B_1
C_1
A
A_2
B_2
B
A_1
c
c_4
c_3 c_2
c_1
a_4
a
a_3
a_2
b_1
a_1
b_2
b_3
b_4
b
b
b_3
a_4 c_2
a_3
a
c_1
b_4
a_2
b_1
a_1
b_2
c_4
c_3
c
A_1
A_2
A
B_1
B_2
B
C_1
C_2
C
a_1
a_2
n
b_1
b_2
c_1
c_2
a_3
a_4
a
b_3
b_4
b
c_3
c_4
c

Fig. 3.35 Connection of D-y-6.

3.17 Voltage Regulation of a Transformer

A three-phase transformer delivers electricity to domestic, commercial and industrial loads. are usually connected with the secondary winding of a transformer. The secondary voltage of a transformer under operation changes due to the voltage drop across the internal impedance and the load. The voltage regulation of a transformer is used to identify the characteristics of the secondary side voltage changes under different loading conditions. The voltage regulation of a transformer is defined as the difference between the no-load terminal voltage (V_2NL) and full-load terminal voltage (V_2FL). Voltage regulation is expressed as either a per unit or a percentage of the full-load terminal voltage. Mathematically, it is expressed as,

$$\text{Voltage regulation} = \frac{V_{2NL} - V_{2FL}}{V_{2FL}} \times 100\% = \frac{E_2 - V_2}{V_2} \times 100\% \tag{3.99}$$

An approximate equivalent circuit referred to the secondary without no-load circuit to find the voltage regulation for different power factors as shown in Fig. 3.36. The phasor diagrams for unity, lagging and leading power factors are shown in Fig. 3.37 where the secondary voltage V_2 is considered as the reference phasor.

The phasor diagram with a unity power factor is shown in Fig. 3.37(a). The phasor form of the secondary induced voltage E_2 for a unity power factor is written as,

$$E_2 = V_2 + I_2(R_{02} + jX_{02}) \tag{3.100}$$

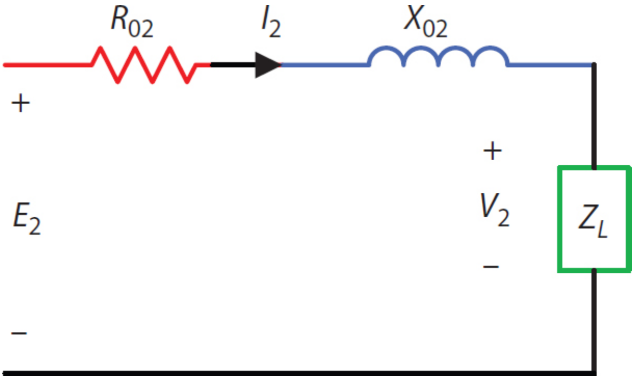
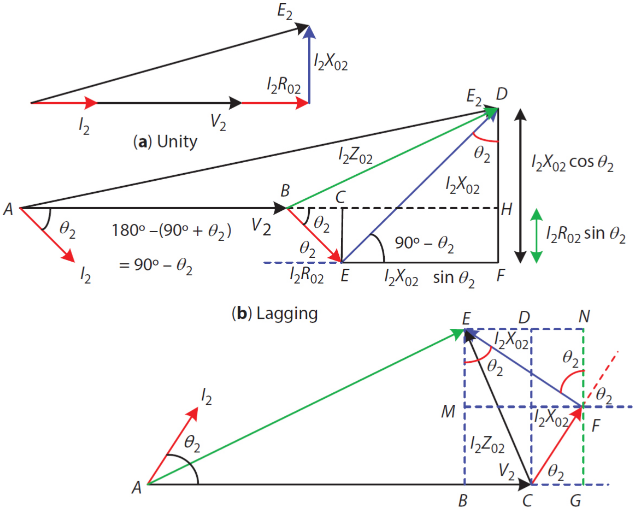


Fig. 3.36 Approximate equivalent circuit referred to secondary.


E_2
I_2X_{02}
I_2R_{02}
I_2
V_2
E_2 D
I_2Z_{02}
θ_2
I_2X_{02}
$I_2X_{02}\cos\theta_2$
A
B
C
H
θ_2
$180° - (90° + \theta_2)$
$= 90° - \theta_2$
V_2
θ_2
θ_2
$90° - \theta_2$
$I_2R_{02}\sin\theta_2$
I_2
I_2R_{02} E
I_2X_{02} $\sin\theta_2$
F
E D N
I_2X_{02}
θ_2
θ_2
I_2
θ_2
M
I_2X_{02}
F
I_2Z_{02}
θ_2
V_2
θ_2
A
B C G

(**a**) Unity

(**b**) Lagging

(**c**) Leading

Fig. 3.37 Phasor diagram for different power factors.

The phasor diagram for a lagging power factor is shown in Fig. 3.37(b). From this diagram, the following expressions are written as,

$$CG = I_2R_{02}\cos\phi_2 \tag{3.101}$$

$$CH = EF = I_2X_{02}\sin\phi_2 \tag{3.102}$$

$$HF = I_2R_{02}\sin\theta_2 \tag{3.103}$$

$$DF = I_2X_{02}\cos\phi_2 \tag{3.104}$$

$$DH = DF - HF = I_2X_{02}\cos\theta_2 - I_2R_{02}\sin\theta_2 \tag{3.105}$$

Applying Pythagoras's theorem to the triangle *ADH* yields,

$$AD^2 = AH^2 + HD^2 \tag{3.106}$$

$$AF^2 = (AB + BC + CH)^2 + (DF - HF)^2 \tag{3.107}$$

Substituting equations from (3.101) to (3.105) into equation (3.107) yields,

$$(3.108)\, E_2 = \sqrt{(V_2 + I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2)^2 + (I_2X_{02}\cos\theta_2 - I_2R_{02}\sin\theta_2)^2}$$

The first term of equation (3.108) is much higher than the second term. Therefore, the second term is neglected and equation (3.108) becomes,

$$(3.109)\, E_2 = \sqrt{(V_2 + I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2)^2 + 0}$$

$$(3.110)\, E_2 = V_2 + I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$$

$$(3.111)\, E_2 - V_2 = I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$$

The phasor diagram for a leading power factor is shown in Fig. 3.37(c). From this diagram, the following expressions are written,

$$(3.112)\, CG = DN = I_2R_{02}\cos\phi_2$$

$$(3.113)\, EN = BG = I_2X_{02}\sin\phi_2$$

$$(3.114)\, ED = EN - DN = I_2X_{02}\sin\phi_2 - I_2R_{02}\cos\phi_2$$

$$(3.115)\, ED = BC$$

$$(3.116)\, BM = FG = I_2R_{02}\sin\phi_2$$

$$(3.117)\, EM = NF = I_2X_{02}\cos\phi_2$$

Applying Pythagoras’s theorem to the triangle *AEB* yields,

$$(3.118)\, AE^2 = AB^2 + EB^2$$

$$(3.119)\, AE^2 = (AC - BC)^2 + (EM + MB)^2$$

Substituting AC = V2, and equations from (3.115) to (3.117) into equation (3.119) yields,

$$(3.120)\, E_2 = \sqrt{(V_2 + I_2R_{02}\cos\phi_2 - I_2X_{02}\sin\phi_2)^2 + (I_2R_{02}\sin\phi_2 + I_2X_{02}\cos\phi_2)^2}$$

The first term of equation (3.120) is much higher than the second term. Therefore, the second term is neglected and equation (3.120) becomes,

$$E_2 = \sqrt{(V_2 + I_2 R_{02}\cos\phi_2 - I_2 X_{02}\sin\phi_2)^2 + 0} \tag{3.121}$$

$$E_2 = V_2 + I_2 R_{02}\cos\phi_2 - I_2 X_{02}\sin\phi_2 \tag{3.122}$$

$$E_2 - V_2 = I_2 R_{02}\cos\phi_2 - I_2 X_{02}\sin\phi_2 \tag{3.123}$$

Substituting equations (3.111) and (3.123) into equation (3.99) yields,

$$\text{Voltage regulation} = \frac{I_2 R_{02}\cos\phi_2 \pm I_2 X_{02}\sin\phi_2}{V_2} \times 100\% \tag{3.124}$$

$$\text{Voltage regulation} = \frac{I_2 R_{02}\cos\phi_2}{V_2} \times 100\% \pm \frac{I_2 X_{02}\sin\phi_2}{V_2} \times 100\% \tag{3.125}$$

$$\%\text{Voltage regulation} = v_r\cos\phi_2 \pm v_x\sin\phi_2 \tag{3.126}$$

where,

$vr = I_2R_{02}/V_2$ is the percentage resistive drop,

$vx = I_2X_{02}/V_2$ is the percentage reactive drop.

Example 3.6

The value of the resistance and reactance of a 200V/400V single-phase transformer are 0.3Ω and 0.6Ω in the primary coil, whereas, the secondary coil resistance and reactance are 0.8Ω and 1.6Ω, respectively. Find the voltage regulation if the secondary current of the transformer is 10A at 0.80 pf lagging.

Solution

The value of the turn ratio is calculated as,

$$a = \frac{V_1}{V_2} = \frac{200}{400} = 0.5 \tag{3.127}$$

The value of the total resistance referred to as the secondary is calculated as, (3.128) $$R_{02} = R_2 + \frac{R_1}{a^2} = 0.8 + \frac{0.3}{0.25} = 2\Omega$$

The total reactance referred to as the primary is determined as,

(3.129) $$X_{02} = X_2 + \frac{X_1}{a^2} = 1.6 + \frac{0.6}{0.25} = 4\Omega$$

The approximate no-load voltage is calculated as,

(3.130) $$E_2 = V_2 + I_2 R_{02}\cos\phi_2 + I_2 X_{02}\sin\phi_2 = 400 + 10\times 2\times 0.8 + 10\times 4\times 0.6 = 440\,\text{V}$$

The voltage regulation for the lagging power factor is determined as,

(3.131) $$\text{Voltage regulation} = \frac{E_2 - V_2}{V_2}\times 100\% = \frac{440-400}{400}\times 100\% = 10\%$$

Practice Problem 3.6

The resistance and reactance of a 110 V/220 V single-phase transformer are 0.2Ω and 0.8Ω in the primary coil. The resistance and reactance in the secondary coil are 0.10Ω and 1.21Ω, respectively. Calculate the voltage regulation, when the secondary current is 3A at a 0.95 power factor leading.

3.18 Efficiency of a Transformer

Efficiency is an important parameter as it is used to identify the performance of any machine and to identify the characteristics of any machine. The efficiency η of any machine is defined as the ratio of its output to the input. Mathematically, it can be expressed as,

(3.132) $$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{Output}}{\text{Ouput+losses}} = \frac{\text{input-losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}}$$

Efficiency is usually calculated based on the input and output powers. In this case, consider the following:

P_0 is the output power in W,

Pi is the input power in W,

PL is the total losses in W.

Equation (3.132) is modified as,

$$\eta = \frac{P_0}{P_{in}} = 1 - \frac{P_L}{P_{in}} \tag{3.133}$$

It is noted that the efficiency of a transformer is generally higher than other electrical machines because the transformer has no moving parts.

3.19 Iron and Copper Losses

There are two losses that are considered in a transformer. These are iron loss and eddy current loss. The iron loss of a transformer is often called a core loss, which is a result of an alternating flux in the core of the transformer. The core loss depends on the voltage. The iron loss is classified as the eddy current loss and the hysteresis loss. In the transformer, most of the flux is transferred from the primary winding to the secondary winding through a low reluctance iron path. A small portion of that alternating flux links with the iron parts of the transformer, which in turn induces emf in the transformer core. As a result of this emf, current will flow in that part of the transformer and it is known as eddy current. The power loss due to this current is known as eddy current loss. The eddy current does not contribute to the output of the transformer but dissipates heat. The eddy current loss (P_e) is directly proportional to the square of the frequency (f) times the maximum magnetic flux density (B_m) and the eddy current loss can be expressed as,

$$P_e = k_e f^2 B_m{}^2 \tag{3.134}$$

Where k_e is the proportionality constant.

Steel is a very good ferromagnetic material which is used for the core of a transformer. This ferromagnetic material contains several domains in the structure and is magnetized easily. But these domains are like small magnets located randomly in the structure.

When an mmf is applied to the core, then those domains change their position. After removing mmf, most of the domains come back to their original position and will remain as it is. As a result, the substance is slightly permanently magnetized. However, an additional mmf is required to change the position of the remaining domains. Therefore, hysteresis loss is defined as the additional energy required to realign the domains in the ferromagnetic material. The hysteresis loss (*Ph*) is directly proportional to the frequency (*f*) and 2.6th power of the maximum magnetic flux density (*Bm*) and the expression of hysteresis loss is,

$$P_h = k_h f B_m{}^{2.6} \tag{3.135}$$

where *kh* is the proportionality constant.

Usually, a magnetic flux density is constant, equations (3.134) and (3.135) can be modified as,

$$P_e \propto f^2 \tag{3.136}$$

$$P_h \propto f \tag{3.137}$$

Practically, hysteresis loss depends on the voltage and the eddy current loss depends on the current. Therefore, the total losses of the transformer depend on the voltage and the current, not on the power factor. That is why the transformer rating is always represented in kVA instead of kW.

In the transformer, copper losses occur due to the primary and the secondary resistances along with the current flowing. The total full load copper losses can be determined as,

$$P_{culoss} = I_1{}^2 R_1 + I_2{}^2 R_2 \tag{3.138}$$

$$P_{culoss} = I_1{}^2 R_{01} = I_2{}^2 R_{02} \tag{3.139}$$

3.20 Maximum Efficiency

It is an important task to monitor the performance regularly of any machine. To analyze the performance, the value of the input and output power is required. The expression of the output power of a transformer is written as,

$$P_{out} = V_2 I_2 \cos\phi_2 \tag{3.140}$$

The expression of the copper loss is written as,

$$P_{cu} = I_2{}^2 R_{02} \tag{3.141}$$

The iron loss consists of eddy current loss and hysteresis loss. An expression of an iron loss is written as,

$$P_{\text{iron}} = P_{\text{eddy}} + P_{\text{hys}} \tag{3.142}$$

According to equation (3.132), the efficiency can be expressed as,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output+losses}} \tag{3.143}$$

Substituting equations (3.140), (3.141) and (3.142) into equation (3.143) yields,

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_{\text{iron}} + P_{cu}} \tag{3.144}$$

Dividing the numerator and denominator by I_2 yields,

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + \dfrac{P_{\text{iron}}}{I_2} + \dfrac{I_2{}^2 R_{02}}{I_2}} \tag{3.145}$$

The terminal voltage at the secondary side of a transformer is usually considered constant. For a given power factor, the secondary current is varied with the variation of load. Therefore, the transformer efficiency will be maximum, if the denominator of equation (3.134) is minimum. The denominator of equation (3.134) will be the minimum if the differentiation of the denominator with respect to the current is set equal to zero. Then this condition is written as,

$$\frac{d}{dI_2}\left(V_2\cos\phi_2+\frac{P_{\mathrm{iron}}}{I_2}+\frac{I_2{}^2R_{02}}{I_2}\right)=0 \tag{3.146}$$

$$-\frac{P_{\mathrm{iron}}}{I_2{}^2}+R_{02}=0 \tag{3.147}$$

$$P_{\mathrm{iron}}=I_2{}^2R_{02} \tag{3.148}$$

From equation (3.148), it is seen that the efficiency of a transformer will be maximum when the iron loss is equal to the copper loss. The expression of the secondary current can also be written from equation (3.148) as,

$$I_2=\sqrt{\frac{P_{\mathrm{iron}}}{R_{02}}} \tag{3.149}$$

For maximum efficiency, the load current can be expressed as,

$$I_{2\eta}=\sqrt{\frac{P_{\mathrm{iron}}}{R_{02}}} \tag{3.150}$$

Mathematically, equation (3.150) can be rearranged as,

$$I_{2\eta}=I_2\sqrt{\frac{P_{\mathrm{iron}}}{I_2{}^2R_{02}}} \tag{3.151}$$

Multiplying both sides of equation (3.151) by the secondary rated voltage V_2 yields,

$$V_2I_{2\eta}=V_2I_2\sqrt{\frac{P_{\mathrm{iron}}}{I_2{}^2R_{02}}} \tag{3.152}$$

$$VI_{\text{max efficiency}} = VI_{\text{rated}} \times \sqrt{\frac{\text{iron loss}}{\text{full load copper loss}}} \tag{3.153}$$

From equation (3.153), the output kVA at maximum efficiency can be calculated if other parameters are given.

Example 3.7

An iron loss and full load copper loss of a 30 kVA transformer are 350 W and 650 W, respectively. Calculate the full load efficiency, output kVA corresponding to maximum efficiency, and maximum efficiency with a 0.6 lagging power factor.

Solution

The total value of the full load loss is calculated as,

$$P_{tloss} = 350 + 650 = 1000\,\text{W} = 1\,\text{kW} \tag{3.154}$$

The output power at full load is calculated as,

$$P_{out} = 30 \times 0.6 = 18\,\text{kW} \tag{3.155}$$

Then the input power at full load is calculated as,

$$P_{in} = 18 + 1 = 19\,\text{kW} \tag{3.156}$$

The efficiency at full load is calculated by using input and output power as,

$$\eta = \frac{18}{19} \times 100 = 94.74\% \tag{3.157}$$

The output kVA corresponding to maximum efficiency is calculated as,

$$kVA_{m\eta} = kVA_{\text{rated}} \times \sqrt{\frac{\text{Iron loss}}{\text{Cu loss at full load}}} = 30 \times \sqrt{\frac{350}{650}} = 22\,\text{kVA} \tag{3.158}$$

The output power is calculated as,

$$P_{o1} = 22 \times 0.6 = 13.2\,\text{kW} \tag{3.159}$$

For maximum efficiency, the iron loss is equal to the copper loss. The total loss is calculated as,

$$P_{tloss-1} = 2 \times 350 = 700\,\text{W} \tag{3.160}$$

Again, the value of the input power is calculated from the output power and the total loss as,

$$P_{in1} = 13.2 + 0.7 = 13.9\,\text{kW} \tag{3.161}$$

Again, the efficiency is calculated as,

$$\eta = \frac{13.2}{13.9} \times 100 = 94.96\% \tag{3.162}$$

Practice Problem 3.7

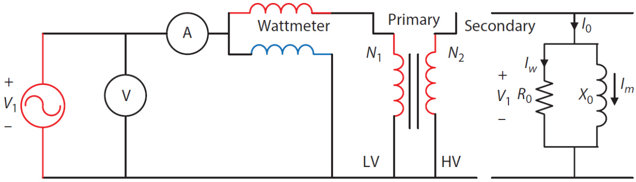
The iron loss and the copper loss of a 200 kVA transformer are 1.5 kW and 5 kW at full load conditions. Calculate the kVA rating at which the efficiency is maximum. Also, calculate the efficiency at a unity power factor.

3.21 Transformer Tests

The equivalent circuit parameters such as equivalent resistance and reactance referred to as the primary (R_{01}, X_{01}) and as the secondary (R_{02}, X_{02}) are very important to determine the efficiency and regulation of the transformer. These parameters of a transformer equivalent circuit can be determined by the open circuit and the short circuit tests.

3.21.1 Open Circuit Test

The main objectives of the open circuit test are to determine the no-load current and iron loss. The components of the no-load current are also used to determine the no-load circuit resistance (R_0) and reactance (X_0). In an open circuit test, the secondary side is considered to be an open circuit, and the primary coil is connected to the source as shown in Fig. 3.38(a).



(a) (b)

Fig. 3.38 Open circuit test with a no-load circuit.

The ammeter, voltmeter and wattmeter are connected to the high voltage or primary side. A rated voltage is applied to the primary winding using a variable transformer known as variac and corresponding readings are recorded. Then the wattmeter will measure the power that refers to the iron loss and small amount of copper loss. The ammeter and voltmeter will measure the no-load current and the voltage, respectively. Since the no-load current is very small (2% to 6% of the full load current), the copper losses can be neglected. Therefore, the wattmeter reading is considered equal to the iron loss. Then the wattmeter reading can be expressed as,

$$P_0 = V_1 I_0 \cos\phi_0 \tag{3.163}$$

From equation (3.163), the no-load power factor can be determined as,

$$\cos\phi_0 = \frac{P_0}{V_1 I_0} \tag{3.164}$$

The working and magnetizing components of the current can be determined as,

$$I_w = I_0 \cos\phi_0 \tag{3.165}$$

$$I_m = I_0 \sin\phi_0 \tag{3.166}$$

Then the no-load circuit resistance and reactance can be determined as,

$$R_0 = \frac{V_1}{I_w} \tag{3.167}$$

$$X_0 = \frac{V_1}{I_m} \tag{3.168}$$

The no-load resistance and reactance are usually placed to the side of the transformer where the measuring instruments are connected.

Example 3.8

A 200V/400V, 50 Hz single-phase transformer has the no-load test data of 200 V, 0.6 A, 80 W. Calculate the no-load circuit resistance and reactance.

Solution

The power factor can be determined as,

$$\cos\phi_0 = \frac{P_0}{V_1 I_0} = \frac{80}{200 \times 0.6} = 0.67 \tag{3.169}$$

$$\sin\phi_0 = \sqrt{1-(0.67)^2} = 0.74 \tag{3.170}$$

The values of the working and magnetizing components of the no-load current are calculated as,

$$I_w = I_0 \cos\phi_0 = 0.6 \times 0.67 = 0.4\ \mathrm{A} \tag{3.171}$$

$$I_m = I_0 \sin\phi_0 = 0.6 \times 0.74 = 0.44\ \mathrm{A} \tag{3.172}$$

The no-load circuit parameters can be determined as,

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.4} = 500\Omega \tag{3.173}$$

$$X_0 = \frac{V_1}{I_m} = \frac{200}{0.44} = 454.5\Omega \tag{3.174}$$

3.21.2 Short Circuit Test

The main objective of the short circuit test is to determine full-load copper loss. This test is also used to determine the equivalent resistance, reactance and impedance. In this test, the supply voltage and the measuring instruments (e.g., wattmeter, voltmeter ammeter) are connected to the primary side, and the secondary winding (LV side) is shorted out by wire or connected with an ammeter as shown in Fig. 3.39.

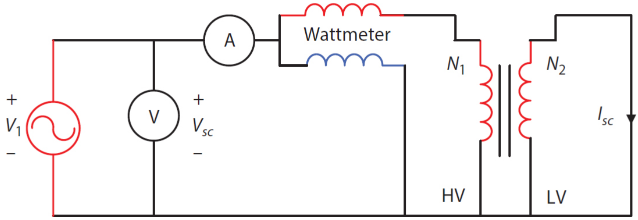


Fig. 3.39 Connection diagram for short circuit test.

A variable voltage is applied to the primary winding of the transformer. The primary voltage is adjusted until the current in the short-circuited winding is equal to the rated primary current. Under this condition, the wattmeter measures the full load copper loss and it can be written as,

$$P_{sc} = I_{sc} V_{sc} \cos\phi_{sc} \tag{3.175}$$

From equation (3.175), the short circuit power factor can be calculated as,

$$\cos\phi_{sc} = \frac{P_{sc}}{I_{sc} V_{sc}} \tag{3.176}$$

The equivalent impedance Z_{01} referred to the HV side is calculated as,

$$Z_{01} = Z_{eq} = \frac{V_{sc}}{I_{sc}} \tag{3.177}$$

However, the equivalent resistance and reactance are calculated as,

$$R_{01} = Z_{01} \cos\phi_{sc} \tag{3.178}$$

$$R_{01} = \frac{P_0}{I_{sc}^{2}} \tag{3.179}$$

$$X_{01} = Z_{01} \sin\phi_{sc} \tag{3.180}$$

$$X_{01} = \sqrt{Z_{01}^{2} - R_{01}^{2}} \tag{3.181}$$

Example 3.9

A 25 kVA, 2200 V/220 V, 50 Hz single-phase transformer's low voltage side is short-circuited and the test data recorded from the high voltage side are $P = 150$ W, $I_1 = 5$A and $V_1 = 40$V. Find the equivalent resistance, reactance and impedance referred to as primary and secondary, and voltage regulation at a unity power factor.

Solution

The parameters referred to as primary are calculated as,

$$Z_{01} = \frac{V_1}{I_1} = \frac{40}{5} = 8\Omega \tag{3.182}$$

$$R_{01} = \frac{P}{I_1^{2}} = \frac{150}{25} = 6\Omega \tag{3.183}$$

$$\phi = \cos^{-1}\left(\frac{6}{8}\right) = 41.4^\circ \tag{3.184}$$

$$X_{01} = Z_{01} \sin\phi = 8 \sin 41.4^\circ = 5.2\Omega \tag{3.185}$$

The turns ratio is calculated as,

$$a = \frac{V_1}{V_2} = \frac{2200}{220} = 10 \tag{3.186}$$

The parameters referred to as secondary are calculated as,

$$Z_{02} = \frac{Z_{01}}{a^2} = \frac{8}{100} = 0.08\,\Omega \tag{3.187}$$

$$R_{02} = \frac{R_{01}}{a^2} = \frac{6}{100} = 0.06\Omega \tag{3.188}$$

$$X_{02} = \frac{X_{01}}{a^2} = \frac{5.2}{100} = 0.052\Omega \tag{3.189}$$

The current in the secondary coil is calculated as,

$$I_2 = \frac{25000}{220} = 113.6\,\text{A} \tag{3.190}$$

The voltage at the secondary coil is calculated as,

$$E_2 = V_2 + I_2 Z_{02} = 220 + 113.6 \times 0.08 = 229\,\text{V} \tag{3.191}$$

The voltage regulation is calculated as,

$$VR = \frac{E_2 - V_2}{V_2} = \frac{229 - 220}{220} = 4\% \tag{3.192}$$

Practice Problem 3.8

The no-load test data of a 220 V/2200 V, 50 Hz single-phase transformer are 220 V, 0.4 A, 75 W. Calculate the no-load circuit resistance and reactance.

Practice Problem 3.9

A 30 kVA, 2200 V/220 V, 50 Hz single-phase transformer's low voltage side is short-circuited by a thick wire. The test data recorded from the high voltage side are $P = 400$W, $I_1 = 8.5$A, $V_1 = 65$V. Find the equivalent resistance, reactance and impedance referred to as primary and equivalent resistance, reactance and impedance referred to as secondary, and voltage regulation at a 0.6

lagging power factor.

3.22 Autotransformer

A small rating transformer with a variable voltage output is usually used in the educational laboratory as well as in the testing laboratory. This type of small rating transformer with variable output is known as an autotransformer. An autotransformer has one continuous winding that is common to both the primary and the secondary. Therefore, in an autotransformer, the primary and secondary windings are connected electrically. The advantages of an autotransformer over a two-winding transformer include lower initial investment, lower leakage reactance, lower losses compared to the conventional transformer and lower excitation current. An autotransformer with primary and secondary windings is shown in Fig. 3.40.

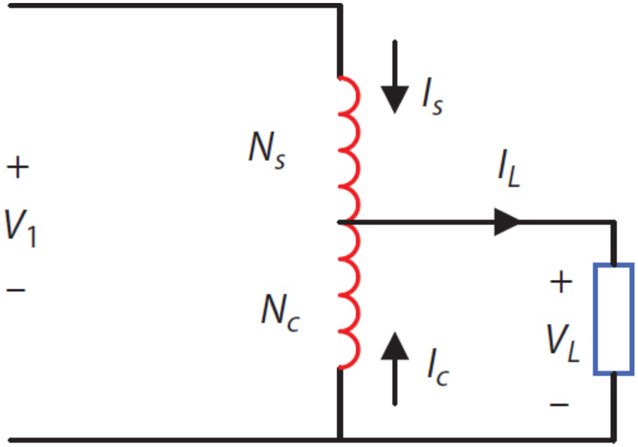


Fig. 3.40 Connection diagram for an autotransformer.

In this connection, the suffix *c* indicates the common winding and the suffix *s* indicates the series winding. From Fig. 3.33, the following equations can be written as,

$$\frac{N_s}{N_c} = \frac{I_c}{I_s} = a \quad (3.193)$$

$$\frac{V_s}{V_c} = \frac{N_s}{N_c} = a \tag{3.194}$$

The total voltage on the primary side is,

$$V_1 = V_s + V_c = V_c\left(1 + \frac{V_s}{V_c}\right) \tag{3.195}$$

Substituting equation (3.194) into equation (3.195) yields,

$$\frac{V_1}{V_c} = (1 + a) \tag{3.196}$$

$$\frac{V_L}{V_1} = \frac{1}{1 + a} \tag{3.197}$$

Where the voltage at the load is equal to the voltage at the common terminals, i.e., $V_L = V_C$.

The expression of the load current can be written as,

$$I_L = I_s + I_c = aI_s + I_s \tag{3.198}$$

$$I_L = (1 + a)I_s \tag{3.199}$$

$$\frac{I_L}{I_s} = (1 + a) \tag{3.200}$$

Example 3.10

Fig. 3.41 shows a single-phase 120 kVA, 2200 V/220 V, 50 Hz transformer which is connected as an autotransformer. The voltages of the upper and lower parts of the coil are 220 V and 2200 V, respectively. Calculate the kVA rating of the autotransformer.

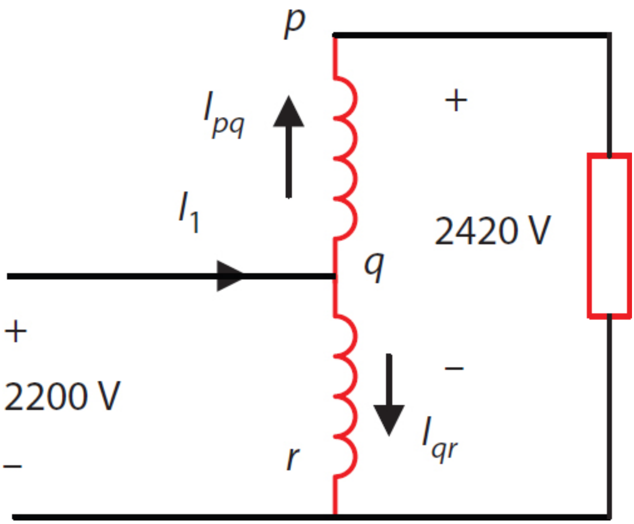


Fig. 3.41 An autotransformer with a specific voltage.

Solution

The current ratings of the respective windings are,

$$I_{pq} = \frac{120{,}000}{220} = 545.5\,\text{A} \tag{3.201}$$

$$I_{qr} = \frac{120{,}000}{2200} = 54.5\,\text{A} \tag{3.202}$$

The current in the primary side is calculated as,

$$I_1 = 545.5 + 54.5 = 600\,\text{A} \tag{3.203}$$

The voltage across the secondary side is calculated as,

$$V_2 = 2200 + 220 = 2420\,\text{V} \tag{3.204}$$

Therefore, the kVA ratings of an autotransformer are calculated as,

$$\text{kVA}_1 = \frac{600 \times 2200}{1000} = 1320\,\text{kVA} \tag{3.205}$$

$$\mathrm{kVA}_2 = \frac{545.5 \times 2420}{1000} = 1320\,\mathrm{kVA} \quad (3.206)$$

Practice Problem 3.10

A single-phase 100 kVA, 1100 V/220 V, 50 Hz transformer is connected as an autotransformer as shown in Fig. 3.42. The voltages of the upper and lower parts of the coil are 220 V and 1100 V, respectively. Determine the kVA rating of the autotransformer.

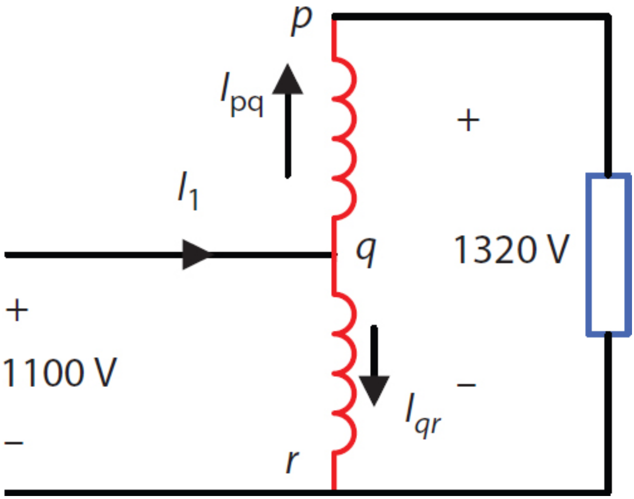


Fig. 3.42 An autotransformer with a specific voltage.

3.23 Parallel Operation of a Single-Phase Transformer

Nowadays, the load demand is increasing with the increase in population and the industrial sector. Sometimes, it is difficult to meet the excess demand for power with the existing single-unit transformer. Therefore, an additional transformer is required to connect in parallel with the existing one. The following points should be considered for making parallel connections of transformers:

- The terminal voltage of both transformers must be the same.

- Polarity must be the same for both transformers.
- For both transformers, the percentage of impedances should be equal in magnitude.
- The ratio of R to X must be the same for both transformers.
- Phase sequences and phase shifts must be the same (for a three-phase transformer).

3.23.1 Equal Voltage Ratio

Transformer T1 and transformer T2 are connected in parallel as shown in Fig. 3.43. The secondary voltage is the same for both the transformers i.e., $E = E_1 = E_2$. The equivalent circuit of two transformers that are connected in parallel can be represented by two impedances in parallel if the no-load circuit is neglected as shown in Fig. 3.44. The total impedance of the circuit is written as,

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} \tag{3.207}$$

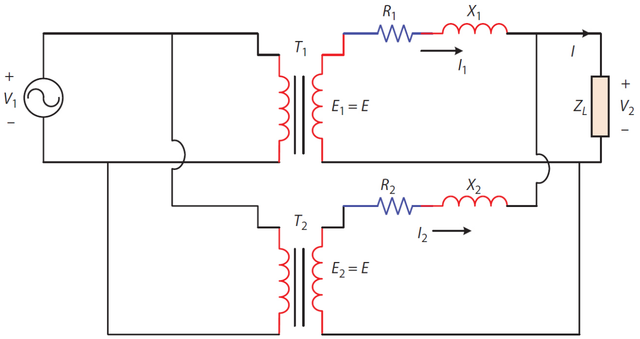


Fig. 3.43 Parallel connection diagram of two transformers with equal voltage ratio.

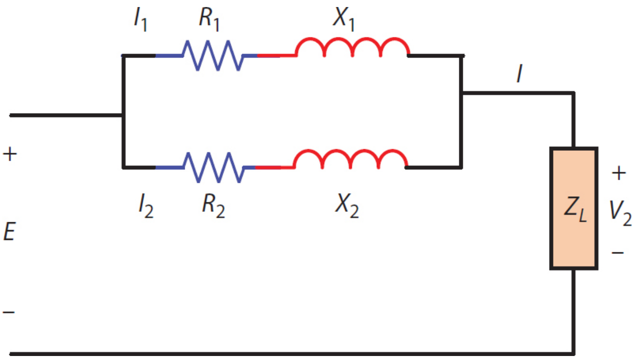


Fig. 3.44 Equivalent circuit.

where $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$. The voltage drop across the parallel circuit is the same and it can be expressed as,

$$IZ = I_1 Z_1 = I_2 Z_2 \tag{3.208}$$

Substituting equation (3.207) into equation (3.208) yields,

$$I_1 Z_1 = IZ = I\frac{Z_1 Z_2}{Z_1 + Z_2} \tag{3.209}$$

$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \tag{3.210}$$

Multiplying both sides of equation (3.210) by V_2 yields,

$$V_2 I_1 = V_2 I\frac{Z_2}{Z_1 + Z_2} \tag{3.211}$$

The load kVA is represented as,

$$S = V_2 I \times 10^{-3} \tag{3.212}$$

The load shared by the first transformer is represented as,

$$S_1 = S\frac{Z_2}{Z_1 + Z_2} \tag{3.213}$$

Again, substituting equation (3.207) into equation (3.208) yields,

$$I_2Z_2 = IZ = I\frac{Z_1Z_2}{Z_1+Z_2} \tag{3.214}$$

$$I_2 = I\frac{Z_1}{Z_1+Z_2} \tag{3.215}$$

Multiplying both sides of equation (3.215) by V_2 yields,

$$V_2I_2 = V_2I\frac{Z_1}{Z_1+Z_2} \tag{3.216}$$

The load shared by the second transformer is represented as,

$$S_2 = S\frac{Z_1}{Z_1+Z_2} \tag{3.217}$$

3.23.2 Unequal Voltage Ratio

An unequal voltage will be induced across the windings if their numbers of turns are not the same. As a result, a circulating current will flow through the windings. This circulating current will create a heating effect with no useful output. Fig. 3.45 shows the connection diagram between two transformers with unequal secondary voltages E_1 and E_2. Apply KVL to the circuit as shown in Fig. 3.45 and the equations are,

$$E_1 = I_1Z_1 + (I_1+I_2)Z_L \tag{3.218}$$

$$E_1 = I_1(Z_1+Z_L) + Z_LI_2 \tag{3.219}$$

$$E_2 = I_1Z_L + (Z_L+Z_2)I_2 \tag{3.220}$$

Multiplying equation (3.219) by $(Z_2 + Z_L)$ and equation (3.219) by Z_L and subtracting yields,

$$E_1(Z_2+Z_L)-Z_LE_2=[(Z_1+Z_L)(Z_2+Z_L)-{Z_L}^2]I_1 \tag{3.221}$$

$$I_1=\frac{E_1(Z_2+Z_L)-Z_LE_2}{Z_1Z_2+Z_2Z_L+Z_1Z_L+{Z_L}^2-{Z_L}^2} \tag{3.222}$$

$$I_1=\frac{E_1Z_2+(E_1-E_2)Z_L}{Z_1Z_2+Z_2Z_L+Z_1Z_L} \tag{3.223}$$

$$E_1Z_L-E_2(Z_1+Z_L)=[{Z_L}^2-(Z_1+Z_L)(Z_2+Z_L)]I_2 \tag{3.224}$$

$$E_2(Z_1+Z_L)-E_1Z_L=[(Z_1+Z_L)(Z_2+Z_L)-{Z_L}^2]I_2 \tag{3.225}$$

$$I_2=\frac{E_2(Z_1+Z_L)-E_1Z_L}{(Z_1+Z_L)(Z_2+Z_L)-{Z_L}^2} \tag{3.226}$$

$$I_2=\frac{E_2Z_1+(E_2-E_1)Z_L}{Z_1Z_2+Z_1Z_L+Z_2Z_L} \tag{3.227}$$

At no load, when E_1 is greater than E_2, the circulating current can be determined as,

$$I_c=\frac{E_1-E_2}{Z_1+Z_2} \tag{3.228}$$

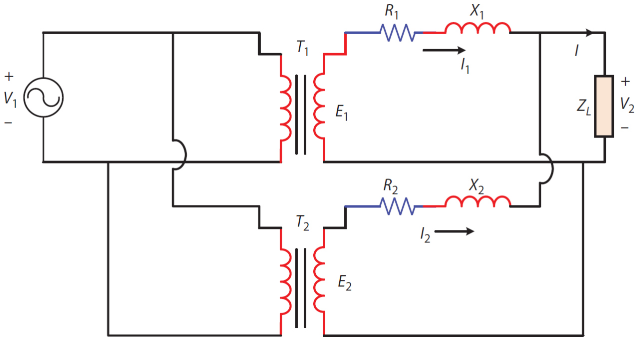


Fig. 3.45 Parallel connection diagram of two transformers with unequal voltage ratio.

Example 3.11

A 100 kVA transformer is connected in parallel with a 200 kVA transformer across the same load. Their secondary induced voltages are 500 V and 450 V, respectively. The percentage impedances of those transformers are 5% and 8%, respectively. Find the circulating current.

Solution

The actual impedance of the first transformer is calculated as,

$$Z_1 = 0.05 \times 500 \times 500 / (100000 / 500) = 0.125 \Omega \tag{3.229}$$

The actual impedance of the second transformer is determined as,

$$Z_2 = 0.08 \times 450 / (100000 / 450) = 0.162 \Omega \tag{3.230}$$

The value of the circulating current can be determined as,

$$I_c = \frac{E_1 - E_2}{Z_1 + Z_2} = \frac{500 - 450}{j(0.125 + 0.162)} = 174.22 \angle -90^\circ \text{ A} \tag{3.231}$$

Practice Problem 3.11

The impedances of the two transformers are $Z_1 = 0.82 + j2.4\ \Omega$ and $Z_2 = 0.75 + j2.1\ \Omega$, respectively. Their secondary induced voltages are 400 V and 350 V respectively. These transformers are connected in parallel across the load $ZL = 10\ \Omega$. Calculate the secondary current in each transformer.

3.24 Three-Phase Transformer Connections

Depending on the load, the primary and secondary windings of the transformer may be connected either by wye (Y) or delta (Δ). Three-phase transformer connections are classified into four possible types; namely, Y-Y (wye-wye), Y-Δ (wye-delta), Δ-Y (delta-wye) and Δ-Δ (delta-delta).

3.24.1 Wye-Wye Connection

Fig. 3.46 shows the Y-Y connection diagram. This type of connection of a three-phase transformer is rarely used for a large amount of power transmission.

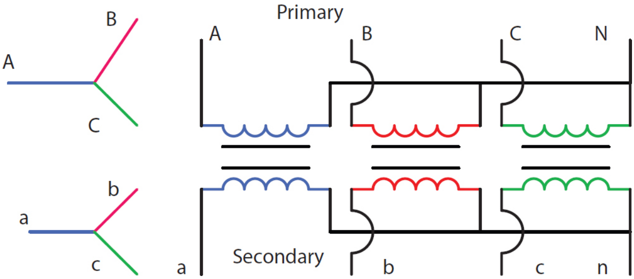


Fig. 3.46 Y-Y connection diagram.

A neutral point is necessary for both the primary and secondary sides in some cases. In balanced loads, this type of connection works satisfactorily and provides neutrality to each side for grounding. On the primary side, the phase voltage can be written as,

$$V_{P1} = \frac{V_{L1}}{\sqrt{3}} \tag{3.232}$$

The secondary phase voltage can be written as,

$$V_{P2} = \frac{V_{L2}}{\sqrt{3}} \tag{3.233}$$

The ratio of the primary line voltage to the secondary line voltage of this connection is,

$$a = \frac{V_{L1}}{V_{L2}} \tag{3.234}$$

$$a = \frac{\sqrt{3}\,V_{P1}}{\sqrt{3}\,V_{P2}} = \frac{V_{P1}}{V_{P2}} \tag{3.235}$$

3.24.2 Wye-Delta Connection

The wye-delta connection is mainly used at the substation where the voltage is stepped down. In this connection, the current in the secondary coil is 57.7% of the load current. At the primary side of this connection, a proper copper wire is used to ground the neutral point. Fig. 3.47 shows the connection diagram of the wye-delta transformer. In this connection, the expression of the primary line voltage is,

$$V_{L1} = \sqrt{3}\, V_{P1} \tag{3.236}$$

On the secondary side, the line voltage is,

$$V_{L2} = V_{P2} \tag{3.237}$$

The ratio of primary phase voltage to secondary phase voltage is,

$$a = \frac{V_{P1}}{V_{P2}} \tag{3.238}$$

The ratio of primary line voltage to secondary line voltage is,

$$\frac{V_{L1}}{V_{L2}} = \frac{\sqrt{3}\, V_{P1}}{V_{P2}} = \sqrt{3}\, a \tag{3.239}$$

The primary phase current is,

$$I_{P1} = I_{L1} \tag{3.240}$$

In this case, the turns ratio is written as,

$$a = \frac{I_{P2}}{I_{P1}} \tag{3.241}$$

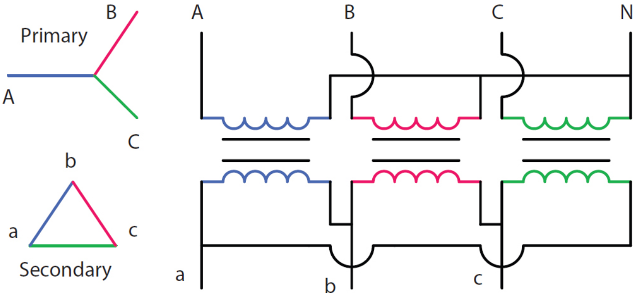

Primary
B
A
C
b
a
c
Secondary
A
B
C
N
a
b
c

Fig. 3.47 Wye-delta connection diagram.

The expression of the secondary phase current is,

$$I_{P2} = aI_{P1} \tag{3.242}$$

The secondary line current is written as,

$$I_{L2} = \sqrt{3}\, I_{P2} = \sqrt{3}\, aI_{P1} \tag{3.243}$$

3.24.3 Delta-Wye Connection

The delta-wye connection is generally used at the power generating station to step the voltage. Fig. 3.48 shows the connection diagram of a delta-wye transformer. In this connection, the expression of the primary line voltage is,

$$V_{L1} = V_{P1} \tag{3.244}$$

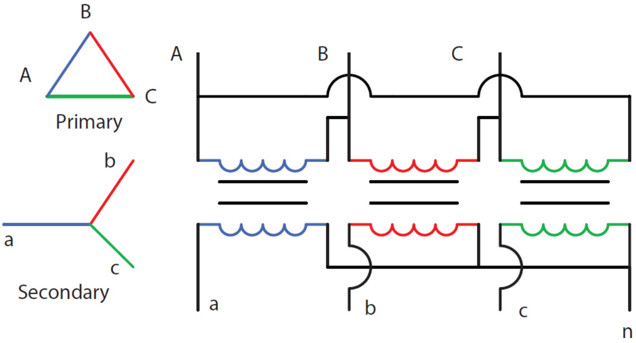

B
A
C
Primary
b
a
c
Secondary
A
B
C
a
b
c
n

Fig. 3.48 Delta-wye connection diagram.

The line voltage at the secondary side is written as,

$$V_{L2} = \sqrt{3}\,V_{P2} \tag{3.245}$$

The ratio of primary line voltage to secondary line voltage is written as,

$$\frac{V_{L1}}{V_{L2}} = \frac{V_{P1}}{\sqrt{3}\,V_{P2}} = \frac{a}{\sqrt{3}} \tag{3.246}$$

The phase current at the primary side is written as,

$$I_{P1} = \frac{1}{\sqrt{3}} I_{L1} \tag{3.247}$$

In this connection, the turns ratio is expressed as,

$$a = \frac{I_{P2}}{I_{P1}} \tag{3.248}$$

In this case, the secondary phase current can be written as,

$$I_{P2} = aI_{P1} \tag{3.249}$$

The secondary line current is expressed as,

$$I_{L2} = I_{P2} = aI_{P1} \tag{3.250}$$

3.24.4 Delta-Delta Connection

The delta-delta connection is generally used for both high-voltage and low-voltage rating transformers where insulation is not an important issue. The connection diagram for the delta-delta transformer is shown in Fig. 3.49.

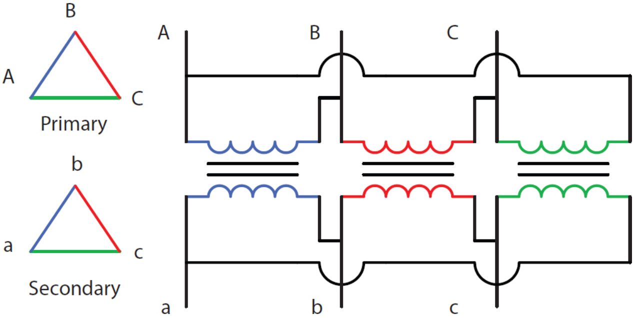


Fig. 3.49 Delta-delta connection diagram.

In this case, the primary line voltage can be written as,

$$V_{L1} = V_{P1} \tag{3.251}$$

The secondary line voltage is expressed as,

$$V_{L2} = V_{P2} \tag{3.252}$$

The ratio of line-to-line voltage of this connection is,

$$\frac{V_{L1}}{V_{L2}} = \frac{V_{P1}}{V_{P2}} = a \tag{3.253}$$

The secondary line current is expressed as,

$$I_{L2} = \sqrt{3}\, I_{P2} \tag{3.254}$$

The output capacity in a delta-delta connection can be expressed as,

$$P_{0\Delta-\Delta} = \sqrt{3}\, V_{L2} I_{L2} \tag{3.255}$$

Substituting equation (3.254) into equation (3.255) yields,

$$P_{0\Delta-\Delta} = \sqrt{3}\, V_{L2} \times \sqrt{3}\, I_{P2} = 3V_{L2} I_{P2} \tag{3.256}$$

The remaining two transformers are able to supply three-phase power to the load terminal if one of the transformers is removed from the connection. This type of three-phase power supply by two transformers is known as an open delta or V-V connection. This open delta connection is able to supply three-phase power at a reduced rate of 57.7%. The connection diagram for an open delta connection is shown in Fig. 3.50.

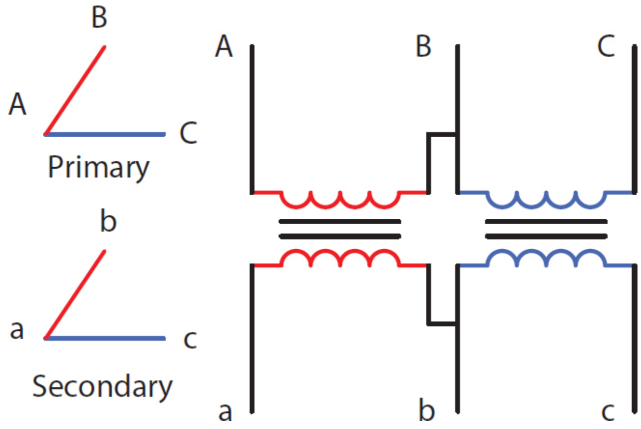


Fig. 3.50 Open delta or V-V connection diagram.

In the V-V connection, the secondary line current is,

$$I_{L2} = I_{P2} \tag{3.257}$$

The output capacity in a V-V connection is expressed as,

$$P_{0\text{V-V}} = \sqrt{3}\, V_{L2} I_{P2} \tag{3.258}$$

The ratio of delta-delta output capacity to the V-V output capacity is,

$$\frac{P_{0\text{V-V}}}{P_{0\Delta-\Delta}} = \frac{\sqrt{3}\, V_{L2} I_{P2}}{3 V_{L2} I_{P2}} = 0.577 \tag{3.259}$$

Example 3.12

A three-phase transformer is connected to an 11 kV supply and draws a 6 A current. Determine the line voltage at the secondary side, and the line current in the secondary coil. Consider the turns ratio of the transformer is 11. Consider delta-wye and wye-delta connections.

Solution

Delta-wye connection:

The phase voltage at the primary side is,

$$V_{P1} = V_{L1} = 11\,\text{kV} \tag{3.260}$$

The phase voltage at the secondary side is calculated as,

$$V_{P2} = \frac{V_{P1}}{a} = \frac{11000}{11} = 1000\,\text{V} \tag{3.261}$$

The line voltage at the secondary is determined as,

$$V_{L2} = \sqrt{3}\,V_{P2} = \sqrt{3} \times 1000 = 1732\,\text{V} \tag{3.262}$$

The phase current in the primary is calculated as,

$$I_{P1} = \frac{I_{L1}}{\sqrt{3}} = \frac{11}{\sqrt{3}} = 6.35\,\text{A} \tag{3.263}$$

The line current in the secondary is calculated as,

$$I_{L2} = I_{P2} = aI_{P1} = 11 \times 6.35 = 69.85\,\text{A} \tag{3.264}$$

Wye-delta connection:

The line voltage at the primary side is,

$$V_{L1} = 11\,\text{kV} \tag{3.265}$$

The phase voltage at the primary side is calculated as,

$$V_{P1} = \frac{V_{L1}}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6350.85\,\text{V} \tag{3.266}$$

The phase voltage at the secondary side is determined as,

$$V_{P2} = \frac{V_{P1}}{a} = \frac{6350.85}{11} = 577.35\,\text{V} \tag{3.267}$$

The line voltage at the secondary is,

$$V_{L2} = V_{P2} = 577.35\,\text{V} \tag{3.268}$$

The phase current in the secondary is calculated as,

$$I_{P2} = aI_{P1} = 11 \times 6 = 66\,\text{A} \tag{3.269}$$

The line current in the secondary is calculated as,

$$I_{L2} = \sqrt{3}\,I_{P2} = \sqrt{3} \times 66 = 114.32\,\text{A} \tag{3.270}$$

Practice Problem 3.12

A three-phase transformer is connected to the 33 kV supply and draws a 15 A current. Calculate the line voltage at the secondary side and the line current in the secondary winding. Consider the turns ratio of the transformer is 8 and the wye-delta connection.

3.25 Instrument Transformers

The magnitude of the voltage and the current are normally high in the power system networks. To reduce the magnitude of the voltage and current, instrument transformers are used. There are two types of instrument transformers; namely, a current transformer (CT) and a potential transformer (PT). If a power system carries an alternating current greater than 100 A, then it is difficult to connect measuring instruments with a lower-range ammeter, wattmeter, energy meter and relay. The current transformer is then connected in series with the line to step down the high magnitude of the current to a rated value of about 5 A for the ammeter and the current coil of the wattmeter. The diagram of a current transformer is shown in Fig. 3.51.

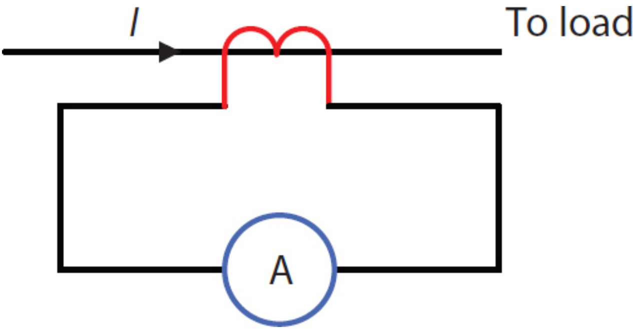


Fig. 3.51 Connection diagram of the current transformer.

If the system voltage exceeds 500 V, then it is difficult to connect measuring instruments directly to the high voltage. The potential transformer is then used to step down to a suitable value of the voltage at the secondary for supplying the voltmeter and the voltage coil of the wattmeter. The secondary of the instrument transformer is normally grounded for safety reasons. The connection diagram of a potential transformer is shown in Fig. 3.52.

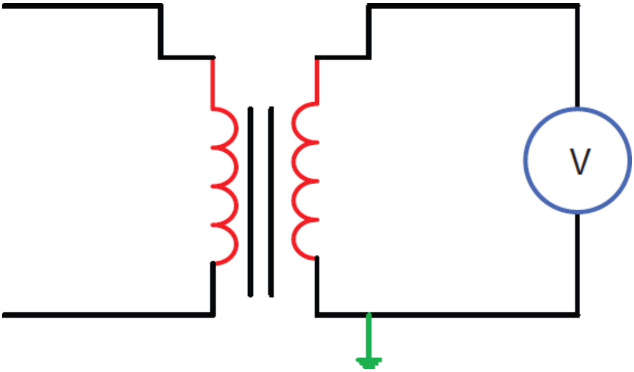


Fig. 3.52 Connection diagram of the potential transformer.

3.26 Transformer Oil Testing and Cooling

Power and distribution transformers are considered to be the most important components for power utility companies in terms of cost and reliability in service. Insulating oil such as Pyranol is used as insulation for the interwinding of a three-phase transformer. Depending on the usage and life cycle of transformers, transformer oil needs to be tested according to various standards such as IEEE Std 62-1995, IEC 60156, ASTM D1816, etc. The percentage saturation of water in oil is shown in Table 3.1. There are a few equipments available to test the electrical breakdown strength of insulating oils. Most of the equipment has a high precision oil breakdown voltage test capacity (0-75 kV ± 1kV) with a very accurate measurement principle performed directly on the HV unit.

Table 3.1 Representation of insulation levels (IEEE Std 62-1995).

Insulation condition	**Saturation of water in oil (%)**
Dry insulation	0-5
Moderate-wet	6-20
Wet insulation	21-30
Extremely wet insulation	>30

Heat is produced inside the transformer by different types of losses. These are small but undesirable for the smooth and reliable operation of the transformer. Therefore, a special cooling system is needed to be provided to reduce the temperature. Transformer cooling is represented by the following symbols:

AA- dry-type self-cooled, AFA- dry-type forced air-cooled, OA- oil immersed self-cooled, OA/FA- oil immersed self-cooled/forced air-cooled, OA/FA/FOA- oil immersed self-cooled/ forced air-cooled/forced oil forced air-cooled.

Natural air blast cooling: Indoor transformers below 200 kVA can be directly cooled by the natural flow of surrounding air. In this method, the transformer core and windings are encased in a metal enclosure through which a blower circulates air.

Self-air cooling: In this method, transformers are normally cooled by surrounding air at atmospheric pressure. The generated heat is removed by natural convection. The self-air-cooling transformer is called the dry type transformer.

Oil-immersed self-cooling: In this type of cooling method, the core and the windings are immersed in the oil-containing tank. The oil gives up its heat to the inner surface of the tank. This heat is then radiated to the outside air.

Oil-immersed forced air-cooling: In this method, the core and the windings are immersed in the oil. The cooling is done by forced air over the oil surface.

Oil-immersed water-cooling: This type of cooling is used where plenty of supply of cool water is available. Here, an extra pipeline is installed near the top of the tank containing the cooling oil. The water is circulated through this pipe and carries away the heat from the oil.

3.27 All-Day Efficiency of Transformer

All-day efficiency is usually taken into account in the case of the distribution transformer. The distribution transformer delivers power to the loads. The loads are classified as residential, commercial and industrial. These loads vary with time such as 24 hours a day. Therefore, all-day efficiency is defined as the ratio of output energy delivered by the transformer to the input energy for 24 hours a day. It is written as,

$$\eta_{\text{All-day}} = \frac{\text{output energy in 24hrs}}{\text{input energy in 24hrs}} = \frac{\text{output energy kWh}}{\text{input energy kWh}} \quad (3.271)$$

The distribution transformers are permanently connected to the electric lines. Sometimes, the secondary sides of those transformers supply full-load or no-load depending on the consumer uses of electrical appliances. Therefore, the core loss occurs throughout the day, whereas the copper losses occur when the transformer is loaded. Hence, all-day efficiency is very important for a distribution transformer and is always less than the normal efficiency. The output kVA of a transformer can be determined as,

$$\text{output kVA} = \frac{\text{output kW}}{pf} \quad (3.272)$$

The copper loss can be determined as,

$$P_{cu1}=\left(\frac{\text{output kVA}}{\text{Rated kVA}}\right)^2\times P_{cu} \tag{3.273}$$

where P_{cu} is the rated copper loss.

Example 3.13

The maximum efficiency of a 12 kVA transformer at a unity power factor is 97%. During the whole day, the transformer is loaded as follows:

i. 8 hours, 10 kW at 0.8 lagging power factor, (ii) 10 hours, 15 kW at 0.90 leading power factor, and (iii) 6 hours at no-load. Calculate all-day efficiency.

Solution

The output power at full load and unity power factor is calculated as,

$$P_{01}=12\times 1=12\,\text{kW} \tag{3.274}$$

The input power at full load is calculated as,

$$P_{in1}=\frac{12}{0.97}=12.371\,\text{kW} \tag{3.275}$$

The total loss is determined as,

$$P_{tloss}=P_{in1}-P_{01}=12.371-12=0.371\,\text{kW} \tag{3.276}$$

The copper loss at 12 kVA is calculated as,

$$P_{cu}=P_i=\frac{0.371}{2}=0.185\,\text{kW} \tag{3.277}$$

The all-day output energy is determined as,

$$P_{024}=10\times 8+15\times 10+6\times 0=230\,\text{kWh} \tag{3.278}$$

The iron loss for 24 hours is calculated as,

$$P_{i24} = 24 \times 0.185 = 4.44\,\text{kWh} \tag{3.279}$$

The copper loss for 24 hours is calculated as,

$$P_{cu24} = 0.185 \times 8 \times \left(\frac{\frac{10}{0.85}}{12} \right)^2 + 0.185 \times 10 \times \left(\frac{\frac{15}{0.9}}{12} \right)^2 + 0 = 5\,\text{kW} \tag{3.280}$$

The all-day input power is determined as,

$$P_{in24} = P_{024} + P_{i24} + P_{cu24} = 230 + 4.44 + 5 = 239.44\,\text{kWh} \tag{3.281}$$

The all-day efficiency is calculated as,

$$\eta_{all-day} = \frac{P_{024}}{P_{in24}} \times 100 = \frac{230}{239.44} \times 100 = 96\% \tag{3.282}$$

Practice Problem 3.13

The maximum efficiency of a 15 kVA transformer at a unity power factor is 98%. During the whole day, the transformer is loaded as follows:

i. 5 hours, 9 kW at 0.85 lagging power factor, (ii) 8 hours, 10 kW at 0.95 leading power factor, and (iii) 11 hours at no-load. Determine all-day efficiency.

Exercise Problems

3.1 A single-phase transformer has a primary voltage of 240 V. The numbers of turns at the primary and secondary coils are 250 and 50, respectively. Determine the secondary voltage.

3.2 A single-phase transformer has a secondary voltage of 100 V. The number of turns at the primary and secondary windings is 500 and 50, respectively. Calculate the primary voltage.

3.3 The 100 A current flows in the primary coil of a single-phase transformer. Calculate the secondary current with a turns ratio of 0.05.

3.4 The number of turns at the primary and secondary windings of a single-phase transformer is 500 and 100, respectively. Find the primary current if the secondary current is 10 A.

3.5 The ratio of the primary current to the secondary current of a single-phase transformer is 1:10. Find the voltage ratio. Also, calculate the secondary voltage if the primary voltage is found to be 1000 V.

3.6 A single-phase transformer has a turns ratio of 1:4. The secondary coil has 5000 turns, and the voltage is found to be 60 V. Determine the voltage ratio, primary voltage and number of turns.

3.7 A single-phase transformer has a voltage ratio of 5:1. The voltage and number of turns at the primary coil are found to be 1100 V and 500, respectively. Calculate the voltage and a secondary number of turns at the secondary coil.

3.8 A single-phase transformer is connected to the 120 V source and draws a 4 A current. Calculate the current in the secondary coil when the transformer steps up the voltage to 500 V.

3.9 The number of turns at the primary and secondary coils of a single-phase transformer is found to be 480 and 60, respectively. The transformer draws a current of 0.6 A when connected to a 120 V supply. Determine the current and the voltage at the secondary coil.

3.10 The turns ratio of a 5 kVA single-phase transformer is found to be 3. The transformer is connected to a 230 V, 50 Hz supply. If the secondary current of a transformer is 6 A, find the primary current and secondary voltage.

3.11 The secondary number of turns of a 30 kVA, 4400 V/440 V single-phase transformer is found to be 100. Find the number of primary turns and full load primary and secondary currents.

3.12 A single-phase transformer of 5 kVA, 1100 V/230 V, 50

Hz is installed near the domestic area of a country. Find the turns ratio and primary and secondary full load currents.

3.13 The number of turns of the primary winding of a single-phase 50 Hz transformer is found to be 50. Find the value of the core flux, if the induced voltage at this winding is 220 V.

3.14 The number of primary turns of a 60 Hz single-phase transformer is found to be 50. Calculate the induced voltage in this winding, if the value of the core flux is 0.05 Wb.

3.15 A 3300 V/330 V, 50 Hz step-down single-phase transformer has a maximum flux of 0.45 Wb. Calculate the number of primary and secondary turns.

3.16 A 5 kVA, 2200 V/220 V, 50 Hz single-phase step-down transformer has a cross-sectional area is 25 cm^2, and a maximum flux density is 4 Wb/m^3. Calculate the primary and secondary turns.

3.17 The number of turns at the primary and secondary of an ideal single-phase transformer is found to be 500 and 250, respectively. The primary of the transformer is connected to a 220 V, 50 Hz source and the secondary coil supplies a current of 5 A to the load. Find the turns ratio, current in the primary and maximum flux in the core.

3.18 A 4 kVA, 1100 V/230 V, 50 Hz single-phase transformer has the primary number of turns of 500. The cross-sectional area of the transformer core is 85 cm^2. Find the turns ratio, number of turns in the secondary and maximum flux density in the core.

3.19 A single-phase transformer has primary and secondary turns of 50 and 500, respectively. The primary winding is connected to a 220 V, 50 Hz supply. Calculate the flux density in the core if the cross-sectional area is 250 cm^2 and the induced voltage at the secondary winding.

3.20 The primary coil number of turns of a single-phase 2200 V/220 V, 50 Hz transformer is found to be 1000. Find the area of the core if the maximum flux density of the core is 1.5 Wb/m^2.

3.21 The maximum flux of a single-phase 1100 V/220 V, 50

Hz transformer is found to be 5 mWb. The number of turns and the voltage at the primary winding are found to be 900 and 1100 V, respectively. Determine the frequency of the supply system.

3.22 A 6 kVA, 2200 V/220 V, 50 Hz single-phase transformer has a maximum flux density of 0.6 Wb/m^2 and the induced voltage per turn of the transformer is 10 V. Calculate the primary number of turns, secondary number of turns, and cross-sectional area of the core.

3.23 A single-phase transformer has a primary voltage per turn is 20 V. Find the magnetic flux density if the cross-sectional area of the core is 0.05 m^2.

3.24 A 1100 V/230 V single-phase transformer has a no-load primary current of 0.5 A and absorbs the power of 350 W from the circuit. Calculate the iron loss and magnetizing components of the no-load current.

3.25 A 3300 V/230 V single-phase transformer draws a no-load current of 0.5 A at a power factor of 0.4 in an open circuit test. Determine the working and magnetizing components of the no-load current.

3.26 A single-phase transformer draws a no-load current of 1.4 A from the 120 V source and absorbs the power of 80 W. The primary winding resistance and leakage reactance are found to be 0.25Ω and 1.2Ω, respectively. Find the no-load circuit resistance and reactance.

3.27 Under the no-load condition, a single-phase transformer is connected to a 240 V, 50 Hz supply and draws a current of 0.3 A at a 0.5 power factor. The number of turns on the primary winding is found to be 600. Calculate the maximum value of the flux in the core, magnetizing and working components of the no-load current, and iron loss.

3.28 The no-load current of a 440 V/110 V single-phase transformer is measured at 0.8 A and absorbs the power of 255 W. Calculate the working and magnetizing components of the no-load current, copper loss in the primary winding if the primary winding resistance is 1.1Ω, and core loss.

3.29 A single-phase 1000V/200V transformer draws a no-load current of 0.7A at 0.4 power factor lagging and the secondary winding supplies a current of 85A at 0.5 power factor lagging. Find the primary current.

3.30 A single-phase 600V/200V transformer supplies a no-load current of 45A at 0.6 power factor lagging to the load, and the transformer draws a no-load current of 3A at 0.4 power factor lagging. Calculate the primary current and its power factor.

3.31 The primary winding resistance and reactance of a 300 kVA, 2200 V/220 V single-phase transformer are 2Ω and 9Ω, respectively. The secondary winding resistance and reactance are found to be 0.02Ω and 0.1Ω, respectively. Determine the equivalent impedance referred to the primary and the secondary.

3.32 The primary winding resistance and reactance of a 200 kVA, 3300 V/220 V single-phase transformer are found to be 5Ω and 12Ω, respectively. The same parameters in the secondary winding are found to be 0.02Ω and 0.05Ω, respectively. Find the equivalent impedance referred to as the primary and the secondary. Also, determine the full load copper loss.

3.33 A 1100 V/400 V single-phase transformer has a resistance of 5Ω and a reactance of 9Ω in the primary winding. The secondary winding resistance and reactance are found to be 0.6Ω and 1.1Ω, respectively. Find the voltage regulation when the secondary delivers a current of 5 A at a 0.9 lagging power factor.

3.34 A 2200 V/220 V single-phase transformer has a resistance of 6Ω and a reactance of 16Ω in the primary coil. The resistance and the reactance in secondary winding are found to be 0.07Ω and 0.05Ω, respectively. Find the voltage regulation when the secondary supplies a current of 6 A at a 0.86 power factor leading.

3.35 A 40 kVA single-phase transformer has an iron loss, and full load copper loss is 450 W and 750 W, respectively. Calculate the full load efficiency, output kVA corresponding to maximum efficiency, and maximum efficiency. Consider the power factor of the load is 0.95 lagging.

3.36 A 40 kVA single-phase transformer has an iron loss of 50% of the full load copper loss. The full load copper loss is found to be 850 W. Find the full load efficiency, output kVA corresponding to maximum efficiency and maximum efficiency. Assume the power factor of the load is 0.9 lagging.

3.37 A 25 kVA single-phase transformer has an iron loss and full load copper loss of 300 W and 550 W, respectively. Find the full load efficiency, output kVA corresponding to maximum efficiency, and maximum efficiency. Assume the power factor of the load is 0.6 lagging.

3.38 A 100 kVA, 2200 V/220 V single-phase transformer has the primary and secondary windings parameters of $R_1 = 0.6\Omega$, $X_1 = 0.9\Omega$, $R_2 = 0.007\Omega$, $X_2 = 0.008\Omega$. The transformer is operating at a maximum efficiency of 75% of its rated load with a 0.9 lagging power factor. Calculate the efficiency of the transformer at full load, and maximum efficiency, if the iron loss is 350 W.

3.39 A 200 kVA, 2400 V/240 V single-phase transformer has the primary and windings parameters $R_1 = 20\Omega$, $X_1 = 27\Omega$, $R_2 = 0.18\Omega$, $X_2 = 0.26\Omega$. The transformer is operating at a maximum efficiency of 80% of its rated load with 0.8 pf lagging. Find the efficiency of the transformer at full load, and maximum efficiency if the iron loss is 400 W.

3.40 The low voltage side of a 30 kVA, 2400 V/220 V, 50 Hz single-phase transformer is short-circuited. The test data recorded from the high voltage side are 200 W, 6 A and 45 V. Find the equivalent resistance, reactance and impedance referred to the primary, equivalent resistance, reactance and impedance referred to the secondary, and voltage regulation at 0.6 power factor lagging.

3.41 The low voltage winding of a 25 kVA, 1100 V/220 V, 50 Hz single-phase transformer is short-circuited, and the test data recorded from the high voltage side are 156 W, 4 A and 50 V. Calculate the equivalent resistance, reactance and impedance referred to the primary, equivalent resistance, reactance and impedance referred to the secondary, and voltage regulation at 0.85 power factor leading.

3.42 The test data of a 10 kVA, 440 V/220 V, 50 Hz single-phase transformer is as follows:

Open circuit test: 220 V, 1.4 A, 155 W;
Short circuit test: 20.5 V, 15 A, 145 W;
Calculate the approximate equivalent circuit parameters.

3.43 The open-circuit test data of a single-phase transformer are recorded as 220 V, 1.2 A and 145 W. Find the no-load circuit parameters.

3.44 A single-phase 100 kVA, 1100 V/220 V transformer is connected as an autotransformer, which is shown in Fig. P3.1. The voltage of the upper portion and the lower portion of the coil is found to be 220 V and 1100 V, respectively. Calculate the kVA rating of the autotransformer.

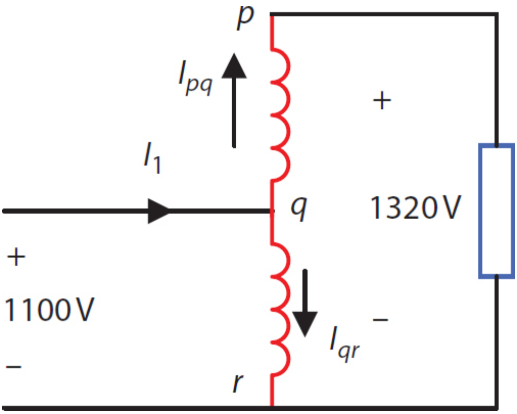


Fig. P3.1 A autotransformer circuit for problem 3.44.

3.45 Two transformers of 100 kVA and 150 kVA are connected in parallel. The impedances of the transformers are $Z_1 = 1.5 + j3.5\Omega$ and $Z_2 = 2 + j4.5\Omega$, respectively. Determine load shared by each transformer when the total load is100 kW at a 0.75 power factor lagging.

3.46 Two transformers of each voltage rating are 1100 V and 220 V respectively. They are connected in parallel to share a load of 100 kVA at 0.6 power factor lagging. The ratings of the transformers are as follows:

Transformer A: 150 kVA, 1% resistance and 9% reactance;
Transformer B: 50 kVA, 1.5% resistance and 6% reactance;
Determine the load shared by each transformer.

3.47 A 400 kVA transformer with a resistance of 2% and a reactance of 5% is connected in parallel with a 200 kVA transformer with a resistance of 1.6% and a reactance of 3%. Calculate how much they will share the load of 600 kVA at 0.9 power factor lagging.

3.48 A 200 kVA three-phase, 50 Hz core-type transformer is connected as delta-wye and has a line voltage ratio of 1100 V/440 V. The core area and the maximum flux density of the transformer is found to be 0.04 m^2 and 3.3 Wb/m^2, respectively. Calculate the number of turns per phase of the primary and secondary coils.

3.49 The number of turns in the primary and secondary coils is found to be 600 and 150, respectively. The transformer is connected to an 11 kV, 50 Hz supply. Find the secondary line voltage when the windings are connected as (a) delta-wye, and (b) wye-delta.

3.50 A 200 kVA, three-phase, 50 Hz delta-wye connection transformer has a line voltage ratio of 33 kV/11 kV and an iron loss of 2850 W. The resistances per phase in the primary and secondary windings are 15Ω and 0.65Ω respectively. Calculate the efficiency of the transformer at (i) full load with a 0.85 power factor lagging, and (ii) 3/4 of full load with a unity power factor.

3.51 The iron loss of a 100 kVA, three-phase, star-star, 11000 V/415 V, 50 Hz transformer is 1600 W. The maximum efficiency of the transformer is occurred at 1/2 of the full load. Determine the (i) efficiency at full load with a 0.90 power factor lagging, and (ii) maximum efficiency at unity power factor.

3.52 The iron loss and full load copper loss of a 10 kVA transformer are 20 W and 40 W, respectively. It operates in the following ways:

i. rated kVA, 0.85 power factor lagging for 8 hours.

ii. 0.75 of rated kVA, 0.75 power factor lagging for 12 hours.
iii. no-load for 4 hours.

Determine the all-day efficiency of the transformer.

4

Direct Current Generators

4.1 Introduction

In our daily life, we use energy in different forms such as electrical and mechanical. Energy is transformed from one form to another. At the power station, electrical energy is generated in the form of alternating current and is transmitted to the consumer terminals with a specific magnitude. There are many applications of electrical energy in the modern world. Electrical energy is used to run all kinds of electrical appliances, such as televisions, DVD players, refrigerators, washing machines, hair dryers, etc. In the industrial sectors, electrical energy is converted into mechanical energy by processes such as electromechanical energy conversion. Later on, this energy is used to run mechanical machines. Depending on the application, energy conversion is performed through the motor and generator actions. Energy converted from the electrical form to the mechanical form is known as motor action, whereas the energy converted from the mechanical form to the electrical form is known as generator action. This process of conversion is shown in Fig. 4.1.

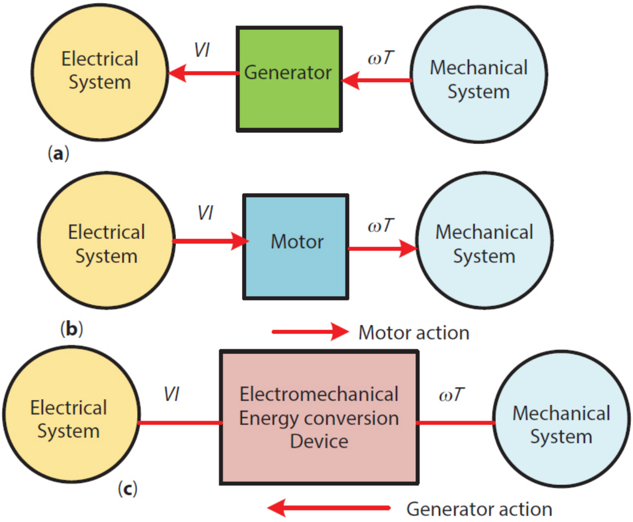

Electrical System
VI
Generator
ωT
Mechanical System
(a)
Electrical System
VI
Motor
ωT
Mechanical System
(b)
Motor action
Electrical System
VI
Electromechanical Energy conversion Device
ωT
Mechanical System
(c)
Generator action

Fig. 4.1 Motor-generator action.

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power). A motor is an electrical machine which converts electrical energy into mechanical energy. Electrical machines are classified as homopolar and heteropolar. In homopolar machines, the induced voltage is low, whereas in the heteropolar machines, the induced voltage is high due to the back connection of the conductor. Most of the DC machines used fall under this category.

4.2 Electromagnetic Induction

In 1831, English scientist Michael Faraday discovered two laws. According to his name, these laws are known as Faraday's laws. The first law can be stated as "whenever a current-carrying conductor is placed in a varying magnetic field, an emf is induced in that conductor". Varying of the magnetic field is obtained by moving the magnetic towards or away from the conductor, by changing the area of the conductor and by changing the rotation of the conductor regarding the magnetic field.

The second law can be stated as "the magnitude of the induced voltage (emf) is equal to the rate of change of flux linkages." This phenomenon is known as electromagnetic induction. Most electrical devices such as generators, transformers and induction machines are based on this principle. Consider a coil with N number of turns. The flux changes from the initial value of ϕ_1 webers to the final value of ϕ_2 webers in time t seconds. Then the expressions of flux linkages can be written in the following ways.

The initial flux linkage is expressed as,

$$\psi_1 = N\phi_1 \tag{4.1}$$

Then the final flux linkage is expressed as,

$$\psi_2 = N\phi_2 \tag{4.2}$$

The change in flux linkage is written as,

$$\psi = \psi_2 - \psi_1 \tag{4.3}$$

The expression of induced voltage is expressed as,

$$e = N\frac{\phi_2 - \phi_1}{t} \tag{4.4}$$

The expression of the induced voltage in differential form is,

$$e = v_{ind} = N\frac{d\phi}{dt} \tag{4.5}$$

where,

v_{ind} is the induced voltage in the conductor in V,

N is the number of turns of the conductor,

$d\phi$ is the magnetic flux cut by the conductor in Wb,

dt is the time required for the conductor to move in the field in sec,

$d\phi/dt$ is the rate of change of flux cut by the conductor.

Example 4.1

The flux in the core is found to be $\phi = 0.05 \sin 314t$ Wb. Determine the induced voltage at the coil terminals if it is wound by 100 turns.

Solution

The induced voltage at the coil terminals is calculated as,

$$e = N\frac{d\phi}{dt} = 100\frac{d(0.05\sin 314t)}{dt} = 1570\cos 314t \text{ V} \tag{4.6}$$

Practice Problem 4.1

An iron core is wound by 50 turns and the flux in the core is found to be $\phi = 0.004 \sin 377t$ Wb. Find the induced voltage at the coil terminals.

4.3 Motional Voltage

The motional emf or voltage is used to measure different electrical quantities when the current-carrying conductor moves from its initial position to the final position in the magnetic field. The initial and final positions of the conductor are shown in Fig. 4.2. A small distance of ds separates the initial and final position of the conductor. The conductor moves at a velocity of v from its initial position to the right in the magnetic field. The magnetic flux can be expressed as,

$$\phi = BA \tag{4.7}$$

Differentiating both sides of equation (4.7) with respect to time t yields,

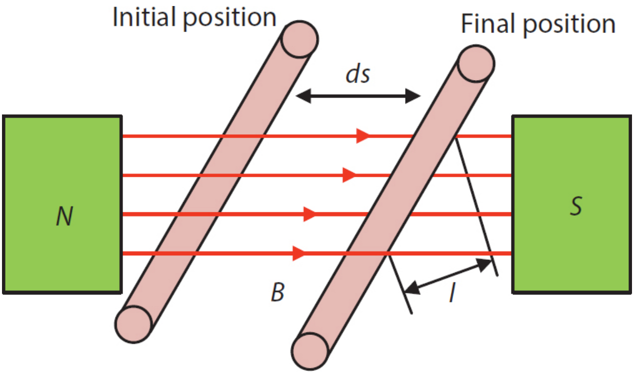


Fig. 4.2 Conductor in a magnetic field.

$$\frac{d\phi}{dt} = B\frac{dA}{dt} \tag{4.8}$$

According to Faraday's law, the magnitude of the induced voltage is expressed as,

$$e = N\frac{d\phi}{dt} \tag{4.9}$$

Substitute equation (4.8) into equation (4.9) yields,

$$e = BN\frac{dA}{dt} \tag{4.10}$$

However, the incremental area *dA* can be expressed as,

$$dA = l\,ds \tag{4.11}$$

Substituting equation (4.11) into equation (4.10) yields,

$$e = NBl\frac{ds}{dt} \tag{4.12}$$

The velocity *v* is defined as,

$$v = \frac{ds}{dt} \tag{4.13}$$

Substituting equation (4.13) into equation (4.12) yields,

$$(4.14)e = NBlv$$

However, for a single loop coil, $N = 1$, the final expression of motional voltage is expressed as,

$$(4.15)e = Blv$$

where,

e is the induced voltage in V,

B is the magnetic flux density in Wb/m^2,

l is the active length of the conductor in m,

v is the speed of the conductor in m/s.

Let the conductor move at an angle of θ through the magnetic field as shown in Fig. 4.3. Then the motional voltage can be expressed as,

$$(4.16)e = Blv \sin\theta$$

From equation (4.16), it is seen that the induced voltage depends on the magnetic flux density, active length and speed of the conductor. If the length of the conductor is increased, the number of lines of the flux cut will be greater and the induced voltage will be more. Similarly, the induced voltage will be small for a smaller length of the conductor. It is also seen that if the conductor moves faster, then the lines of flux cut by the conductor will be increased. As a result, the induced voltage will be higher.

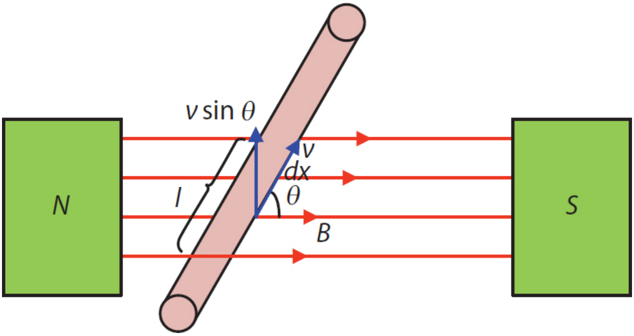


Fig. 4.3 Rotating conductor with an angle.

Example 4.2

A 0.65 m conductor moves in a magnetic field with a speed of 35 m/s. If the magnetic flux density of the field is 0.8 Tesla, find the induced voltage at the conductor terminals.

Solution

The induced voltage at the conductor is,

$$e = Blv = 0.8 \times 0.65 \times 35 = 18.2\,\mathrm{V} \tag{4.17}$$

Example 4.3

A 1.5 m conductor moves in the magnetic field with a velocity of 20 m/s and an angle of 35°. Calculate the induced voltage if the magnetic flux density is 0.9 $\mathrm{Wb/m^2}$.

Solution

The induced voltage is,

$$e = Blv\sin\theta = 0.9 \times 1.5 \times 20 \times \sin 35^\circ = 15.5\,\mathrm{V} \tag{4.18}$$

Practice Problem 4.2

A conductor of 16 m in length moves in a magnetic field with a speed of 69 m/s which induces a voltage of 100 V. Find the magnetic flux density.

Practice Problem 4.3

A voltage of 200 V is induced at the 16 m length conductor when moves with a velocity of 25 m/s and an angle of 65º. Calculate the magnetic flux density.

4.4 Lenz's Law

In 1834, Russian physicist Heinrich Emil Lenz (1804-1865) formulated a law that relates the definite direction of induced voltage and current when the flux is changing. It is known as

Lenz's law, and it is all about the conservation of energy. According to Faraday's laws, the voltage is induced if the conductor moves in the magnetic field and cuts the flux lines. As a result of this induced voltage, current flows through the conductor if the circuit is closed. The magnetic field produced by this current always opposes the motion of the conductor. This is known as Lenz's law, which is explained in Fig. 4.4.

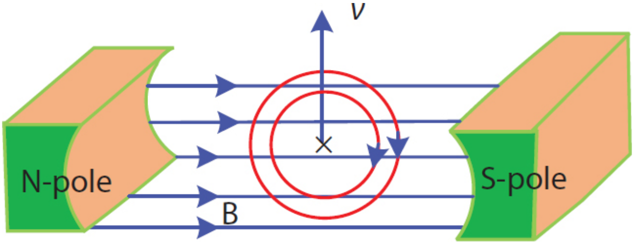


Fig. 4.4 Conductor with the current.

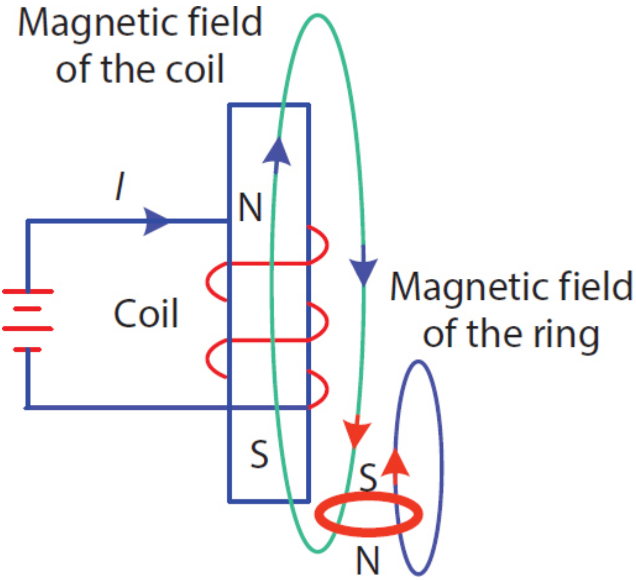


Fig. 4.5 Coil with a ring metal.

A coil is wound with a core and carries a current *I* that creates the magnetic field as shown in Fig. 4.5. The change in magnetic field is due to the change in current in the coil that induces a current in the metal ring. As a result of this current, a magnetic field is created in the ring metal. A magnetic field is due to the metal ring acting opposite to the magnetic field of the coil, which is known as Lenz's law.

4.5 Working Principle

The voltage induced in a conductor depends on the three parameters as shown in Fig. 4.6. These parameters are categorized as:

- Magnetic field,
- A conductor or a group of conductors,
- Rotation of a conductor with respect to the magnetic field.

If the length of the conductor and magnetic flux density are greater, then the induced voltage will be more. The induced voltage will be zero if there is no magnetic field. The induced voltage will be increased if the conductor moves in a magnetic field with a higher speed and vice versa. Let us consider a conductor rotates in a clockwise direction within a uniform magnetic field with a constant speed as shown in Fig. 4.6.

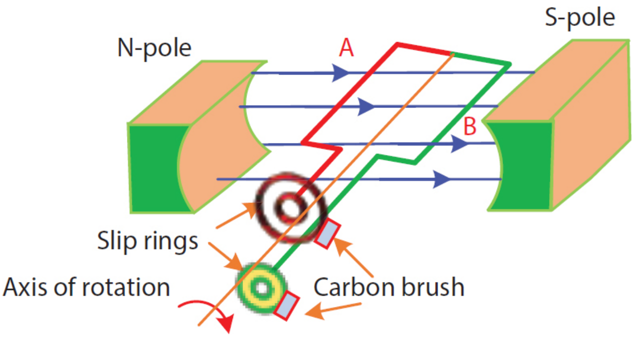


Fig. 4.6 Coil in a magnetic field.

A two-pole DC generator with a single turn of a coil is considered to explain the working principle of a DC generator. The two ends of a coil are connected to the slip rings which act as a commutator. The brushes are attached to the slip rings. The following steps are considered to explain the induced voltage in the generator.

Step 1: At position 1 of Fig. 4.7(a), the conductors are moving in parallel to the lines of flux and therefore, the sides of the conductors did not cut any lines of flux. As a result, the rate change of flux is zero and the generated emf is zero.

Step 2: At position 2 of Fig. 4.7(b), the conductor sides move at right angles to the lines of flux and the conductor cuts the maximum flux. Hence, the rate of change flux is maximum and the induced voltage in the coil is maximum.

Step 3: At position 3 of Fig. 4.7(c), the conductor sides change position i.e., conductor side B is at the top position instead of the bottom. The conductor sides move in parallel to the magnetic field and the generated voltage is again zero because position 3 is opposite to position 1.

Step 4: At position 4 of Fig. 4.7(d), the induced voltage is again maximum because position 4 is just opposite to position 2.

Step 5: At position 5 of Fig. 4.7(e), the conductors are moving in parallel to the lines of flux and therefore, the sides of the conductors did not cut any lines of flux. As a result, the rate change of flux is zero and the generated emf is zero.

From Fig. 4.7(f), it is seen that the voltage induced in a DC generator is in an alternating form. Fleming's right-hand rule is used to find the direction of the induced voltage. The induced voltage across the generator terminals is then converted into direct voltage by a device called the commutator. The commutator ensures that the current from the generator flows in one direction to the load. The output of the commutator is a pulsating DC as shown in Fig. 4.8. However, the pulsating DC output can be smooth DC if the number of armature conductors increases.

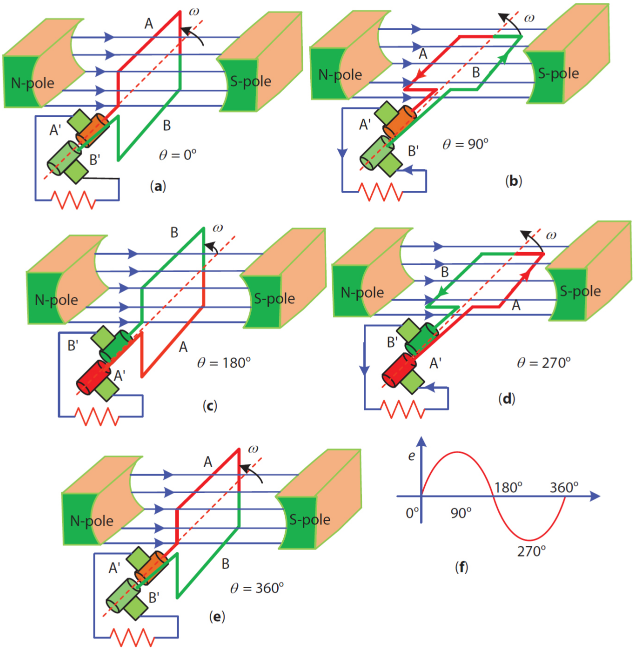

N-pole
S-pole
A
B
A'
B'
ω
θ = 0°
(a)
θ = 90°
(b)
θ = 180°
(c)
θ = 270°
(d)
θ = 360°
(e)
e
0°
90°
180°
270°
360°
(f)

Fig. 4.7 Different positions of a conductor in a magnetic field.

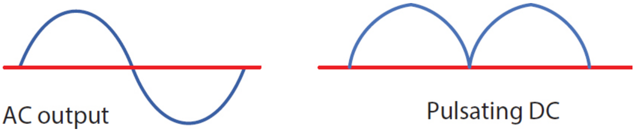


Fig. 4.8 Pulsating DC output.

4.6 Construction of a DC Generator

The schematic diagram of a DC generator is shown in Fig. 4.9. The main components of a DC generator are discussed here.

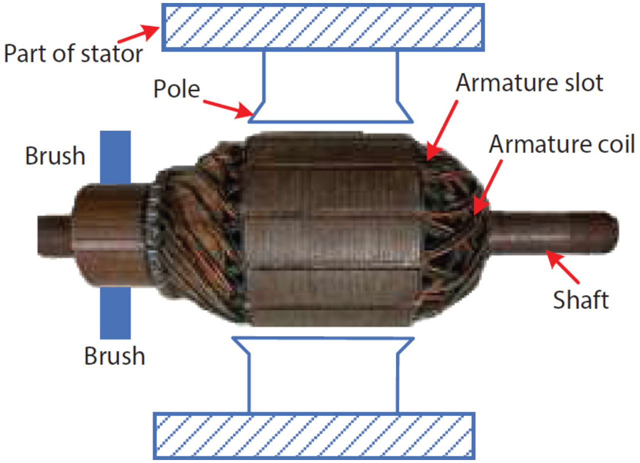


Fig. 4.9 Parts of a DC generator.

Stator: It is the stationary part of the DC generator, which provides mechanical support for the machine. It consists of a core and slots. The core is laminated to reduce the eddy current loss. The stator contains the field poles. The field poles produce the main flux and spread out to the air gap. Field poles are rectangular cross-sections made up of sheet steel. The poles are attached to the yoke, which completes the magnetic path. The yoke is another part of a stator, which provides a high permeable path for the magnetic flux.

Rotor: It is the inner part of the DC machine that normally rotates within the poles and provides the slots in its outer periphery. Armature windings are put on the slots of the rotor.

Armature winding: Armature windings are the insulated copper conductors that are placed inside the armature slots. Within the slots, the armature winding is connected in either a lap winding or a wave winding. The armature winding consists of several identical coils carried in the slots, which are uniformly distributed around the periphery of the rotor. The coils are interconnected through the commutator segments.

Commutator: The commutator is the main component of the DC generator used as a converter. It is made up of hard copper segments. The main function of the commutator is to convert the alternating voltage generated in the armature windings into a direct voltage across the load circuit. The commutator is also known as a mechanical rectifier. It is attached to the generator shaft and rotates with the armature.

Brushes: The brushes are made of carbon and are touched to the commutator by a suitable pressure of the control spring. The main function of the brushes is to provide a proper electrical connection between the commutator and the load circuit.

4.7 Armature Coils and Windings

The windings are required in a DC generator both for the pole and the armature to generate the voltage. In a DC generator, the field windings are placed on the stator slots to excite the pole and the armature windings are put on the rotor slots. Three important terms, turn, coil and winding, are necessary to explain clearly in this section. A turn consists of two sides of the wire. A coil consists of many turns connected in series. A winding is the combination of a certain number of coils that are connected either in series or in parallel. A single turn, coil and winding are shown in Fig. 4.10. When two conductors are connected in series in a magnetic field this is known as the turn. Two conductors with their end connection constitute a coil as shown in Fig. 4.10(a). The coil either be a single-turn or multi-turn. There are two sides in a single-turn coil, whereas there are many conductors in a multi-turn coil as shown in Fig. 4.10(b). A winding is formed when the finish end of one multi-turn coil is connected to the starting end of another multi-turn coil as shown in Fig. 4.10(c).

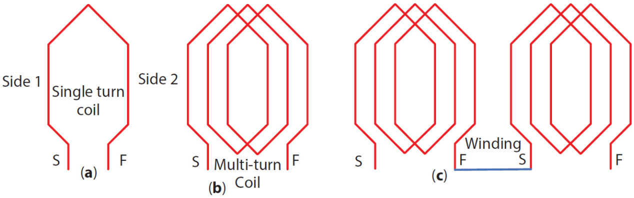


Fig. 4.10 Turn, coil and winding.

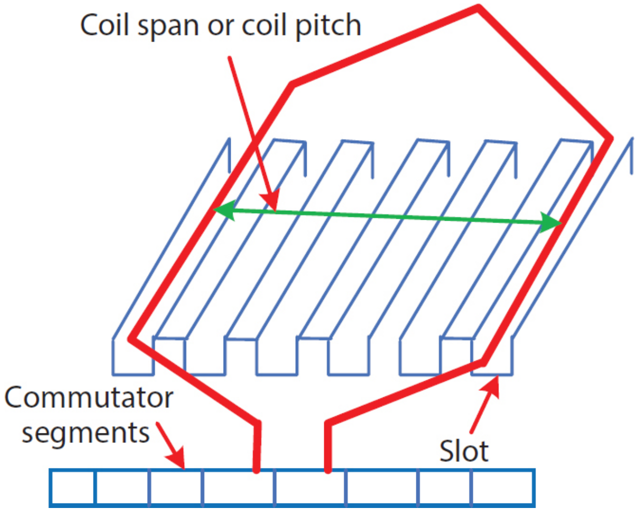


Fig. 4.11 Schematic of coil span.

In the generator winding, the following terms are important and should have sufficient knowledge, which is illustrated in Fig. 4.11.

Coil span: The distance between the two sides of the same coil is known as coil span or coil pitch as shown in Fig. 4.11.

Pole pitch: The distance between the center line of one pole to the center line of the next pole is known as pole pitch. The pole pitch is always 180º regardless of the number of poles. If the two sides of the coil are placed in the slots in such a way that the coil span is obtained 180 electrical degrees, then the span is known as pole pitch.

Full-pitch coil: If the coil covers a span, which is equal to pole pitch, i.e., spanned to 180 electrical degrees, then it is known as a full-pitch coil. Simply, when the coil pitch is equal to the pole pitch is known as full pitch coil as shown in Fig. 4.12(a).

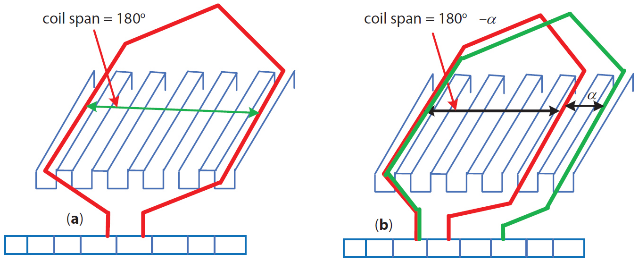


Fig. 4.12 Schematic of full-pitch and short-pitch coil.

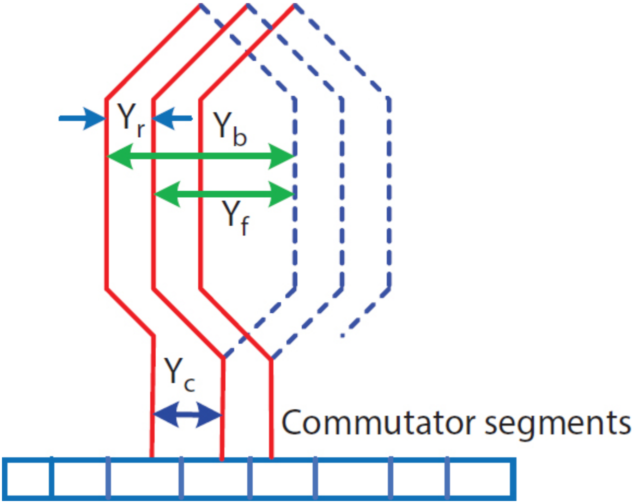


Fig. 4.13 Types of pitches.

Fractional-pitch coil: If the coil covers a span, which is less than 180 electrical degrees, then it is known as a fractional-pitch coil. Simply, when the coil pitch is less than the pole pitch it is known as a short-pitch or fractional-pitch coil as shown in Fig. 4.12(b).

Front pitch: The distance between the second conductor of one coil and the first conductor of the next coil is known as the front pitch. In other words, the distance between the two sides of the coil, when these two sides are connected to the same commutator segment, is shown in Fig. 4.13. It is denoted by Y_f.

Back pitch: The distance between two sides of the coil, when these two sides are not connected to the same commutator segment, i.e., coil sides are spanned by back-end connections. In other words, the distance between the first and the last conductors of a coil is known as the back pitch as shown in Fig. 4.13 and it is denoted by Y_b.

Resultant pitch: The distance between the beginning of one coil to the beginning of the next coil is known as commutator pitch. The commutator pitch is denoted by Y_r.

Commutator pitch: The distance between the commutator segments to which the two ends of a coil are connected is known as commutator pitch and it is denoted by Y_C.

The armature winding is classified into two types, namely: (i) wave winding, and (ii) lap winding. These are discussed here.

Wave winding: The coils of this winding are connected in series through the segments of the commutator in such a way that the number of conductors is divided into two parallel paths (A). Therefore, in a simplex wave winding $A = 2$, regardless of the number of poles. Consider that Z number of conductors are put on the slots of the generator armature. The value of the series conductor in each parallel path would be half of the total number of conductors, i.e., Z/2. The connection diagram of the wave winding is shown in Fig. 4.14.

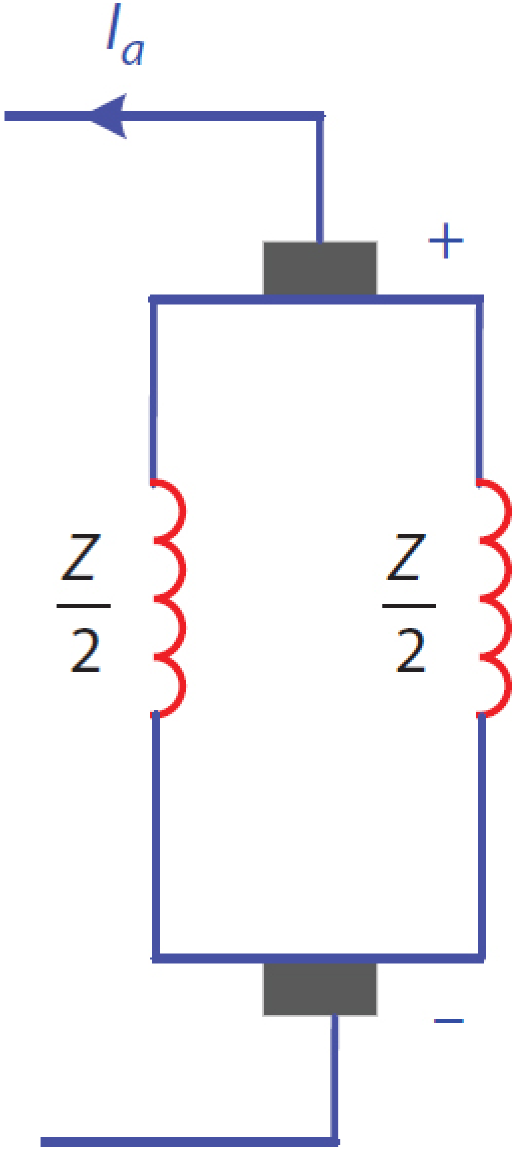


Fig. 4.14 Connection diagram of a wave winding.

Lap winding: The coils of this winding are connected in series through the segments of the commutator in such a way that the number of poles divides the number of the total conductors. Consider that the total number of conductors is Z put on the slots of the armature of a 4-pole generator. The value of the series conductors in each parallel path would be Z/P. The connection diagram of the lap winding is shown in Fig. 4.15.

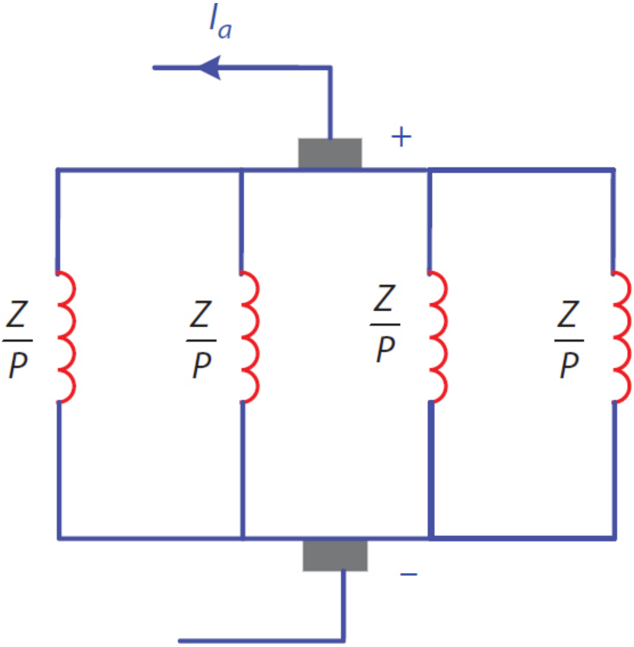


Fig. 4.15 Connection diagram of a lap winding.

4.8 Induced EMF of a DC Generator

Consider a single turn of a coil that is rotating in a uniform magnetic field. This coil cuts the magnetic flux lines which induces the voltage. Let us define the following parameters to derive the induced voltage of a DC generator.

Ec is the induced emf per conductor in V,

Eg is the induced emf in V,

P is the number of pairs of poles of the generator,

ϕ is the flux per pole in Wb,

N is the speed in rpm,

Z is the total number of armature conductors,

A is the number of parallel paths,

Z/A is the number of conductors in series per parallel path.

According to Faraday’s law of electromagnetic induction, the induced emf per conductor for a single loop ($N = 1$) is,

$$E_c = \frac{d\phi}{dt} \tag{4.19}$$

The flux cut by one conductor in one revolution of the armature is expressed as,

$$d\phi = \frac{\text{Flux}}{\text{Pole}} \times \text{Pole} = P\phi \quad \text{Wb} \tag{4.20}$$

The total time to complete one revolution is expressed as,

$$dt = \frac{60}{N} \quad \text{sec} \tag{4.21}$$

Substitute equations (4.20) and (4.21) into equation (4.19) to yield the expression of generated voltage per conductor,

$$E_c = \frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} \tag{4.22}$$

$$E_c = \frac{P\phi N}{60} \tag{4.23}$$

The net emf generated is calculated by the product of emf per conductor and the number of conductors in series per parallel path. Therefore, the expression of net generated voltage becomes,

$$E_g = E_c \times \frac{Z}{P} \tag{4.24}$$

Substituting equation (4.23) into equation (4.24) yields,

$$E_g = \frac{P\phi N}{60}\frac{Z}{A} \tag{4.25}$$

Alternative approach

The mean induced voltage of the conductor with a length of l which rotates at a speed of v is expressed as,

$$E_{mean} = B_{mean}\, l\, v \tag{4.26}$$

The total surface area (S_t) of a conductor with a diameter of D and a length of l is expressed as,

$$S_t = \pi D l \tag{4.27}$$

The mean magnetic flux density is,

$$B_{mean} = \frac{\text{total flux}}{\text{total surface area}} = \frac{P\phi}{\pi D l} \tag{4.28}$$

The velocity of the conductor is expressed as,

$$v = \pi D \frac{N}{60} \tag{4.29}$$

Substitute equations (4.28) and (4.29) into equation (4.26) yields the mean induced voltage per conductor,

$$E_{mean} = \frac{P\phi}{\pi D l} \times l \times \frac{\pi D N}{60} \tag{4.30}$$

$$E_{mean} = E_c = \frac{P\phi N}{60} \tag{4.31}$$

Substitute equation (4.31) into equation (4.24) yields the expression of induced voltage as,

$$E_g = \frac{P\phi N}{60}\frac{Z}{A}$$

Let the generator supply a current I_a to the load and the electrical power developed by the generator armature is expressed as,

$$P_d = E_g I_a \tag{4.32}$$

Again, consider that the total electric power is converted to mechanical power and this expression becomes,

$$P_{md} = T_{em}\omega_m \tag{4.33}$$

Where T_{em} is the electromagnetic torque, which is developed by the generator armature and ωm is the angular velocity of the generator in radians per sec. The expression of angular velocity is expressed as,

$$\omega_m = \frac{2\pi N}{60} \tag{4.34}$$

Substituting equations (4.25), (4.32) and (4.34) into equation (4.33) yields,

$$\frac{PZ\phi N}{60A} \times I_a = T_{em} \times \frac{2\pi N}{60} \tag{4.35}$$

$$T_{em} = \frac{PZ\phi}{2\pi A} I_a \tag{4.36}$$

From equation (4.36), it is seen that the electromagnetic torque is directly proportional to the armature current, number of poles, flux and the total number of conductors.

Example 4.4

The number of slots of a 4-pole wave-wound generator is 40 and 10 conductors are placed per slot. The flux per pole is 0.02 Wb. Calculate the generated emf when the generator is driven at 1200 rpm.

Solution

The total number of conductors is calculated as,

$$Z = 10 \times 40 = 400\Omega \tag{4.37}$$

The generated voltage can be determined as,

$$E_g = \frac{P\phi ZN}{60A} = \frac{4\times 0.02\times 400\times 1200}{60\times 2} = 320\,\text{V} \tag{4.38}$$

Example 4.5

The number of conductors of a 6-pole lap-wound DC generator is 600. The flux per pole is 0.05 Wb. Calculate the generated voltage when the speed of the generator is 1000 rpm. Also, find the electromagnetic torque, if the generator supplies a current of 120 A.

Solution

The generated voltage can be determined as,

$$E_g = \frac{P\phi ZN}{60A} = \frac{6\times 0.05\times 600\times 1000}{60\times 6} = 500\,\text{V} \tag{4.39}$$

The value of the electromagnetic torque is,

$$T_{em} = \frac{PZ\phi}{2\pi A} I_a = \frac{600\times 0.05\times 120}{2\pi} = 572.9\,\text{N-m} \tag{4.40}$$

Practice Problem 4.4

A 4-pole, lap-wound generator rotates at a speed of 300 rpm which generates a voltage of 220 V. The useful flux per pole is 0.02 Wb and the armature has 100 slots. Calculate the number of conductors per slot.

Practice Problem 4.5

The armature of a 6-pole lap-wound generator has 90 slots. The armature rotates at a speed of 600 rpm to generate a voltage of 200 V. If each coil has 5 turns, calculate the flux per pole of the generator. Also, find the electromagnetic torque, if the armature current is 115 A.

4.9 Classification of a DC Generator

DC generators are classified into two types based on the method of field excitation. These are separately excited and self-excited DC generators.

4.10 Separately Excited DC Generator

The term “separate excitation” is given because the field winding is energized from a separate DC supply. The field winding is connected to an independent external DC source through a variable resistance, R_f. The connection diagram of a separately excited DC generator is shown in Fig. 4.16. The following parameters are defined as:

R_f is the field resistance,

R_a is the armature resistance,

R_L is the load resistance,

I_a is the armature current,

IL is the load current,

V_t is the terminal voltage,

E_g is the generated voltage.

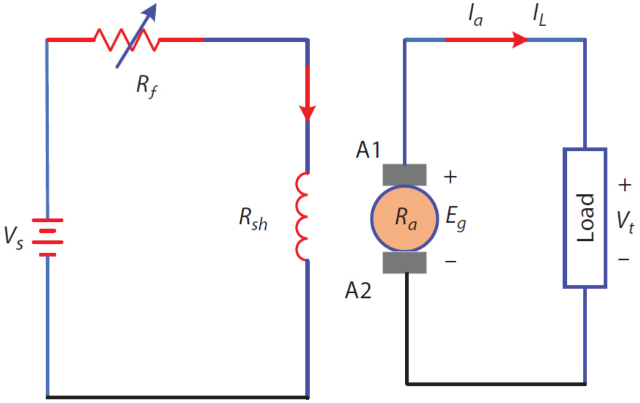


Fig. 4.16 Separately excited DC generator.

The armature current and load current are the same and it is written as,

$$I_a = I_L \tag{4.41}$$

The expression of terminal voltage is expressed as,

$$V_t = E_g - I_a R_a \tag{4.42}$$

The electric power developed by the armature is written as,

$$P_{ad} = E_g I_a \tag{4.43}$$

The power loss due to the armature resistance is written as,

$$P_{lossar} = I_a^2 R_a \tag{4.44}$$

The net power delivered to the load (shaft) is expressed as,

$$P_{load} = P_{ad} - P_{lossar} \tag{4.45}$$

Substituting equations (4.43) and (4.44) into equation (4.45) yields,

$$P_{load} = E_g I_a - I_a^2 R_a \tag{4.46}$$

$$P_{load} = I_a (E_g - I_a R_a) \tag{4.47}$$

Substituting equations (4.42) into equation (4.37), yields,

$$P_{load} = V_t I_a \tag{4.48}$$

4.11 Self-Excited DC Generator

The field winding of this generator is energized by the output of the generator itself. Three types of self-excited generators are available: series, shunt, and compound generators. The type is based on how their field windings are connected to the armature. Brief descriptions and analysis are presented here.

4.12 Series Generator

A series generator is a machine whose field winding is connected in series with the armature as well as with the load. Therefore, the same armature current flows through the field winding and the load. The series field winding is excited when the load is connected to this circuit. The connection diagram of a series generator is shown in Fig. 4.17. In this connection, the armature current can be written as,

$$I_a = I_{se} = I_L \tag{4.49}$$

Apply KVL to the circuit in Fig. 4.14 yields,

$$V_t = E_g - (R_a + R_{se})I_a \tag{4.50}$$

The electric power developed by the armature is expressed as,

$$P_{sead} = E_g I_a \tag{4.51}$$

The power loss due to the armature and series field resistances is written as,

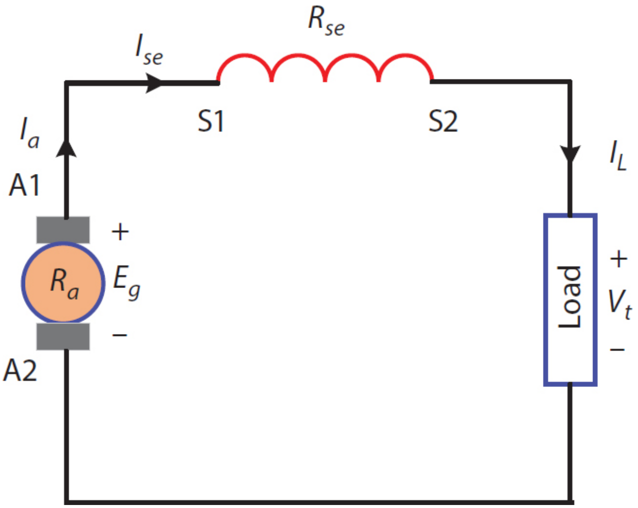


Fig. 4.17 Connection diagram of a series generator.

$$P_{lossar} = I_a^2 (R_a + R_{se}) \tag{4.52}$$

The net power delivered to the load is calculated as,

$$P_{sel} = P_{sead} - P_{lossar} \tag{4.53}$$

$$P_{segl} = E_g I_a - I_a^2 (R_a + R_{se}) \tag{4.54}$$

$$P_{segl} = I_a[E_g - I_a(R_a + R_{se})] \tag{4.55}$$

Substituting equation (4.50) into equation (4.55) yields,

$$P_{segl} = V_t I_a \tag{4.56}$$

4.13 Shunt Generator

A shunt generator is a machine whose shunt field winding is connected in parallel with the armature as well as across the supply. The terminal voltage of a shunt generator is reduced sharply with the load increased. As a result, the field current falls as the terminal voltage drops. The schematic circuit diagram of a shunt generator is shown in Fig. 4.18. From Fig. 4.18, the shunt field current is determined as,

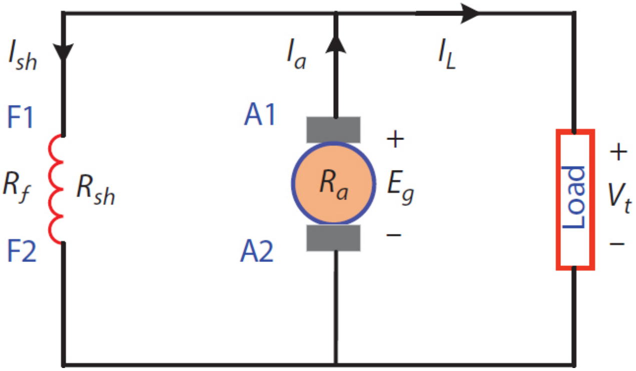


Fig. 4.18 Connection diagram of a shunt generator.

$$I_{sh} = \frac{V_t}{R_{sh}} \tag{4.57}$$

Applying KCL to find the expression of armature current, it becomes,

$$I_a = I_{sh} + I_L \tag{4.58}$$

However, the terminal voltage is determined as,

$$V_t = E_g - I_a R_a \tag{4.59}$$

The power developed by the armature is calculated as,

$$P_{shad} = E_g I_a \tag{4.60}$$

The power delivered to the load is expressed as

$$P_{shgl} = V_t I_L \tag{4.61}$$

Example 4.6

A 220 V DC shunt generator delivers a current of 125 A to the load. The shunt field and armature resistances are 100 Ω and 0.03 Ω, respectively. Determine the generated voltage.

Solution

The shunt field current is determined as,

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{220}{100} = 2.2\,\text{A} \tag{4.62}$$

The armature current is calculated as,

$$I_a = 125 + 2.2 = 127.2\,\text{A} \tag{4.63}$$

The generated voltage is determined as,

$$E_g = V_t + I_a R_a = 220 + 127.2 \times 0.03 = 223.82\,\text{V} \tag{4.64}$$

Example 4.7

A 220 V DC shunt generator delivers a power of 15 kW to the load. The shunt field and armature resistances are 120 Ω and 0.05 Ω, respectively. Determine the generated voltage if the total brush voltage drop is 2V.

Solution

The load current is determined as,

$$I_L = \frac{P}{V_t} = \frac{15000}{200} = 75\,\text{A} \tag{4.65}$$

The value of the shunt field current is determined as,

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{220}{120} = 1.83\,\text{A} \tag{4.66}$$

The value of the armature current is calculated as,

$$I_a = 75 + 1.83 = 76.83\,\text{A} \tag{4.67}$$

The value of the generated voltage is calculated as,

$$E_g = V_t + I_a R_a + \text{brush drop} = 220 + 76.2 \times 0.05 + 2 = 225.81\,\text{V} \tag{4.68}$$

Practice Problem 4.6

A 40 kW, 230 V shunt generator has a field resistance of 60 Ω and an armature resistance of 0.04 Ω. Calculate the generated voltage.

Practice Problem 4.7

A 220 V DC shunt generator delivers a power of 25 kW to the load. The shunt field and armature resistances are 100 Ω and 0.04 Ω, respectively. Calculate the generated voltage.

4.14 Compound Generator

A DC generator which has both a series field winding and a shunt winding is known as a compound generator. The compound generator was developed to prevent the terminal voltage from decreasing with increasing load. Depending on the connection of the field winding, it is divided into two types: short shunt and long shunt generators.

Short shunt: In the short compound generator, the shunt field winding is connected in parallel with the armature winding only and the series field winding is in series with the load. The connection diagram of the short shunt compound generator is shown in Fig. 4.19. In a short shunt compound generator connection, the series field current is the same as the load current and it is written as,

$$I_{se} = I_L \tag{4.69}$$

The shunt field current is calculated as,

$$I_{sh} = \frac{E_g + I_a R_a}{R_{sh}} = \frac{V_t + I_{se} R_{se}}{R_{sh}} \tag{4.70}$$

The terminal voltage is calculated as,

$$V_t = E_g - I_{se} R_{se} - I_a R_a \tag{4.71}$$

The power developed in the short shunt compound motor armature is calculated as,

$$P_{cssdcg} = E_g I_a \tag{4.72}$$

The power delivered to the load is calculated as,

$$P_{cssdl} = V_t I_L \tag{4.73}$$

Long shunt: In the long shunt compound generator, the shunt field winding is in parallel combined with a series field and the armature winding as well with the load. The connection diagram of the long shunt compound generator is shown in Fig. 4.20. The series field current is expressed as,

$$I_{se} = I_a = I_L + I_{sh} \tag{4.74}$$

The shunt field current can be determined as,

$$I_{sh} = \frac{V_t}{R_{sh}} \tag{4.75}$$

The expression of terminal voltage is written as,

$$V_t = E_g - I_a (R_a + R_{se}) \tag{4.76}$$

From Figs. 4.18 and 4.19, it is seen that in the short shunt connection, the load current excites the series field, whereas, in the long shunt connection, the armature current excites the series field.

The power developed by the armature is expressed as,

$$(4.77)\ P_{lsgd} = E_g I_a$$

The power delivered to the load is expressed as,

$$(4.78)\ P_{lsgl} = V_t I_L$$

Example 4.8

The terminal voltage of a 25 kW compound generator is 220 V. The armature, series and shunt field windings have a resistance of 0.07 Ω, 0.05 Ω and 50 Ω, respectively. Calculate generated emf when the generator is connected in a long shunt and short shunt. Consider that the voltage drop per brush is 1 V.

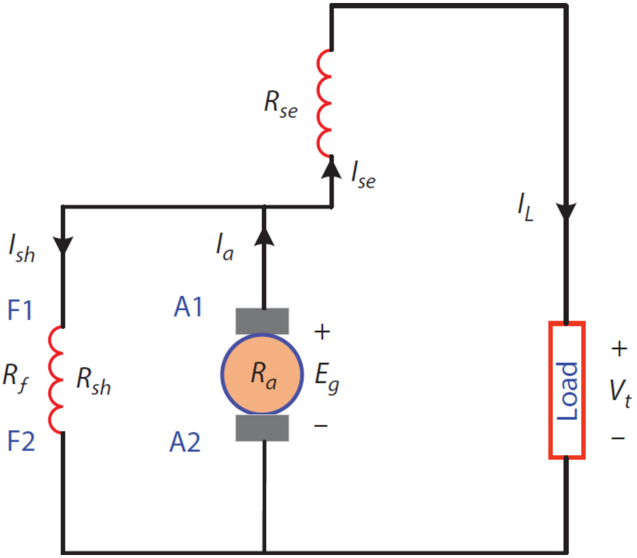


Fig. 4.19 Connection diagram of a short shunt generator.

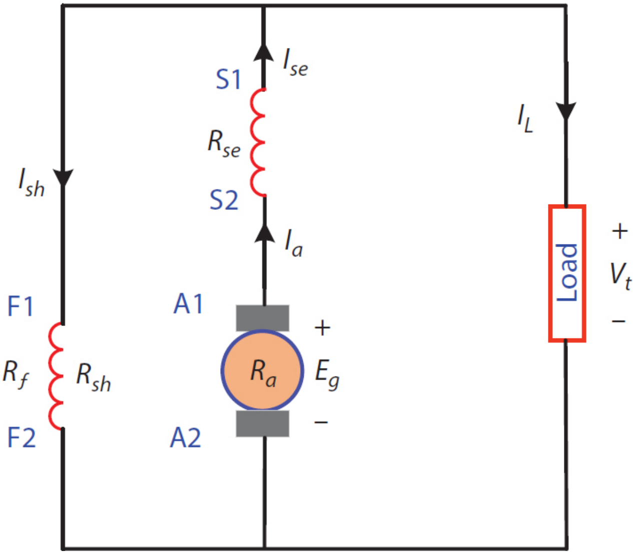

S1
I_{se}
R_{se}
S2
I_a
I_{sh}
I_L
F1
A1
R_f
R_{sh}
R_a
+
E_g
−
F2
A2
Load
+
V_t
−

Fig. 4.20 Connection diagram of a long shunt generator.

Solution

The value of the load current is calculated as,

$$I_L = \frac{25000}{220} = 113.6\,\text{A} \tag{4.79}$$

The value of the shunt field current is determined as,

$$I_{sh} = \frac{220}{50} = 4\,\text{A} \tag{4.80}$$

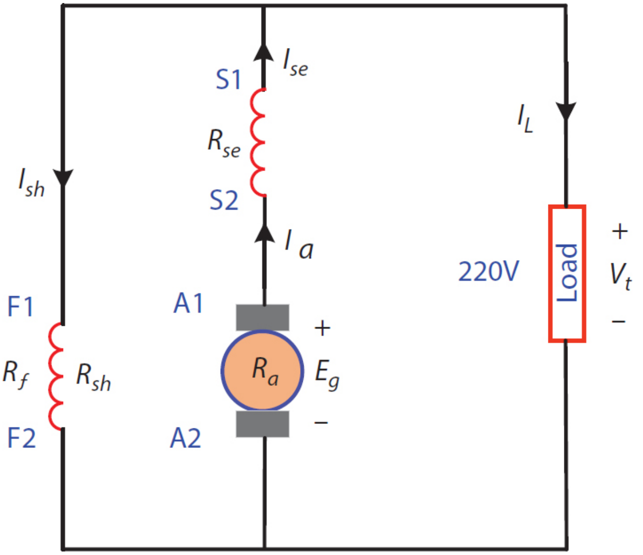


Fig. 4.21 A long shunt generator for example 4.8.

The value of the armature current is calculated as,

$$I_a = I_{sh} + I_L = 113.6 + 4 = 117.6\,\text{A} \tag{4.81}$$

Fig. 4.21 represents the long shunt generator and the value of the generated voltage of a long shunt generator is calculated as,

$$E_g = V_t + I_a(R_a + R_{se}) + \text{brush drop} = 220 + 117.6 \times 0.12 + 2 \times 1 = 236\,\text{V} \tag{4.82}$$

Fig. 4.22 shows the connection diagram for a short shunt generator. The voltage across the shunt field is calculated as,

$$V_{ab} = V_t + I_L R_{se} = 220 + 113.6 \times 0.05 = 225.7\,\text{V} \tag{4.83}$$

The value of the current in the shunt field is calculated as,

$$I_{sh} = \frac{V_{ab}}{R_{sh}} = \frac{225.7}{50} = 4.51\,\text{A} \tag{4.84}$$

The value of the armature current is calculated as,

$$I_a = I_{sh} + I_L = 113.6 + 4.51 = 118.11\,\text{A} \tag{4.85}$$

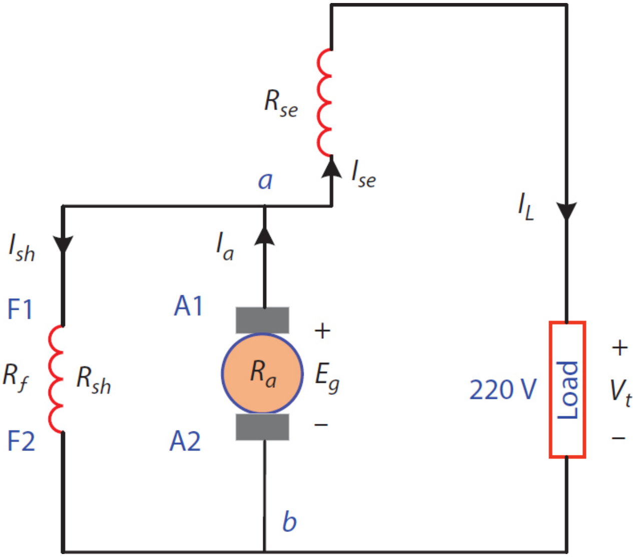


Fig. 4.22 A short shunt generator for example 4.8.

The generated voltage is calculated as,

$$E_g = V_{ab} + I_a R_a + \text{brush drop} = 225.7 + 118.11 \times 0.07 + 2 \times 1 = 235.97\ \text{V} \tag{4.86}$$

Example 4.9

A 220 V long shunt generator delivers a current of 146 A to the load. The shunt field, series field, diverter and armature circuit resistances are 55 Ω, 0.02 Ω, 0.03 Ω and 0.01 Ω, respectively. Find the generated voltage and power distribution in different branches.

Solution

The shunt field current is calculated as,

$$I_{sh} = \frac{220}{55} = 4\ \text{A} \tag{4.87}$$

The value of the armature current is calculated as,

$$I_a = I_{sh} + I_L = 146 + 4 = 150\ \text{A} \tag{4.88}$$

The diverter resistance is always connected in parallel with the series field winding resistance as shown in Fig. 4.23. Therefore, the combined resistance is,

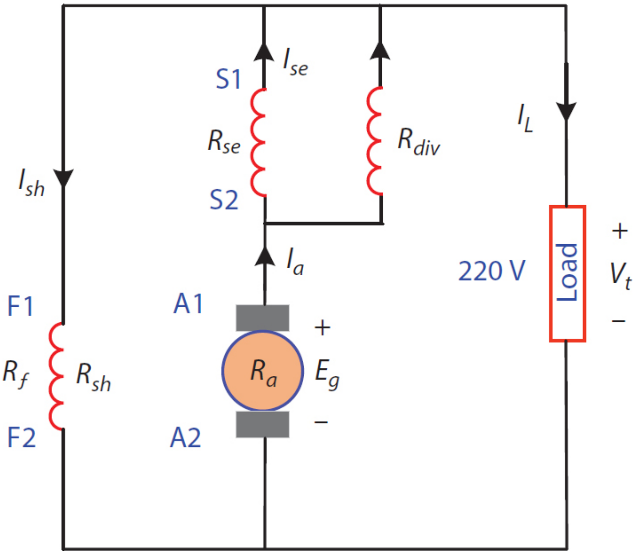


Fig. 4.23 Circuit for example 4.9.

$$R_{com} = \frac{0.02\times 0.03}{0.05} = 0.012\,\Omega \tag{4.89}$$

The value of the generated voltage is calculated as,

$$E_g = V_t + I_a(R_a + R_{com}) = 220 + 150(0.012 + 0.012) = 223.6\,\text{V} \tag{4.90}$$

The power loss in the series field and diverter is determined as,

$$P_{lsd} = I_a{}^2 R_{com} = 150^2 \times 0.012 = 270\,\text{W} \tag{4.91}$$

The power loss in the armature circuit resistance is calculated as,

$$P_{la} = I_a{}^2 R_a = 150^2 \times 0.01 = 225\,\text{W} \tag{4.92}$$

The power loss in the shunt field resistance is determined as,

$$P_{lsh} = V_t I_{sh} = 220 \times 4 = 880\,\text{W} \tag{4.93}$$

The power delivered to the load is calculated as,

$$P_{dl} = V_t I_L = 146 \times 220 = 32{,}120\,\text{W} \tag{4.94}$$

Practice Problem 4.8

The terminal voltage of a 20 kW short shunt compound generator is 230 V. The armature, series and shunt field windings have a resistance of 0.04 Ω, 0.03 Ω and 50 Ω, respectively. Calculate the generated emf and assume the voltage drop per brush is 1.5 V.

Practice Problem 4.9

A 200 V long shunt compound generator delivers a current of 140 A to the load. The shunt field, series field, diverter and armature circuit resistances are 45 Ω, 0.05 Ω, 0.07 Ω and 0.01 Ω, respectively. Find the generated voltage and power distribution.

4.15 Saturation Curve of a DC Generator

A separate DC generator is considered to explain the saturation curve. An ammeter is connected in series with the field winding and the voltmeter is connected across the generator armature terminals as shown in Fig. 4.24. The voltage induced in a generator mainly depends on the field flux and the speed of the rotation. If the speed of the conductor is fixed then the induced voltage can vary with the variation of the field flux. The saturation curve is obtained by plotting the field current on the x-axis and the induced voltage on the y-axis. The field flux is directly proportional to the shunt field current (*If*). A field rheostat is used to vary this field current.

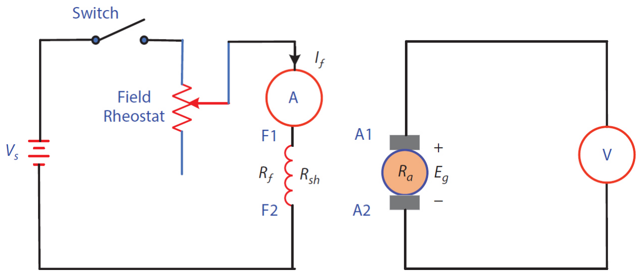


Fig. 4.24 Separately excited generator for saturation curve.

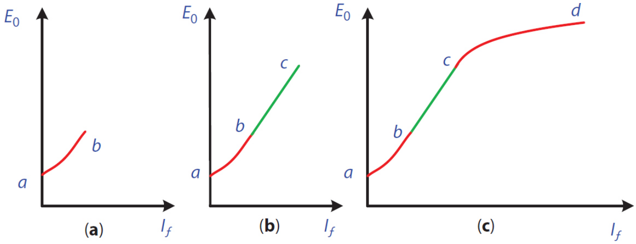


Fig. 4.25 Formation of the saturation curve.

When the current through the field coil is zero, i.e., $If = 0$, a small magnitude of voltage will be recorded through the voltmeter obtained due to residual magnetism. This small magnitude of voltage is shown in Fig. 4.25(a) by *ab* on the y-axis. As the field rheostat decreases, the shunt field current increases. As a result, the induced voltage increases proportionally, which is represented by points *b* and *c* as shown in Fig. 4.25(b). After point *c*, if the field current increases further, the induced voltage increases in a smaller magnitude due to the property of a magnetic material. This magnitude of the voltage is represented by point *d* as shown in Fig. 4.25(c). In Fig. 4.25(c), the point *d* represents the saturation point. After this point, if the shunt field current increases, the magnitude of the induced voltage will remain the same.

In this case, there is no need to increase the field current. If the shunt field current decreases, the induced voltage will decrease and it will return through a different path and meet with the voltage line at the point *e*. This action is also due to the property of the magnetic circuit. Finally, the hysteresis loop in the positive x-y axes is formed as shown in Fig. 4.26.

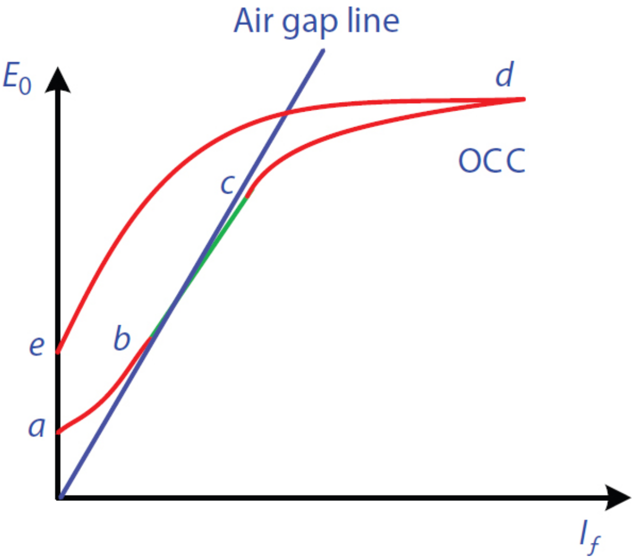


Fig. 4.26 Saturation curve.

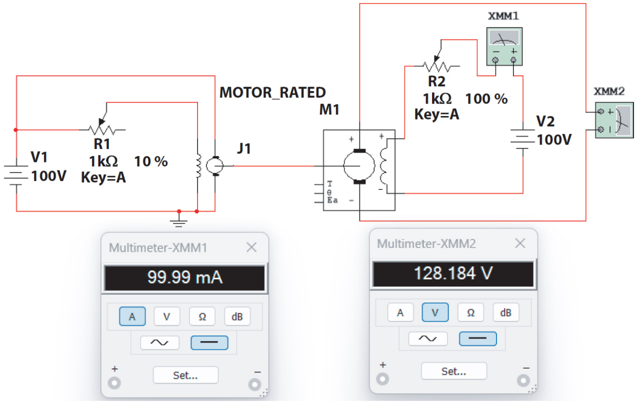

XMM1
R2
1kΩ 100 %
Key=A
XMM2
V2
100V
MOTOR_RATED
M1
J1
V1
100V
R1
1kΩ 10 %
Key=A
Multimeter-XMM1
99.99 mA
A V Ω dB
Set...
Multimeter-XMM2
128.184 V
A V Ω dB
Set...

Fig. 4.27 Simulation circuit.

Table 4.1 Data for DC machine wound field.

Parameter	Value
Armature inductance	0.1mH
Armature resistance	0.2Ω
Field winding inductance	0.2H
Field winding resistance	0.1Ω

Armature-field mutual inductance	500 mH
Shaft inertia	0.0001 kg.m2
Shaft friction	1e-6 N-ms/rad
Initial angular speed	0 rad/s
Initial angle	0 rad

The separately excited DC generator and a prime mover as "Motor-Rated" along with measuring instruments are connected to simulate using Multisim software.

The prime mover and the DC generator are shown in Fig. 4.27. Before starting the simulation, the parameters of the DC machine are modified as shown in Table 4.1. The parameters of the DC generator are also revised based on Table 4.2.

The value of the potentiometer R1 is set to 10% and then the value of the potentiometer R2 to 100%. Then the simulation is performed and records the generated output voltage across the armature terminals. Again, the generated voltage is measured by varying the potentiometer R2 to 90%, 80%, 70%, 60%, 50%, 40% and 30%. All the measured voltages are recorded in Table 4.3.

Table 4.2 Data for prime mover motor-rated.

Parameter	**Value**
Animation showdown factor	5
Rated armature voltage (Vf)	200V
Maximum armature voltage	200V
Rated armature current (Ia)	8.8A
Maximum armature current	200A
Rated field voltage (Vf)	200V
Maximum field voltage	200V
Armature resistance (Ra)	1.1Ω
Armature inductance (La)	1mH
Field resistance (Rf)	128 Ω
Field reactance (Lf)	1mH
Shaft friction (Bf)	0.01N-m-s/rad
Rotational inertia (J)	0.01N-m-s/rad

Rated rotational speed (N)	1500 rpm
Load torque (T)	0 N-m

Table 4.3 Generated voltage across the armature terminals.

% R_2	I_{fg} (mA)	E_g (V)
100	99.99	128.184
90	111.09	142.426
80	124.984	160.228
70	142.837	183.113
60	166.639	213.627
50	199.96	256.343
40	249.938	320.413
30	333.222	427.182

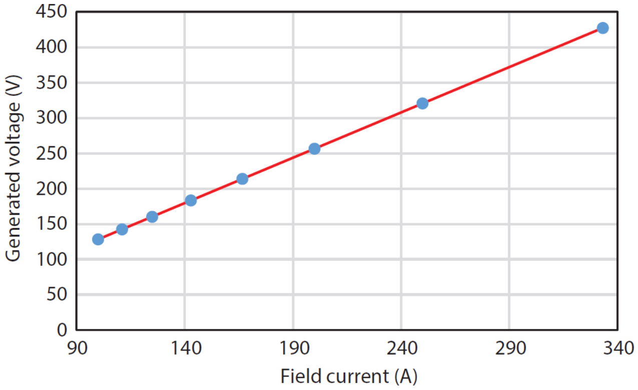


Fig. 4.28 Generated voltage versus field circuit.

The measured generated voltages across the armature terminals are plotted versus field current as shown in Fig. 4.28, which shows that the generated voltage increases with decreasing field resistance.

4.16 Generator Build-Up Process

The shunt field winding of a shunt generator is excited by the voltage generated by the generator. Consider a shunt generator, which runs at a constant speed to build up a voltage. The build-up process is described in the following steps:

- The armature of the generator is driven at a certain speed without the shunt field winding, i.e., the switch is disconnected with the field winding. A small magnitude of voltage E_r appears across the generator terminals due to residual flux in the poles cuts by the armature conductors.
- Now the switch is connected to the field winding which means the generator is driven with a shunt field winding. As a result, a small magnitude of current I_{f1} flows through the field winding, which creates the magnetic flux. This flux aids the residual flux of the poles which increases the magnitude of the total flux.
- As a result of the increased flux in the poles, the generated voltage will now increase from Er to E_1.
- The increased generated voltage E_1 will further increase the field current to I_{f2}. As a result of this increased current, the magnetic flux will be increased which will increase the generated voltage to E_2.
- This build-up process will continue until the open circuit characteristics (OCC) curve is obtained where the OCC curve intersects the shunt field resistance (OF). However, the no-load voltage can be obtained from the point of intersection between the lower value of the field resistance line and the saturation curve. After the intersection point, if the field current increases, the generated voltage will not increase anymore. It is due to the property of the magnetic material. The whole build-up process is shown in Fig. 4.29.

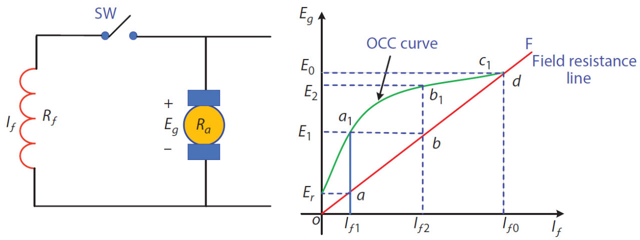


Fig. 4.29 Generator voltage build-up process.

4.17 Voltage Build-Up Failure Reasons

The voltage build-up process of the DC generator depends on the residual flux in the poles, shunt field resistance, field connection and commutator. This voltage build-up process may be interrupted due to the following main reasons:

Absence of residual flux: Initially, the residual flux is required to build up the voltage from zero to a small magnitude. In this regard, a separate DC voltage source is used to get a residual flux for a short period. The voltage across the generator terminals will not be generated without a residual flux.

Higher value of shunt field resistance: A higher value of shunt field resistance does not intersect the saturation curve. If the shunt field resistance is higher than the critical resistance, the generator will not build up any voltage. Therefore, the shunt field resistance must be lower than the critical field resistance to build up the voltage.

Reversed field connections: If the field winding is connected in such a way that the flux produced by its magnetomotive force opposes the residual flux, the generated voltage will decrease rather than increase.

Polluted commutator: The polluted commutator does not provide a good connection with brushes. This loose connection provides a higher resistance to the flow of the current in the field winding and produces the same effect as produced by the high field winding resistance.

4.18 Field Circuit Resistance Lines

The field circuit resistance is the relationship between the induced voltage at the generator terminal voltage and the current in the shunt field winding. The field circuit resistance line is obtained by plotting the terminal voltage as the y-axis and the field current as the x-axis. The shunt field resistance is normally considered the field circuit resistance. In a self-excited DC generator, the voltage build-up process depends on the field circuit resistance as well as on the point of intersection with the saturation curve. The line corresponding to the lower resistance Rf_1 intersects the saturation curve at point c_1. At this point, the magnitude of the voltage is oc, i.e., the maximum voltage that the shunt generator can generate at a specified speed. The field circuit resistance is increased to Rf_2, which intersects the saturation curve at point b_1. According to this point, the magnitude of the voltage is ob which is slightly lower than oc.

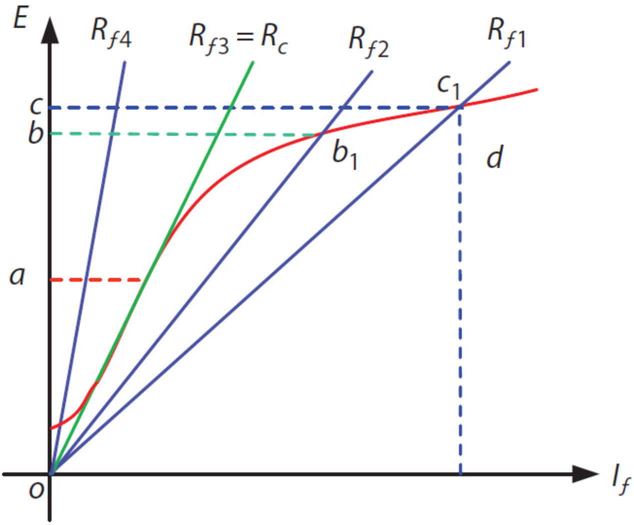


Fig. 4.30 Field resistance lines.

If the field circuit resistance increases gradually, the magnitude of the generated voltage buildup will be reduced. After some time, when the field circuit resistance line becomes tangent to an open circuit characteristic or saturation curve, the generator will not build any more voltages. This is the critical condition of the dc generator. The generated voltage under critical conditions is oa. Therefore, the critical resistance is the value of the field circuit resistance, which makes it tangent to the magnetization or saturation curve. If the resistance line is increased further beyond this critical resistance as represented by Rf_4, the generator will not induce any voltage, because this resistance line does not intersect the saturation curve. All the field circuit resistance lines from lower to higher values are shown in Fig. 4.30.

The critical speed is the speed at which the field circuit resistance becomes the critical resistance.

4.19 Theory of Commutation

The coils or conductors move from one pole to another pole during the operation of a generator. The conductors are short-circuited by the brushes through the commutator segments for a fraction time while moving from one pole to another pole. This action is known as commutation. The duration for which a coil remains short-circuited is known as the commutation period. The direction of the current in the coil is reversed by the combined action of the commutator and brushes. The brushes are normally fixed on the commutator segments. The brushes move from one segment to another segment of the commutator, when the coil rotates in a clockwise direction. The coils under the commutation are shown in Fig. 4.31. In Fig. 4.31(a), the brush is fixed on segment 2 and the current I is flowing through the coil x from left to right (L-R). The brush on segment 2 carries current $2I$ equally coming from the left and right of the coils. In Fig. 4.31(b), the brush is fixed both on segments 2 and 3. In this case, the current through coil x is zero, i.e., the coil x is short-circuited by the brush. However, the total current $2I$ is collected by the brush equally from segments 2 and 3. In Fig. 4.31(c), the brush is fixed on segment 3. In this case, the current I is flowing from right to left (R-L) through the coil x.

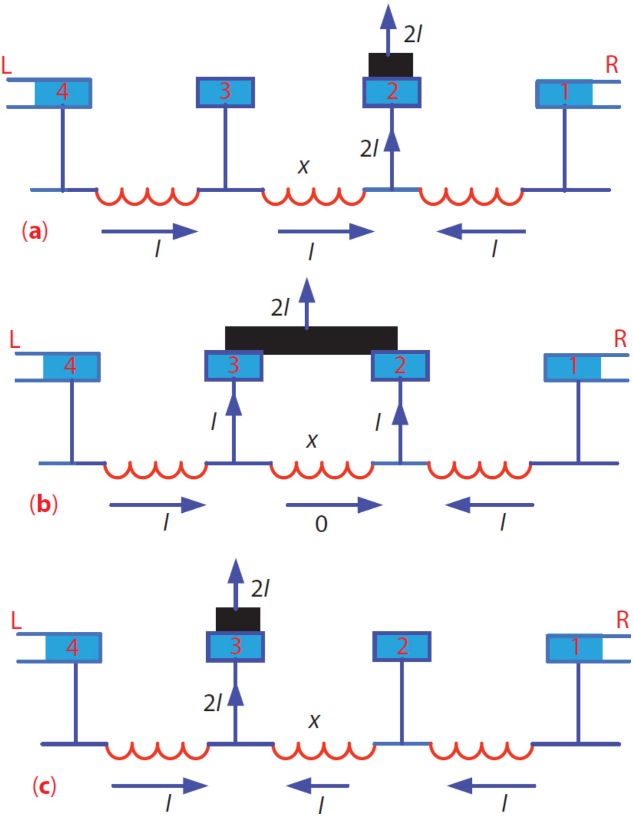


Fig. 4.31 Coils under commutation.

Therefore, in Fig. 4.31(c), the direction of this current is opposite to the current in the first case. As a result, a short period is required for the reversal of the current. The current of the coil x is shown in Fig. 4.32. The coil undergoing commutation will induce the voltage due to the rate of change of current Ldi/dt. This voltage is known as reactance voltage and it always opposes the reversal of the current flowing in it. This reactance voltage is very small and it produces a large current in the coil as it has a smaller resistance during short circuits. Due to this large current, a spark occurs at the brushes of the machine during running.

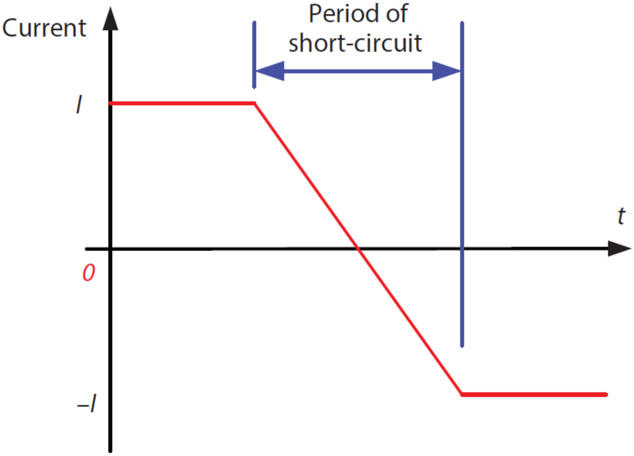


Fig. 4.32 Commutation process of the coil x.

4.20 Armature Reaction

In a DC generator, current in the armature conductors produces the magnetic field or the flux. There is an axis or plane such as MNA (Magnetic neutral axis) and GNA (Geometrical neutral axis) along which the armature conductors move either parallel or perpendicular to the main field flux to generate the voltage. MNA is defined as the axis along which no voltage is generated as the armature conductors move parallel to the main field, whereas the GNA is defined as the axis which is perpendicular to the main field. The flux due to the main poles acts inside the field system of the generator to generate the voltage. The armature flux acts in the opposite direction to the main flux, which affects the flux pattern. As a result, the resultant flux is distorted and weakened. This distortion causes a shift in the neutral plane, which affects the whole commutation process. Therefore, the opposite action of the armature flux on the main flux and the change in the neutral plane is known as the armature reaction. The terminal voltage is slightly reduced due to the armature reaction. The armature reaction can be explained in the following ways.

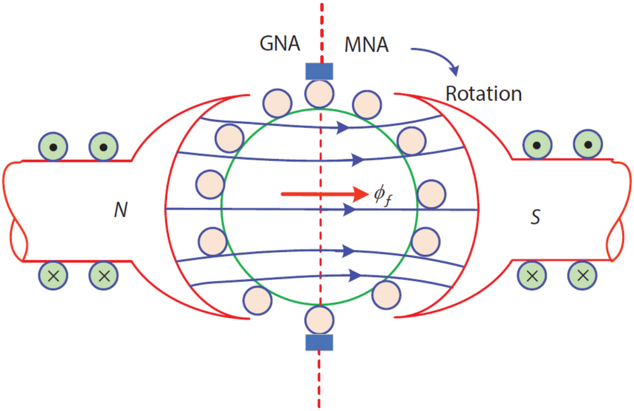


Fig. 4.33 Distribution of field flux under no-load condition.

Consider the operation of a two-pole DC generator and in a commutation, the coil short-circuited by the brushes must be placed in the neutral planes. At no-load conditions, the current in the armature conductors is zero. There is only one flux ϕf produced by the field winding that is distributed uniformly and symmetrically over the pole phases as shown in Fig. 4.33.

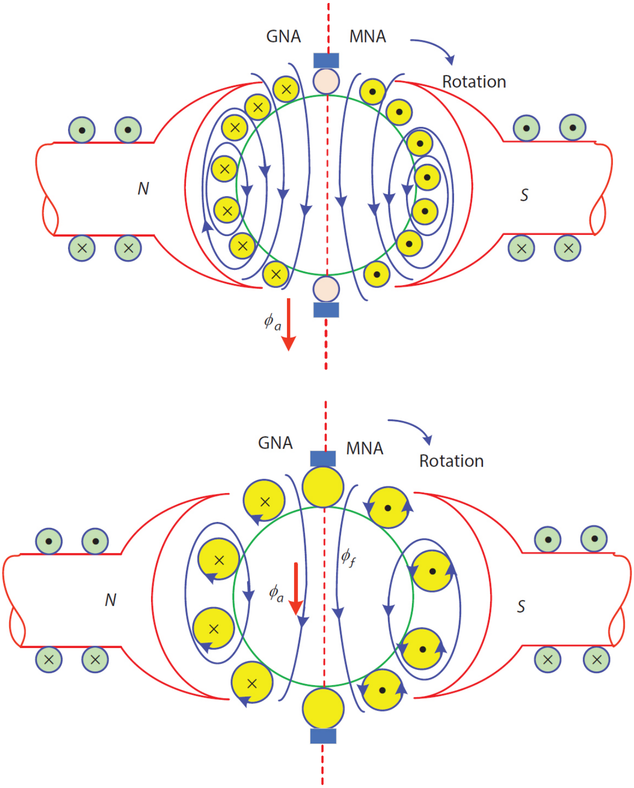


Fig. 4.34 Distribution of field flux and armature flux.

Now, consider a prime mover drives the generator in a clockwise direction and the field coils are not energized. The direction of the currents in the armature conductors under load conditions is shown in Fig. 4.34. When armature conductors rotate in the clockwise direction, the conductors under the N-pole carry current into the direction towards the inside as indicated by the cross mark. The flux produced by the armature current under the N-pole creates the flux ϕa in the downward direction. Similarly, the conductors under the S-pole carry current in the direction out of the plane that is marked by the dot. The flux produced by the armature current under the S-pole also creates the flux ϕa in the downward direction. Therefore, all the armature conductors both under the N-pole and S-pole create the flux ϕa which acts in the downward direction. This flux is normally created by the armature current which is known as armature flux. The magnetic field is generated around each side of the coil as shown in Fig. 4.34. This field increases the strength of each wire in the armature conductor and sets up a flux ϕa which is almost perpendicular to the main field flux ϕf. For this reason, the armature flux is known as the cross-magnetizing flux.

Finally, the armature and the field coils are energized simultaneously and the generator is running. Under this condition, the current in the armature conductors will create the flux ϕa and the current in the field coils will create the ϕf. These two fluxes create the resultant flux as shown in Fig. 4.35(b). The main flux from the pole is superimposed to the armature flux which is distorted and shifted. As a result of this, the direction of the resultant flux is shifted to the direction of the rotation of the generator. However, the MNA is always perpendicular to the resultant flux. As a result of the shift of MNA to the new axis, the main flux decreases which decreases the generated emf ($Eg \propto N\phi$) with an increase in load. Fig 4.35 represents how the armature flux distorts the main field flux and how the magnetic neutral plane is shifted in the direction of rotation. If the brushes are kept in the old neutral plane, they will be short-circuiting the coils that have a voltage induced in them. Consequently, there will be an arcing between the commutator and the brushes.

Because of the brush shift, the armature conductors as well as armature currents are redistributed. This armature flux can be resolved into two components, namely, cross-magnetizing flux and demagnetizing flux. Cross-magnetizing flux is at right angles to the main field flux. Demagnetizing flux is in the same plane as the main field flux, but it is in the opposite direction to the field flux. Due to nonuniform flux distribution, the resultant flux is distorted, which reduces the generated voltage. The armature reaction has a demagnetizing effect on the machine. Especially, in the large machine, its effect is more under loaded conditions.

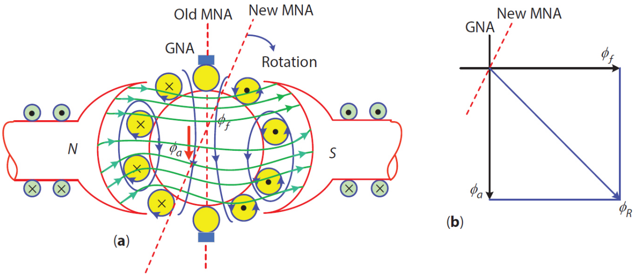

Old MNA
New MNA
GNA
Rotation
ϕ_f
ϕ_a
N
S
(a)
GNA
New MNA
ϕ_f
ϕ_a
ϕ_R
(b)

Fig. 4.35 Distribution of field flux and armature flux.

4.21 Demagnetizing and Cross-Magnetizing

In a DC generator, the armature reaction causes cross-magnetizing and demagnetizing effects. The flux produced by the armature conductor, which is perpendicular to the main field flux is known as a cross-magnetizing flux. This cross-magnetizing flux distorts the main field flux and is known as the cross-magnetizing effect. Demagnetizing flux is in the same plane as the main field flux but is in the opposite direction to the main field flux. This demagnetizing flux reduces the main field flux and is known as the demagnetizing effect. The MNA line *AB* is shifted ahead which makes an angle α with the geometrical neutral axis (GNA) as shown in Fig. 4.36. The other line CD is drawn in the opposite direction of line AB with the same angle as GNA.

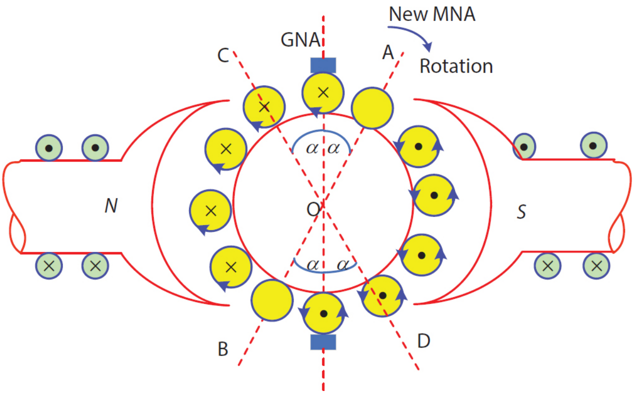

New MNA
GNA
C
A
Rotation
α α
N
O
S
α α
B
D

Fig. 4.36 Conductors under cross-magnetizing and demagnetizing.

Let,

Z is the total number of armature conductors,

I_c is the current in each armature conductor,

$I_c = I_a/2$ for the wave winding,

A is the number of parallel paths,

$I_c = I_a/A$ for the lap winding.

The conductors under AOC and BOD carry current in such a direction that opposes the main field flux, which is known as the demagnetizing effect. Each turn of the conductor has two sides. Therefore, for Z number of conductors, the number of turns is Z/2. The total demagnetizing ampere-turns is equal to the ampere-turns under the angles of AOC and BOD and this can be expressed as,

$$AT_d = I_c \frac{Z}{2} \frac{\left[\lfloor\underline{BOD} + \lfloor\underline{AOC}\right]}{360} \tag{4.95}$$

$$AT_d = \frac{ZI_c}{360} \frac{\left[2\alpha + 2\alpha\right]}{2} \tag{4.96}$$

$$AT_d = \frac{ZI_c}{360} 2\alpha \tag{4.97}$$

The demagnetizing ampere-turn per pole is expressed as,

$$AT_d \,/\, \text{pole} = \frac{1}{2} \times AT_d \tag{4.98}$$

Substituting equation (4.97) into equation (4.98) yields,

$$AT_d \,/\, \text{pole} = \frac{1}{2} \times ZI_c \frac{2\alpha}{360} \tag{4.99}$$

$$AT_d \,/\, \text{pole} = ZI_c \frac{\alpha}{360} \tag{4.100}$$

The current per parallel path is expressed as,

$$I_c = \frac{I_a}{A} \tag{4.101}$$

Total armature ampere-turn is expressed as,

$$AT_t = I_c \frac{Z}{2} \tag{4.102}$$

Total armature ampere-turn per pole is expressed as,

$$AT_t \,/\, \text{pole} = I_c \frac{Z}{2P} \tag{4.103}$$

The cross-magnetizing ampere-turn per pole is expressed as,

$$AT_c \,/\, \text{pole} = AT_t \,/\, \text{pole} - AT_d \,/\, \text{pole} \tag{4.104}$$

Substituting equations (4.100) and (4.103) into equation (4.104) yields,

$$AT_c = ZI_c\left(\frac{1}{2P} - \frac{1}{360}\alpha\right) \tag{4.105}$$

Example 4.10

A 4-pole wave-connected DC generator has 500 conductors and delivers a current of 30 A. The brushes have been displaced through an angle of 6° from the geometrical axis. Calculate the (i) demagnetizing ampere-turns, and (ii) cross-magnetizing ampere-turns.

Solution

The current per conductor is calculated as,

$$I_c = \frac{I_a}{A} = \frac{30}{2} = 15\,\text{A} \tag{4.106}$$

The demagnetizing ampere-turns per pole can be determined as,

$$AT_d = ZI_c \frac{\alpha}{360} = 500 \times 15 \frac{6}{360} = 125\,\text{At} \tag{4.106}$$

The cross-magnetizing ampere-turns per pole can be determined as,

$$AT_c = ZI_c\left(\frac{1}{2P}-\frac{1}{360}\alpha\right)500\times 15\left(\frac{1}{2\times 4}-\frac{1}{360}6\right)=812.5\,\mathrm{At} \tag{4.107}$$

Practice Problem 4.10

A 6-pole wave-connected DC generator has 600 conductors and delivers a current of 100 A. The brushes have been displaced through an angle of 10º from the geometrical axis. Find the demagnetizing ampere-turns and cross-magnetizing ampere-turns.

4.22 Cancellation of Armature Reaction

The problem of armature reaction cannot be solved by shifting the brush position to the new magnetic neutral plane. The effect of the armature reaction varies with the load current. So, each time a new load current magnetic neutral plane is needed to be shifted to the new position. This means that the brush position must be changed. There are a few methods to cancel or neutralize the effect of armature reaction. Here, two methods, namely, compensating windings and interpoles are discussed.

Compensating Windings: Compensating windings are used to cancel the armature reaction due to armature flux on a large generator. These windings are placed in the slots of the pole facing the main field poles.

The compensating winding is connected in series with the armature in such a way that the current through the compensating winding is in the opposite direction to the current in the armature conductors. The compensating windings are connected in series to produce a magnetic field, which varies directly with the armature current. The flux produced by the compensating winding is equal and opposite to the armature flux. As a result, the armature flux is cancelled completely. The connection of the compensating winding is shown in Fig. 4.37.

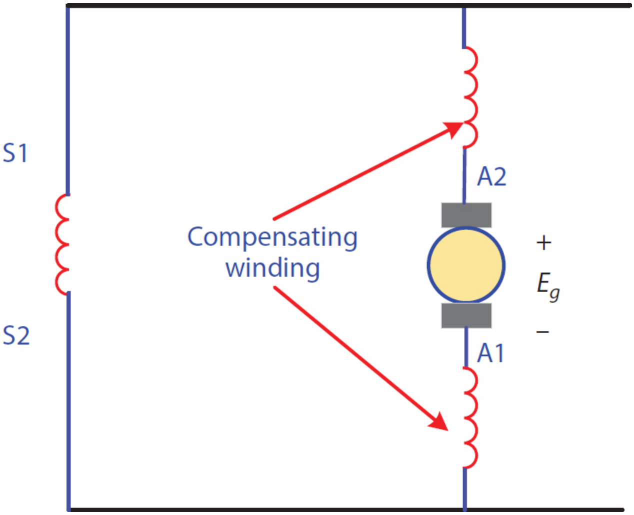

S1
S2
A2
A1
Compensating winding
+
E_g
−

Fig. 4.37 Application of compensating winding.

Interpoles: Interpoles, often called commutating or narrow poles, are placed in between the main field poles. The cross-section of the main field poles along with the interpoles is shown in Fig. 4.38. The interpoles have a few turns of large wire and are connected in series with the armature winding. The current through the interpoles sets up a flux. This flux is equal and opposite to the flux due to armature winding. As a result, the armature flux is cancelled completely. A shunt generator with an interpole is shown in Fig. 4.38(b).

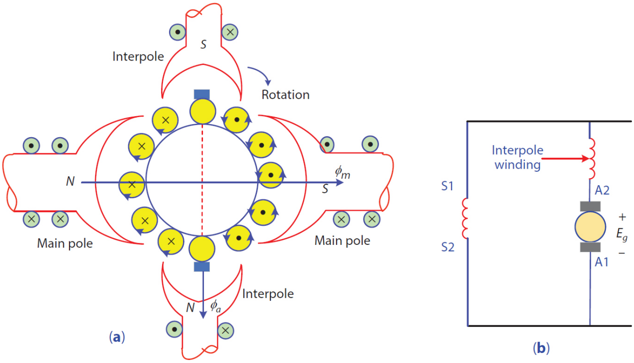

Interpole
S
Rotation
N
ϕ_m
S
Main pole
Main pole
Interpole
N
ϕ_a
(a)
Interpole winding
S1
S2
A2
A1
+
E_g
−
(b)

Fig. 4.38 Position of interpoles.

4.23 Series Generator Characteristics

There are three types of characteristics in a DC generator. These are no-load, external and internal characteristics. The no-load characteristics often refer to open-circuit characteristics that find the relationship between the generated emf E_0 versus field current I_f. In external characteristics, it establishes the relationship between the terminal voltage V_t and the load current I_L, whereas in internal characteristics, it finds the relationship between the generated emf E_g and the armature current I_a.

In a self-excited DC series generator, the field winding is connected in series with the armature as well as with the load circuit. Therefore, the same current will flow through the whole circuit. Initially, the generator is driven without a load, no current will flow through the series field winding. As a result, the magnetomotive force in the series field winding is zero. In this case, the terminal voltage will be equal to E_r, which is produced by the residual flux. Again, the generator is driven with a load and it will deliver a current to the load through the series field. The series field winding will produce a magnetomotive force. This magnetomotive force will produce the flux that aids the residual flux, which will increase the total flux. Therefore, the generated voltage will be increased to E_0. The terminal voltage of the series generator is normally reduced due to series field drop, armature resistance drop and armature reaction (AR) drop as shown in Fig. 4.39.

The series generator is used for welding and electro-plating purposes. It is also used for boosting DC transmission lines to compensate for the transmission line drop.

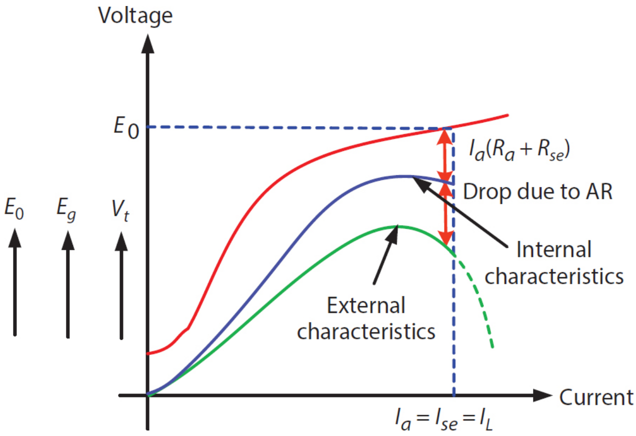


Fig. 4.39 Series generator characteristics.

4.24 Shunt Generator Characteristics

A load is connected across the terminals of the generator. An ammeter and voltmeter are connected to the load circuit to measure the load current and the terminal voltage. The characteristics between the terminal voltage V_t and the load current I_L are known as the external characteristics of a DC shunt generator. The field resistance of a shunt generator is higher than the armature circuit resistance. Therefore, less current will flow through the shunt field winding; most of the current will pass through the load circuit. The terminal voltage of the shunt generator can be calculated by the following formula:

$$V_t = E_g - I_a R_a \tag{4.108}$$

A rheostat and ammeter are connected to the field circuit as shown in Fig. 4.40. Initially, run the generator with its rated speed without the load and measure the no-load generated voltage E_0. Then run the generator with the load. Then run the generator by gradually increasing the load and keeping the field current constant with the help of field rheostat.

The current through the armature conductors sets up the flux that distorts and weakens the main field flux. As a result, the resultant flux is reduced. Therefore, the terminal voltage of the generator reduces slightly. The terminal voltage also reduces due to the drop of the armature circuit resistance, i.e., $I_a R_a$ drop. The terminal voltage is reduced when the load is increased. This reduced terminal voltage reduces the field current. As a result, terminal voltage will be reduced slowly until the saturation region of the magnetic materials is reached. Again, the load current is increased to reduce the terminal voltage until the breakdown point is obtained. At the breakdown point, the magnetic circuit falls on its saturation property. Therefore, the terminal voltage reduces sharply towards zero with a change in the field current as shown in Fig. 4.41.

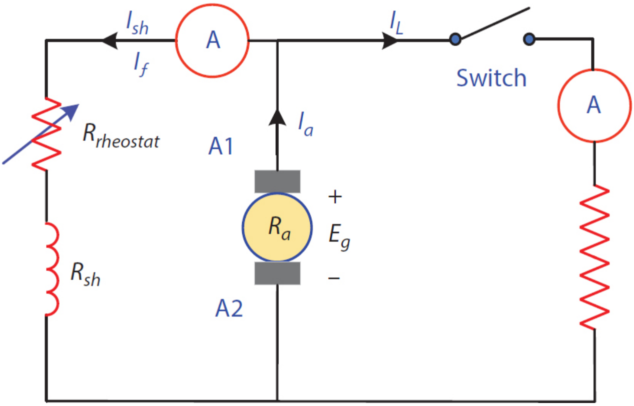


Fig. 4.40 Shunt generator with a resistive load.

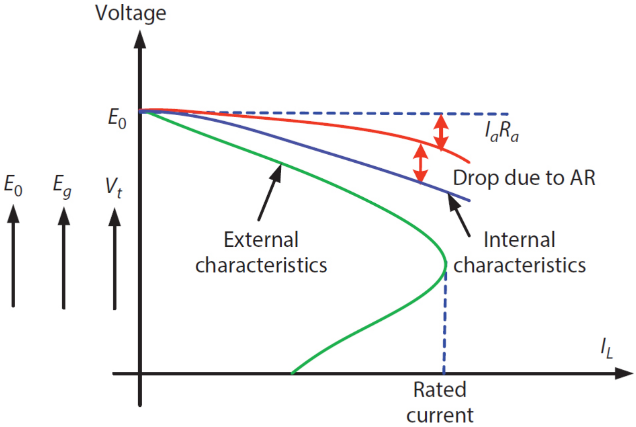


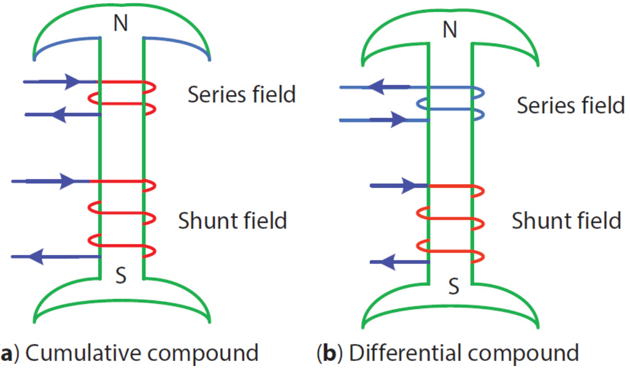
Fig. 4.41 Shunt generator characteristics.

4.25 Compound Generator Characteristics

The characteristics of a series generator and shunt generator have already been discussed. It is seen that the characteristic of a series generator is rising and that of the shunt generator is dropping. However, it is difficult to use these generators due to their characteristics. Therefore, there is a need to combine these generators for a unique operation with the achievement of good characteristics. This can be done by winding both series and shunt field windings on each pole as shown in Fig. 4.42.

When a flux produced by the series field winding aids the flux produced by the shunt field winding it is known as a cumulative compound generator, whereas when a flux produced by the series field winding opposes the flux produced by the shunt field winding it is known as a differential compound generator.

In a cumulative compound generator, the number of turns of the shunt field winding is higher than the series field winding. The terminal voltage of the differential compound generator is less than the cumulative and shunt generator. The degrees of compounding of the generators are shown in Fig. 4.43 and are discussed in the following ways.



(**a**) Cumulative compound (**b**) Differential compound

Fig. 4.42 Schematics of a compound generator.

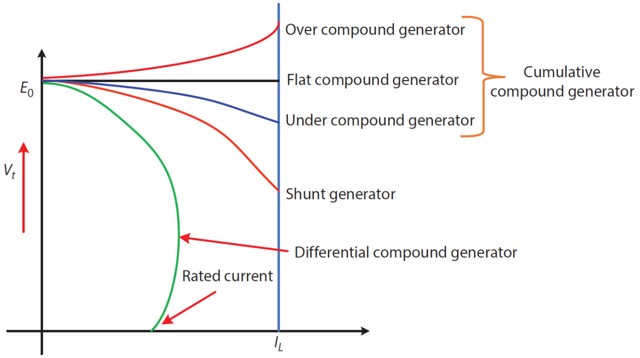


Fig. 4.43 Degree of a compound.

Over compound generator: The generator whose total flux due to series and shunt field is of such magnitude that the terminal voltage rises with an increase in the load is known as an over compound generator.

Flat compound generator: In the generator, the terminal voltage decreases with the increase of the load. However, when the no-load terminal voltage is equal to the load terminal voltage, the generator is known as a flat compound generator. This generator is used when the distance between the generator and the load is short.

Under compound generator: When the full load terminal voltage of the compound generator is lower than that of the no-load terminal voltage, it is known as an under compound generator.

4.26 Applications of a DC Generator

According to their characteristics, DC generators are used for different purposes and are discussed here.

Separately excited generators: This type of generator is used for laboratory equipment testing, electroplating, electrorefining, etc.

Shunt generator: This generator has good characteristics with a constant terminal voltage. It is used for charging storage batteries. It is also used for running normal lights.

Series generator: The series generator has rising characteristics. Therefore, it is used as a series booster to increase the magnitude of the voltage across the feeder line.

Flat compound generator: This type of generator is used to supply electrical power where the distance between the generator and the load is small.

Over compound generator: This type of generator is used to supply electrical power where the distance between the generator and the load is long.

Differential compound generator: This generator is used for arc welding where a high voltage drop occurs with an increase in current.

4.27 Voltage Regulation of DC Generator

Power rating, voltage, efficiency and voltage regulation are used to compare DC generator performance. The voltage regulation of a DC generator is used to identify the change in terminal voltage with an increase in the generator load. The generator is said to have good voltage regulation if there is a small change in voltage from no load to full load. Otherwise, it is considered a poor voltage regulation. Voltage regulation is defined as the change in terminal voltage from no-load to full load and it is expressed as a percentage of full load terminal voltage. Mathematically, it can be defined as,

$$\text{(4.109)} \quad \mathrm{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100$$

where,

VR is the voltage regulation,

V_{nl} is the no-load terminal voltage,

V_{fl} is the full load terminal voltage.

The voltage regulation of a DC generator is directly proportional to the speed and inversely proportional to the torque.

Example 4.11

A 4-pole DC shunt generator delivers 15A current to a 20 Ω resistive load. The armature resistance and shunt field resistance are 0.4 Ω and 100 Ω, respectively. The total brush drop is 3V. Calculate the voltage regulation.

Solution

The full load terminal voltage is calculated as,

$$V_t = E_{fl} = I_L R = 15 \times 20 = 300\text{V} \tag{4.110}$$

The shunt field current is calculated as,

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{300}{100} = 3\text{A} \tag{4.111}$$

The armature current is calculated as,

$$I_a = I_{sh} + I_L = 3 + 15 = 18\text{A} \tag{4.112}$$

The no-load voltage is calculated as,

$$E_g = E_0 = V_t + I_a R_a + \text{Brush drop} = 300 + 18 \times 0.4 + 3 = 310.2\text{V} \tag{4.113}$$

The voltage regulation is calculated as,

$$\text{VR} = \frac{E_g - V_t}{V_t} \times 100 = \frac{310.2 - 300}{300} \times 100 = 3.4\% \tag{4.114}$$

Practice Problem 4.11

A 4-pole DC shunt generator delivers 10A current to a 20 Ω resistive load. The armature resistance and shunt field resistance are 0.5 Ω and 90 Ω, respectively. The voltage drop per brush is 1V. Determine the voltage regulation.

4.28 Losses of a DC Generator

There are three major losses in a dc generator. These are copper loss, mechanical loss and magnetic or iron loss. The higher values of these losses decrease the efficiency of the generator. Therefore, the main objectives are to reduce the losses to increase the efficiency of the generator.

Copper losses: When current flows through the copper wire, a loss of I^2R occurs, which is known as copper loss. The copper losses for the different windings are named in the following,

- Armature copper loss, $I_a{}^2R_a$,
- Shunt field copper loss, $I_{sh}{}^2R_{sh}$,
- Series field copper loss, $I_{se}{}^2R_{se}$,
- Interpoles field copper loss ($I_a{}^2R_i$) and compensating field copper loss ($I_a{}^2R_c$),
- Brush drop (2 V_{bIa}).

Mechanical loss: Mechanical loss occurs due to some associated components. These are friction between the shaft and bearings, friction between the commutator and brushes and friction between the rotating armature and surrounding air.

Iron loss: The losses that occur due to the iron parts of the generator are known as iron loss. This loss is often known as core loss or magnetic loss. Iron loss is classified into two types such as eddy current loss and hysteresis loss.

Eddy current loss: A voltage will be induced due to the rotation of the armature in the magnetic field. Because of this voltage, a small magnitude of current will flow through the body of the armature core. This current is known as eddy current and the power loss due to this current is known as eddy current loss. Using thin laminations of varnish can reduce the eddy current loss. The eddy current loss depends on the magnetic flux density, frequency of the induced voltage, thickness and volume of the magnetic material. The eddy current loss can be expressed as,

$$P_e = k_e B_m{}^2 f^2 t^2 V \tag{4.115}$$

where,

f is the frequency of the induced voltage,

t is the thickness of the lamination,

V is the volume of the lamination,

B_m is the maximum magnetic flux density.

Hysteresis loss: The armature conductors are changing their position between the poles. Due to the new position of the conductor, the magnetic field also changes. Therefore, this loss occurs in the armature due to the reversal of the magnetic field. The hysteresis loss is expressed as,

$$(4.116)\quad P_h = \eta B_m^{1.6} f V$$

4.29 Efficiency of a DC Generator

Efficiency is used to identify the efficiency of a DC generator. The term efficiency is defined as the ratio of the output power to the input power. The efficiency of a generator depends on the load, which is connected across its terminals. Therefore, the efficiency of the DC generator varies with the load as well as the losses. Consider a shunt generator whose terminal voltage is V_t and delivers a load current of I_L.

The generator output power is expressed as,

$$(4.117)\quad P_o = V_t I_L$$

The expression of copper loss or variable loss is expressed as,

$$(4.118)\quad P_{cu} = I_a^2 R_a$$

The constant loss is represented as,

$$(4.119)\quad \text{Constant loss} = P_c$$

The input power can be determined as,

$$(4.120)\quad P_{in} = P_o + P_{cu} + P_c$$

Substituting equations (4.117) and (4.118) into equation (4.120) yields,

$$(4.121)\quad P_{in} = V_t I_L + I_a^2 R_a + P_c$$

The generator efficiency is expressed as,

$$\eta = \frac{P_0}{P_{in}} \tag{4.122}$$

Substituting equations (4.117) and (4.121) into equation (4.122) yields,

$$\eta = \frac{P_0}{P_{in}} = \frac{V_t I_L}{V_t I_L + {I_a}^2 R_a + P_c} \tag{4.123}$$

The shunt field current is very small compared to the load current. Therefore, the armature current is equal to the load current. Therefore, equation (4.123) can be modified as,

$$\eta = \frac{1}{1 + \frac{I_a R_a}{V_t} + \frac{P_c}{V_t I_a}} \tag{4.124}$$

The efficiency will be maximum when the denominator of the equation (4.88) is minimum. The denominator will be minimum if the differentiation of the denominator of equation (4.124) with respect to the armature current is equal to zero. Then it can be expressed as,

$$\frac{d}{dI_a}\left(1 + \frac{I_a R_a}{V_t} + \frac{P_c}{V_t I_a}\right) = 0 \tag{4.125}$$

$$\frac{R_a}{V_t} - \frac{P_c}{V_t {I_a}^2} = 0 \tag{4.126}$$

$$P_c = {I_a}^2 R_a \tag{4.127}$$

$$\text{Constant loss} = \text{Variable loss} \tag{4.128}$$

From equation (4.128), it is seen that the efficiency of a generator will be maximum when the constant loss is equal to the variable loss.

Example 4.12

A 12 kW, 250 V, 4-pole lap wound shunt generator runs at a speed of 1000 rpm. The armature has 500 conductors and the full load copper loss is 600 W. Determine the flux per pole. Neglect the shunt current and consider the total brush drop as 2V.

Solution

The armature current is calculated as,

$$I_a = \frac{12000}{230} = 48\,\text{A} \tag{4.129}$$

The value of the armature resistance is calculated from the copper loss as,

$$I_a^{\,2} R_a = 600\,\text{W} \tag{4.130}$$

$$R_a = \frac{600}{48^2} = 0.26\,\Omega \tag{4.131}$$

The generated voltage is then determined as,

$$E_g = V_t + I_a R_a + \text{brush drop} = 250 + 48 \times 0.26 + 2 = 264.5\,\text{V} \tag{4.132}$$

The flux per pole is determined as,

$$\phi = \frac{E_g 60 A}{PZN} = \frac{264.5 \times 60 \times 4}{4 \times 500 \times 1000} = 0.032\,\text{Wb} \tag{4.133}$$

Example 4.13

A 4-pole DC generator is delivering 25 A current from its 230 V terminal voltage. The armature resistance is 0.2 Ω and the shunt field resistance is 55 Ω. The voltage drop per brush is 1 V. Calculate the induced voltage and efficiency of the generator.

Solution

The shunt field current is calculated as,

$$I_{sh} = \frac{230}{55} = 4.2\ \text{A} \tag{4.134}$$

The armature current is then calculated as,

$$I_a = I_L + I_{sh} = 25 + 4.2 = 29.2\ \text{A} \tag{4.135}$$

The generated voltage is calculated as,

$$E_g = V_t + I_a R_a + \text{brush drop} = 230 + 29.2 \times 0.2 + 2 = 237.8\ \text{V} \tag{4.136}$$

The power generated in the armature is calculated as,

$$P_{arm} = E_g I_a = 237.8 \times 29.2 = 6944.9\ \text{W} \tag{4.137}$$

The power absorbed by the load is determined as,

$$P_L = V_t I_L = 230 \times 25 = 5750\ \text{W} \tag{4.138}$$

The efficiency of the generator is calculated as,

$$\eta = \frac{P_L}{P_{arm}} = \frac{5750}{6944.9} = 82\% \tag{4.139}$$

Practice Problem 4.12

The speed of a 10 kW, 220 V, 4-pole wave wound generator is 1000 rpm. The armature has 400 conductors and the full load copper loss is 500 W. Determine the flux per pole and the efficiency of the generator. Consider the shunt field resistance of 60 Ω and the total brush drop of 2 V.

Practice Problem 4.13

A 4-pole DC generator is delivering 15 A current from its 220 V terminal voltage. The armature resistance is 0.5 Ω and the shunt field resistance is 85 Ω. The voltage drop per brush is 1.5 V. Find the efficiency of the generator.

4.30 Parallel Operation of a DC Generator

Parallel operation of the DC generators is necessary when customer demand exceeds the current load. The continuous power supply is also important for the parallel operation of the DC generators. Efficiency is another parameter to run the generators to their maximum rated condition to satisfy excess customer demand. Generators usually run 24 hours a day, 30 days a month. Therefore, maintenance and repair are also important for the parallel operation of DC generators. For parallel operation of the generators, the following conditions must be considered.

- The terminal voltage of the connected generators must be the same.
- The polarities of the terminal voltage of the connected generators must be the same.
- The prime movers for both generators must be the same.
- The change of the terminal voltage with the load for both generators must have the same characteristics.

Two shunt generators 1 and 2 are connected to a busbar to supply the load as shown in Fig. 4.44. Generator 1 runs at its full-load rated speed by a prime mover. Then turn on the switch SW1 that delivers power to the busbar. Now, run generator 2 by a prime mover and adjust the field circuit rheostat to indicate the same voltage as the busbar voltage. Once the voltmeter reading is the same as the busbar voltage then turn on the switch SW2. Hence, both generators are in parallel to supply the load.

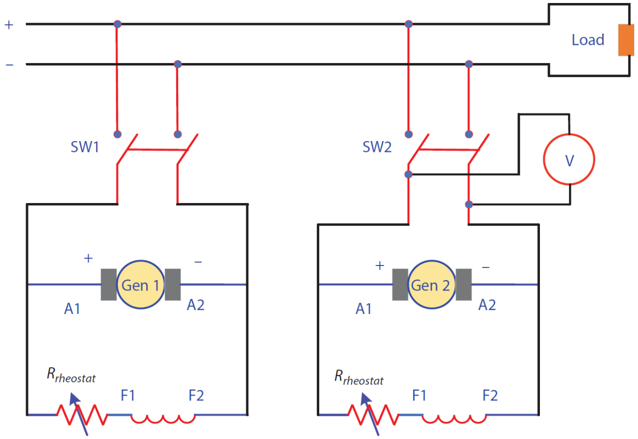


Fig. 4.44 Shunt generators in parallel.

Again, disconnect the switch SW2 and adjust the field rheostat of generator 1 to supply the entire load. If the capacity of the generator 1 is enough to provide power to the load then there is no need to connect the generator 2. In this case, generator 2 is considered in a floating condition. Similarly, disconnect the switch SW1 and adjust the field rheostat of generator 2 so that generator 2 can supply the entire load. In this condition, generator 1 is in a floating stage.

Again, consider two identical series generators are connected in parallel as shown in Fig. 4.45. Initially, consider that the generated emf E_{g1} is slightly higher than the E_{g2}. As a result, the current in the series field winding of generator 1 will be strengthened which will further increase the generated emf E_{g1} whereas the series field winding of generator 2 will be weakened. In this condition, the generated E_{g2} will be decreased further. Finally, generator 1 will deliver power to the load as well as generator 2 and generator 2 will run as a motor. To avoid this situation and for the smooth operation of the DC generators connected in parallel, a conductor with negligible resistance known as an equalizer bar is connected at the end armature of the two generators.

The function of the equalizer bar is to make an equal flow of currents through the series field windings of the two generators.

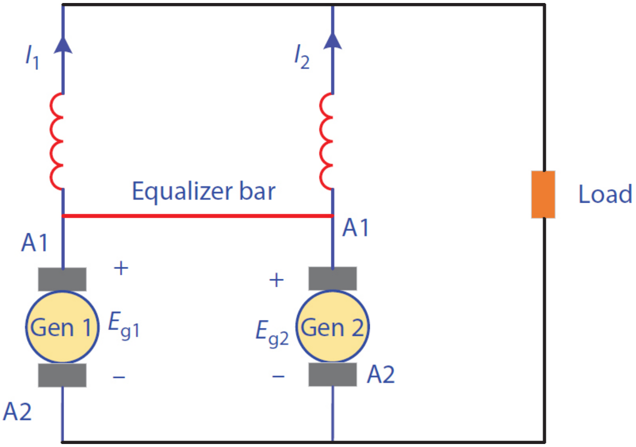


Fig. 4.45 Series generators in parallel.

Exercise Problems

4.1 A conductor of 10 m in length is moving in a magnetic field with a speed of 25 m/s. The magnetic flux density of the field is 0.5 Tesla. Determine the induced voltage in the conductor.

4.2 A conductor of 75 m in length is moving in a magnetic field with a speed of 85 m/s. The induced voltage in the conductor is 124 V. Determine the magnetic flux density.

4.3 A conductor is moving in a magnetic field with a speed of 50 m/s. The induced voltage in the conductor is 100 V. Determine the length of the conductor if the magnetic flux density is 0.05 T.

4.4 A conductor of 16 m in length is moving in a magnetic field at a certain speed. The induced voltage in the conductor is 256 V. Determine the speed of the conductor if the magnetic flux density is 0.5 T.

4.5 A conductor of 0.8 m in length is moving in a uniform magnetic field at an angle of 45°. Find the induced voltage if the speed of the conductor is 30 m/s and the magnetic flux density is 1.5 T.

4.6 A conductor of 15 m in length is moving in a magnetic field. The magnetic flux density of the conductor is 2.2 T at a certain angle. The conductor induced a voltage of 160 V with a velocity of 10 m/s. Find the angle of the conductor.

4.7 A 4-pole wave-wound generator has 30 slots and 5 conductors are placed per slot. The flux per pole is 0.05 Wb. Calculate the generated emf when the generator is driven at 1200 rpm.

4.8 The number of armature conductors of a 4-pole DC generator is 500. The conductors are wave connected and the flux per pole is 0.022 Wb. Determine the speed of the generated voltage is 220 V.

4.9 A 6-pole, lap wound generator is supplying a load current of 115 A. The number of conductors in the generator is 500. Find the electromagnetic torque if the flux per pole is 0.06 Wb.

4.10 The electromagnetic torque of a 4-pole wave wound generator is 300 N-m. The generator has 200 conductors and delivers a current of 105 A to the load. Find the flux per pole.

4.11 The armature of a 4-pole lap-wound generator has 60 slots. The armature is rotating at a speed of 600 rpm to generate a voltage of 220 V. If each coil has 4 turns, calculate the flux per pole of the generator.

4.12 The induced voltage of a wave-connected generator is 250 V and the flux per pole is 0.025 Wb. The number of conductors in the generator is 500. The generator is driven at a speed of 600 rpm. Determine the number of generator poles.

4.13 The induced voltage of a wave-connected generator is 200 V and the flux per pole is 0.02 Wb. The number of conductors in the generator is 500. The generator is driven at a speed of 600 rpm. Determine the number of poles of a generator.

4.14 The flux per pole of a 4-pole lap-connected generator is 0.05 Wb. The generator is driven at a speed of 600 rpm and the induced voltage is 220 V. Determine the number of conductors. Also, determine the electromagnetic torque if the generator delivers a current of 115 A.

4.15 The flux per pole of a 6-pole wave-connected generator is 0.05 Wb. The generator is driven at a speed of 800 rpm and the induced voltage is 480 V. Determine the number of conductors.

4.16 The number of conductors of a 400 V DC generator is 800. The flux per pole is 0.02 Wb and the number of parallel paths is 2. Determine the number of poles if the generator is driven at 500 rpm.

4.17 A shunt generator with a terminal voltage of 220 V delivers 200 A current to the load. The shunt field and armature resistances are 50Ω and 0.02Ω, respectively. Calculate the generated voltage.

4.18 The field resistance of the shunt generator is 55Ω and the terminal voltage is 220 V. It delivers a current of 175 A to the load. The armature circuit resistance is 0.001 times the shunt field resistance. Determine the generated voltage.

4.19 A 10 kW, 230 V shunt generator has an armature resistance of 0.02Ω and a field resistance of 65Ω. Determine the generated voltage if the total brush drop is 3 V.

4.20 The armature resistance of a 120 V shunt generator is 0.12Ω. This generator is loaded progressively until the terminal voltage is zero. The load current is 85 A and the field current is 5 A. Determine the generated voltage.

4.21 The armature resistance of a shunt generator is 0.15Ω. This generator is loaded progressively until the terminal voltage is equal to 100 V. The load current is 95 A and the field current is 5 percent of the load current. Find the generated voltage.

4.22 A 230 V shunt generator delivers 195 A current to the load. The shunt field and armature resistances are 55Ω and 0.02Ω respectively. Calculate the generated voltage. Consider the voltage drop per brush is 2 V.

4.23 A 25 kW, 220 V shunt generator has a field resistance of 52Ω and an armature resistance of 0.012 Ω. Calculate the generated voltage. Consider the total brush drop is 0.2% of the terminal voltage.

4.24 A 220V shunt generator is delivering a current of 155 A to the load and the shunt field resistance is 55Ω. Find the armature resistance if the generated voltage is 240 V.

4.25 The series field resistance of a 10 kW, 200 V series generator is 0.05Ω and the armature resistance is 0.03Ω. Determine the generated voltage if the total brush drop is 3 V.

4.26 A 12 kW, 220 V series generator has a series field resistance of 0.04Ω. Find the armature resistance if the induced voltage is 250 V.

4.27 A short shunt compound generator is delivering 70 A current from its 230 V terminal voltage. The armature, series, and shunt field resistance are 0.04Ω, 0.03Ω and 50Ω respectively. Voltage drop per brush is 1.5 V. Calculate the induced voltage.

4.28 A long shunt generator is delivering 146 A load current from its 230 V terminal voltage. The shunt field, series field, diverter and armature circuit resistances are 45Ω, 0.04Ω, 0.023Ω and 0.02Ω, respectively. Find the generated voltage and power distribution.

4.29 A short shunt generator is delivering 150 A load current from its 220 V terminal voltage. The shunt field, series field, diverter and armature circuit resistances are 55Ω, 0.02Ω, 0.03Ω and 0.1Ω, respectively. Calculate the generated voltage. The diverter is connected in parallel with the series field.

4.30 A 230 V shunt generator is delivering 185 A current to the load. The armature resistance is 0.01 Ω and the shunt field resistance is 50Ω. The iron and friction losses are 800 W. Determine the induced voltage, copper losses, output power and efficiency.

4.31 The armature conductor of a 4-pole lap-wound DC generator is 500 and delivers a current of 100 A. If the brushes lead at an angle of 20°, then determine the cross and demagnetizing components.

4.32 The full load current of the 230 V shunt generator is 175 A. The shunt field resistance is 55Ω and the stray losses are 560 W. Find the armature resistance if the generator efficiency is 80%. Also, determine the load current corresponding to maximum efficiency.

4.33 A 230 V short shunt generator is delivering 120 A current to the load. The resistances of the shunt field, series field and armature circuit are 55Ω, 0.02Ω and 0.03Ω, respectively. The iron and friction losses are 800 W and the total brush drop is 3V. Determine the induced voltage, copper losses, output power and efficiency.

5

Direct Current Motors

5.1 Introduction

Electrical power is normally generated, transmitted, and distributed in alternating current (AC) form. Household electrical appliances and industrial sectors use alternating power. However, there are some places where AC supply is not available. In those places, it is difficult to use an AC motor. As an alternative, a DC motor is used to supply electrical power. A DC motor is a machine which converts DC power into mechanical power. Normally, the horsepower is used to represent the unit of the DC motor and its size varies from fractional horsepower to over 1000 horsepower. The speed of the DC motor can be controlled over a wide range. Because of this facility, DC motors are extensively used in a control system as a positioning device. The more suitable applications of a DC motor are airplanes, computers, plotters, printers, robots, toys, electric shavers, movie cameras, audio, video, DVD players, rolling mills, hoist cranes, the mining industry, printing presses, etc. Its operation is based on the principle that when a current-carrying conductor is placed in a magnetic field, the conductor experiences a force. This force plays an important role in producing a powerful torque. The magnitude of the force is given by the equation,

$$F = BIl \tag{5.1}$$

where,

B is the flux density in Wb/m^2,

l is the length of the conductor in meter,

I is the current in amperes.

The working principle, classification, characteristics, and starting and speed control of a DC motor are discussed in this chapter.

5.2 Working Principle

A DC supply is fed to the armature through the brush and commutator as shown in Fig. 5.1. The commutator changes the direction of the current in the armature conductors as it passes across the field poles. Whenever a current-carrying conductor is placed in a magnetic field it would experience a force. The fields produced by the main poles as indicated from the N-pole to the S-pole. The current entering the conductor (cross) creates a magnetic field that opposes the main field. The interaction of the two fields due to the main pole and the conductor creates the resultant upward force (F_u). The current coming out from the conductor (dot) creates the magnetic field that aids the main field. The interaction of the two fields due to the main pole and the conductor creates the resultant downward force (F_d). However, the downward force is higher than the upward force that rotates the motor in the clockwise direction. The direction of this force is determined by Fleming's left-hand rule. The magnitude of the force is expressed as,

$$(5.2)\quad F_{u,d} = BlI$$

where,

B is the magnetic flux density in T,

l is the active length of the conductor in m,

I is the current flowing through the conductor in A,

F is the Lorentz force in N.

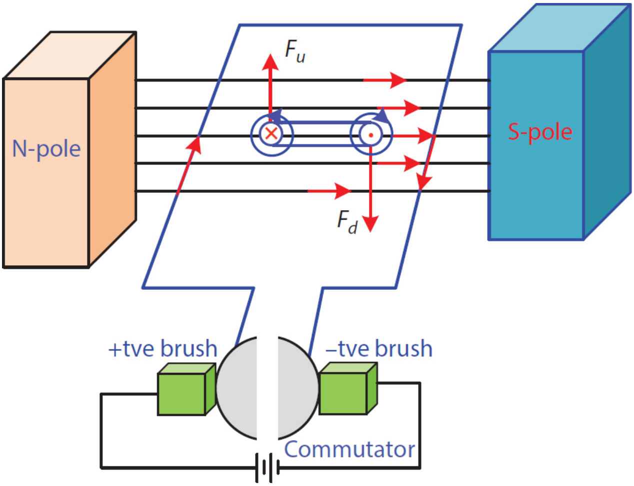


Fig. 5.1 Schematic of a DC motor.

Example 5.1

A conductor of 10 m in length is rotating in the magnetic field. The current in the conductor is 2 A and the magnetic flux density is 0.56 T. Determine the magnitude of the force.

Solution

The magnitude of the force can be determined as,

$$F = BlI = 0.56 \times 10 \times 2 = 11.2\,\mathrm{N} \tag{5.3}$$

Practice Problem 5.1

A current in the 50 m conductor is 1 A and it is rotating in a magnetic field whose flux density is 0.89 T. Determine the force.

5.3 Construction

The main parts of a DC motor are the yoke, stator (magnetic field system), rotor (armature core), armature winding, commutator, spring, and brushes as shown in Fig. 5.2. The yoke is made up of steel which supports the poles and provides overall mechanical protection. The magnetic field system is the stationary part of the motor bolted to the yoke and is wound with filed coils.

A field pole is always an even number and produces the main field flux. The armature core (rotor) is mounted to the shaft and provides slots in its outer periphery. The armature windings are placed in these slots. The armature winding is made up of copper conductors and is classified as lap winding and wave winding. The commutator is made of wedge-shaped copper segments that are insulated from each other by the mica. Each segment of the commutator is connected to the end of each coil. The commutator is a mechanical rectifier that converts the DC voltage to an alternating voltage to the coils. The brushes are made up of carbon and are attached to the commutator through a suitable spring pressure.

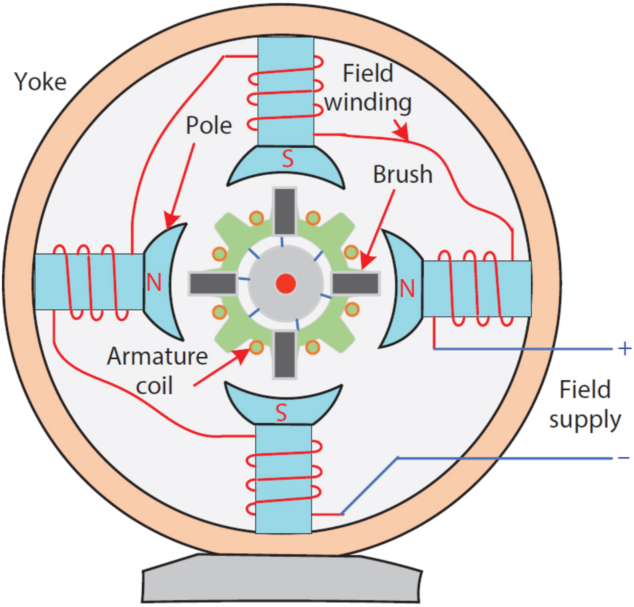


Fig. 5.2 DC motor components.

5.4 Back EMF

When the armature of the motor rotates, the conductors also rotate in the magnetic field. Therefore, these conductors are cut by the magnetic flux and according to the laws of electromagnetic induction, an emf will be induced in the conductors. The direction of the induced voltage can be determined by Fleming's right-hand rule and it is found that this induced emf opposes the applied voltage. Due to its opposite action, this induced voltage is known as the back emf or counter emf of the motor. The back emf is always less than the applied voltages and it is denoted by E_b. The connection diagram of the shunt motor is considered to explain the back emf, which is shown in Fig. 5.3.

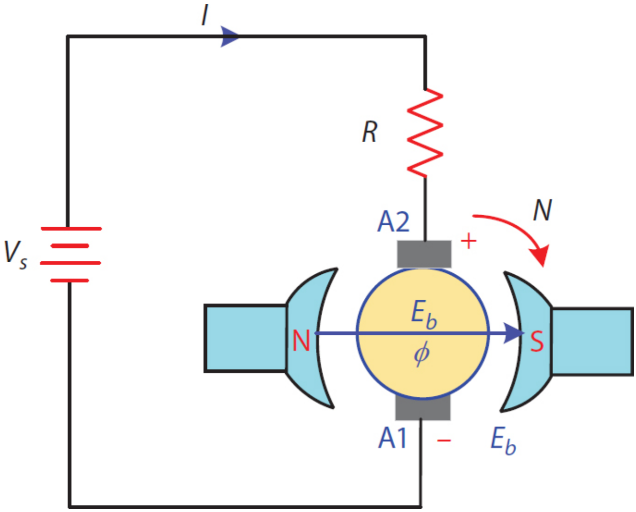


Fig. 5.3 DC motor with back emf.

The following parameters can be defined as follows,

E_b is the back emf of the motor,

V_s is the applied or source voltage.

The expression of back emf is written as,

$$E_b = \frac{PZ\phi N}{60A} \tag{5.4}$$

But if the number of poles P, number of conductors Z and number of parallel paths A are constant, then equation (5.4) is modified as,

$$E_b \infty \phi N \tag{5.5}$$

Apply KVL in the circuit in Fig. 5.3 yields,

$$-V_s + I_a R + E_b = 0 \tag{5.6}$$

The current through the armature circuit resistance is,

$$I_a = \frac{V_s - E_b}{R_a} \tag{5.7}$$

5.5 Necessity of Back EMF

The back emf plays an important role in controlling the additional load in a DC motor. Consider the expression of armature current as shown in equation (5.7) to explain the importance of the back emf. During the starting time, the back emf is zero because the armature does not rotate. For a small magnitude of the armature circuit resistance, the current in the armature will be extremely high, which can be explained by the following equation,

$$I_a = \frac{V_s - 0}{R_a} \uparrow \tag{5.8}$$

This armature current is 25 times higher than the full load current of the motor. This high magnitude of the armature current may damage the fuses or trip the circuit breaker of the motor. This may also increase the force, which produces powerful torque that accelerates the armature rapidly. The back emf increases as the speed of the motor increases, which decreases the net voltage across the armature circuit. As a result, the armature current decreases, which decreases the torque. Because of this reduced torque, the motor will stop. Therefore, the back emf in a DC motor regulates the flow of the armature current depending on the additional connected load.

5.6 Classification of a DC Motor

DC motors are operated from an external power supply. Based on the external power supply, the DC motor is considered as separately excited. According to the field arrangements, DC motors are classified as shunt, series and compound motors.

5.6.1 Shunt Motor

A shunt motor is one whose field winding is connected across the armature circuit as well as across the supply as shown in Fig. 5.4. The field winding consists of large turns with fine wire. The current supplied to the motor is divided into two paths. One part of the current is flowing through the shunt field and the other part is flowing through the armature circuit. The expression of the terminal voltage is,

$$V_t = E_b + I_a R_a \tag{5.9}$$

The expression of the back emf is,

$$E_b = V_t - I_a R_a \tag{5.10}$$

The expression of the shunt field current is,

$$I_{sh} = \frac{V_t}{R_{sh}} \tag{5.11}$$

The armature current can be determined as,

$$I_a = I_L - I_{sh} \tag{5.12}$$

The expression of the input power is,

$$P_{in} = V_t I_L \tag{5.13}$$

The expression of the shunt field copper loss is,

$$P_{sh} = V_t I_{sh} \tag{5.14}$$

The expression of the armature copper loss is,

$$P_{acu} = I_a^2 R_a \tag{5.15}$$

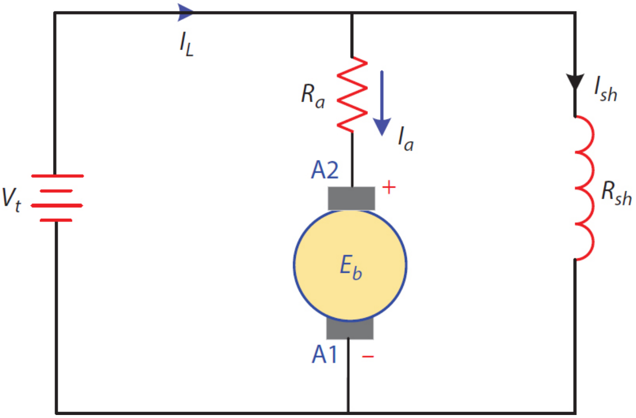


Fig. 5.4 Connection diagram of a DC shunt motor.

The expression of total loss is written as,

$$P_{tloss} = V_t I_{sh} - I_a^2 R_a \tag{5.16}$$

The power developed can be determined as,

$$P_{dshm} = \text{input power-losses} \tag{5.17}$$

$$P_{dshm} = P_{in} - P_{tloss} \tag{5.18}$$

Substituting equations (5.13) and (5.16) into equation (5.18) yields,

$$P_{dshm} = V_t I_L - V_t I_{sh} - I_a^2 R_a \tag{5.19}$$

$$P_{dshm} = V_t (I_L - I_{sh}) - I_a^2 R_a \tag{5.20}$$

Substituting equation (5.12) into equation (5.20) yields,

$$P_{dshm} = V_t I_a - I_a^2 R_a \tag{5.21}$$

$$P_{dshm} = (V_t - I_a R_a) I_a \tag{5.22}$$

Substitute equation (5.10) into equation (5.22) and it becomes,

$$P_{dshm} = E_b I_a \tag{5.23}$$

5.6.2 Series Motor

In a series motor, the field winding is connected in series with the armature. Normally, the series field is wound by a few turns of thick wire, which can carry the rated full load current of the motor. The schematic diagram of the series motor is shown in Fig. 5.5. The same current will flow through the series field and the armature windings. The armature current can be written as,

$$(5.24)\, I_a = I_L$$

The expression of the terminal voltage can be written as,

$$(5.25)\, V_t = E_b + I_a(R_a + R_{se})$$

$$(5.26)\, E_b = V_t - I_a(R_a + R_{se})$$

The expression of the power developed in a series motor is derived as,

$$(5.27)\, P_{dsem} = P_{in} - P_{tloss}$$

$$(5.28)\, P_{dsem} = V_t I_a - I_a^2(R_a + R_{se})$$

$$(5.29)\, P_{dsem} = I_a[V_t - I_a(R_a + R_{se})]$$

Substituting equation (5.26) into equation (5.29) yields,

$$(5.30)\, P_{dsem} = E_b I_a$$

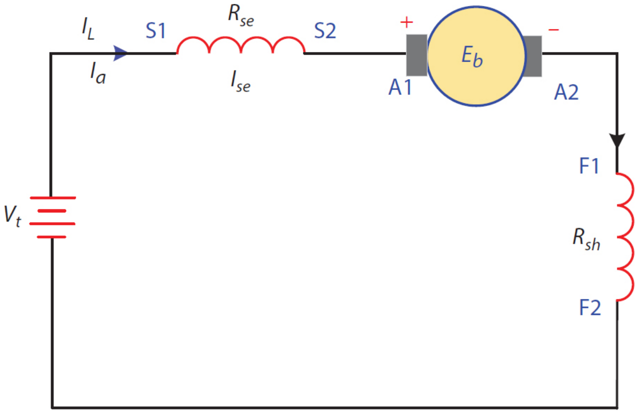


Fig. 5.5 Connection diagram of a DC series motor.

5.6.3 Compound Motor

There are two field windings in the compound motor. These field windings are arranged in different ways. According to the field arrangement, the compound motor can be classified as a short shunt compound motor and a long shunt compound motor. These motors are discussed below.

5.6.3.1 Short Shunt Compound Motor

Here, the field winding is connected in parallel with the armature. Fig. 5.6 shows the connection diagram of a short shunt compound motor. The voltage across the shunt field circuit can be determined as,

$$-V_{ab}+E_b+I_aR_a=0 \tag{5.31}$$

$$V_{ab}=E_b+I_aR_a \tag{5.32}$$

$$-V_t+I_LR_{se}+V_{ab}=0 \tag{5.33}$$

$$V_{ab}=V_s-I_LR_{se} \tag{5.34}$$

The shunt field current can be determined as,

$$I_{sh}=\frac{V_{ab}}{R_{sh}}=\frac{V_s-I_LR_{se}}{R_{sh}} \tag{5.35}$$

Alternately, the shunt field current can be determined as,

$$I_{sh}=\frac{V_{ab}}{R_{sh}}=\frac{E_b+I_aR_a}{R_{sh}} \tag{5.36}$$

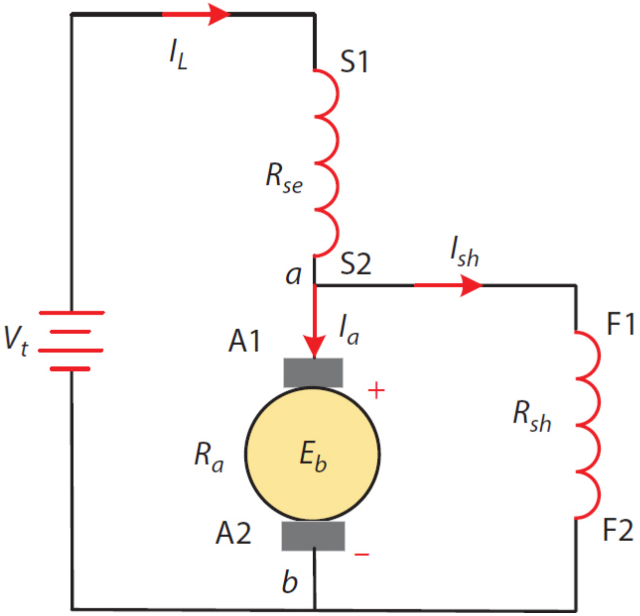


Fig. 5.6 Connection diagram of a short shunt compound motor.

5.6.3.2 Long Shunt Compound Motor

Here, field winding is connected in parallel with the series combination of the series field and the armature windings. The connection diagram of a long shunt compound motor is shown in Fig. 5.7. The current through the series field and armature is the same.

Applying KVL to the circuit as shown in Fig. 5.7 yields,

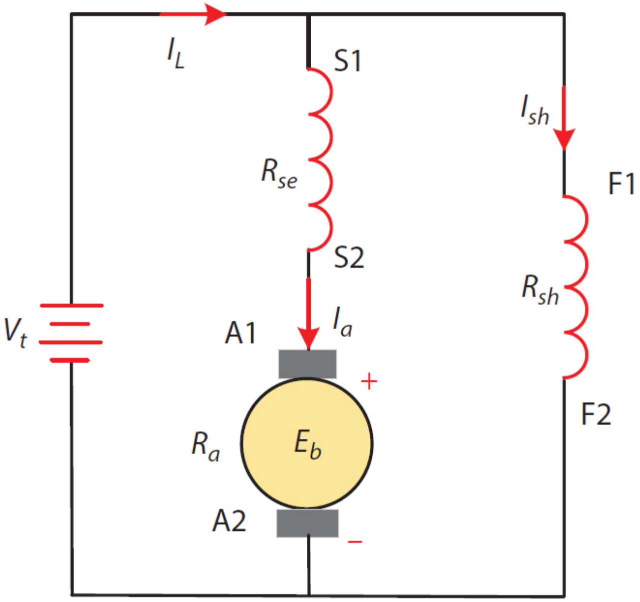


Fig. 5.7 Connection diagram of a long shunt compound motor.

$$-V_t + I_a(R_a + R_{se}) + E_b = 0 \tag{5.37}$$

$$E_b = V_t - I_a(R_a + R_{se}) \tag{5.38}$$

The shunt field current can be determined as,

$$I_{sh} = \frac{V_s}{R_{sh}} \tag{5.39}$$

Alternately, the shunt field current can be determined as,

$$I_{sh} = \frac{I_a(R_a + R_{se}) + E_b}{R_{sh}} \tag{5.40}$$

5.7 Mechanical Power of a DC Motor

Consider the connection diagram of a shunt motor as shown in Fig. 5.8 to find out the mechanical power. Here, the following variables are defined as follows:

V_t is the applied voltage,

E_b is the back emf,

R_a is the armature resistance,

R_{sh} is the shunt field resistance,

I_a is the armature current.

Applying KVL to the circuit as shown in Fig. 5.8 yields,

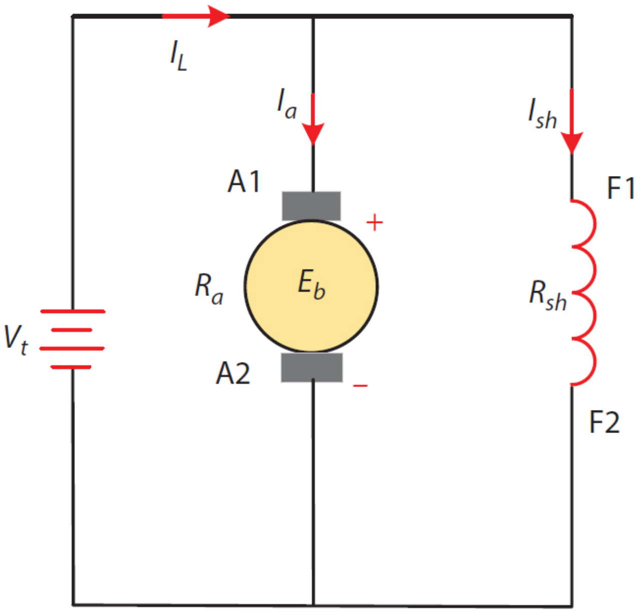


Fig. 5.8 Connection diagram of a shunt motor.

$$-V_t + R_a I_a + E_b = 0 \tag{5.41}$$

$$V_t = R_a I_a + E_b \tag{5.42}$$

Equation (5.42) is known as the voltage equation of the DC motor. Again, multiplying equation (5.42) by the armature current I_a yields,

$$V_t I_a = R_a I_a^2 + E_b I_a \tag{5.43}$$

Equation (5.43) is known as the power equation of the DC motor.

Here, the following parameters are defined as,

$V_t I_a$ is the electric power supplied to the armature,

$E_b I_a$ is the mechanical power developed by the armature,

$I_a{}^2 R_a$ is the armature copper loss.

The small amount of power will be wasted due to copper loss and the remaining portion will be converted into mechanical power within the armature. From Fig. 5.9, the mechanical power developed by the motor armature is,

$$P_{md} = E_b I_a \tag{5.44}$$

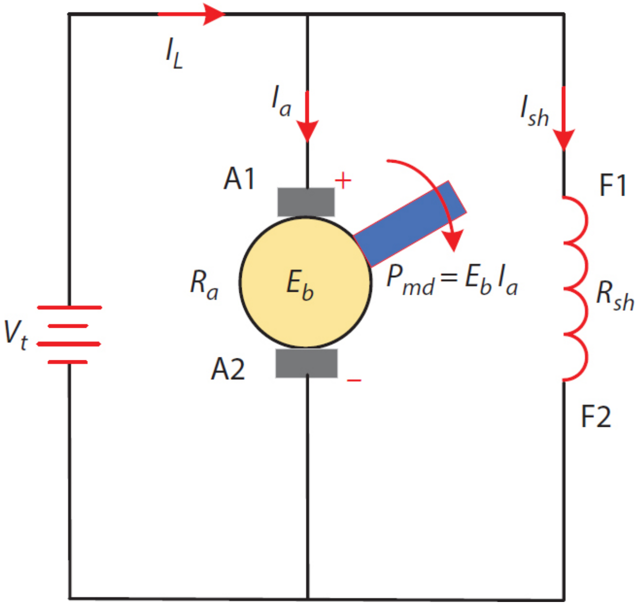


Fig. 5.9 DC shunt motor with mechanical power.

Substituting equation (5.43) into equation (5.44) yields,

$$P_{md} = V_t I_a - R_a I_a{}^2 \tag{5.45}$$

Normally, the value of the armature circuit resistance and the applied voltage is fixed in the shunt motor. Therefore, the power developed by the motor depends on the armature current. The mechanical power developed by the motor will be maximum if the differentiation of the mechanical power with respect to the armature current is taken to be zero. From equation (5.45), this relation can be derived as,

$$\frac{d}{dI_a}(P_{md}) = 0 \tag{5.46}$$

$$V_t - 2R_a I_a = 0 \tag{5.47}$$

$$R_a I_a = \frac{V_t}{2} \tag{5.48}$$

Substituting equation (5.48) into equation (5.42) yields,

$$V_t = \frac{V_t}{2} + E_b \tag{5.49}$$

$$E_b = \frac{V_t}{2} \tag{5.50}$$

From equation (5.50), it is seen that the mechanical power developed by the motor armature will be maximum when the back emf is equal to half of the applied voltage.

Example 5.2

A shunt motor takes a total current of 20 A from the 250 V supply. The shunt field and armature circuit resistances are 200 Ω and 0.3 Ω, respectively. Determine the value of back emf and mechanical power developed in the motor armature.

Solution

The value of the shunt field current is calculated as,

$$I_{sh} = \frac{250}{200} = 1.25\,\text{A} \tag{5.51}$$

The value of the armature current is determined as,

$$I_a = 20 - 1.25 = 18.13\,\mathrm{A} \tag{5.52}$$

The back emf is determined as,

$$E_b = V_t - R_a I_a = 250 - 18.13 \times 0.3 = 244.6\,\mathrm{V} \tag{5.53}$$

The mechanical power developed can be calculated as,

$$P_{md} = E_b I_a = 244.6 \times 18.13 = 4433.9\,\mathrm{W} \tag{5.54}$$

Example 5.3

A shunt motor has an armature circuit resistance of 0.7 Ω and the applied voltage is 240 V. The no-load and full-load armature currents are 5 A and 30 A, respectively. Find the change in back emf from no-load to full load.

Solution

The value of the back emf under no-load is determined as,

$$E_{bnl} = V_t - I_a R_a = 240 - 5 \times 0.7 = 236.5\,\mathrm{V} \tag{5.55}$$

The value of the back emf under full load is calculated as,

$$E_{bfl} = V_t - I_a R_a = 240 - 35 \times 0.7 = 215.5\,\mathrm{V} \tag{5.56}$$

The change in back emf from no-load to full load is determined as,

$$E_{bc} = E_{bnl} - E_{bfl} = 236.5 - 215.5 = 21\,\mathrm{V} \tag{5.57}$$

Practice Problem 5.2

A 230 V shunt motor takes a total current of 25 A from the supply. The shunt field and armature resistances are 55 Ω and 0.5 Ω, respectively. Find the back emf and mechanical power developed in the motor armature.

Practice Problem 5.3

A 4-pole shunt motor is energized by the 450 V supply and takes an armature current of 55 A. The resistance of the armature circuit is 0.4 Ω. The armature winding is wave-connected by 650 conductors and a useful flux per pole is 0.025 Wb. Calculate the speed of the motor.

5.8 Torque of a DC Motor

Consider the conductor, which is placed in a magnetic field. The current through the conductor produces a force, which is shown in Fig. 5.10. Because of this force, the conductor will tend to rotate in a clockwise direction. This tendency to produce the rotation of the conductor is known as torque. Mathematically, the torque is defined as the product of the force and the perpendicular distance from the center of the rotation of the conductor. The expression of the torque can be written as,

$$T = F \times r \tag{5.58}$$

where,

T is the torque in N-m,

F is the force in N,

r is the perpendicular distance in m.

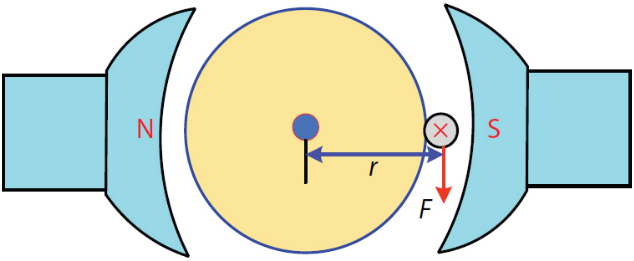


Fig. 5.10 Force on a conductor.

In a DC motor, normally two types of torques are considered. These are the armature torque and the shaft torque. A brief descriptions of those torques are mentioned below.

5.8.1 Armature Torque

The torque produced by the armature conductors is known as armature torque. The armature torque is denoted by Ta. Let us consider the motor runs at a speed of N rpm. The mechanical power developed by the motor is,

$$(5.59)\quad P_{md} = T_a \times \omega$$

From the equations (5.50) and (5.51), the expression of the armature torque can be derived as,

$$(5.60)\quad T_a \times 2\pi \frac{N}{60} = E_b I_a$$

$$(5.61)\quad T_a = 9.55 \times \frac{E_b I_a}{N}$$

Substituting equation (5.4) into equation (5.61) yields,

$$(5.62)\quad T_a = 9.55 \times \frac{PZ\phi N I_a}{60AN}$$

$$(5.63)\quad T_a = 0.159 \frac{PZ\phi I_a}{A}$$

If P, Z and A are constant, then equation (5.63) is modified as,

$$(5.64)\quad T_a \infty \phi I_a$$

5.8.2 Shaft Torque

The whole armature torque is not transferred at its output terminals. Some percentage of this torque is wasted due to iron and friction losses. After the deduction of these losses, the torque, which is available at the shaft of the motor for doing useful work is known as shaft torque. This torque is represented by Ts_h. Consider the motor runs at a speed of N rpm and the output power P_0 of the motor is given by,

$$(5.65)\quad P_o = T_{sh} \times \omega = T_{sh} \times 2\pi \frac{N}{60}$$

$$(5.66)\quad T_{sh} = 9.55 \frac{P_o}{N}$$

Example 5.4

A shunt motor takes a current of 40 A from the 230 V supply and runs at a speed of 1100 rpm. Find the torque developed by the armature, if the armature and the shunt field resistances are 0.25 Ω and 230 Ω respectively.

Solution

The value of the shunt field current is calculated as,

$$I_{sh} = \frac{230}{230} = 1\,\text{A} \tag{5.67}$$

The value of the armature current is determined as,

$$I_a = 40 - 1 = 39\,\text{A} \tag{5.68}$$

The back emf is determined as,

$$E_b = V_t - I_a R_a = 230 - 39 \times 0.25 = 220.25\,\text{V} \tag{5.69}$$

The armature torque is calculated as,

$$T_a = 9.55 \times \frac{E_b I_a}{N} = 9.55 \frac{220.25 \times 39}{1100} = 74.57\ \text{N-m} \tag{5.70}$$

Example 5.5

A 6-pole shunt motor is energized by the 230 V DC supply. The motor has 450 conductors that are wound in a lap configuration and the armature resistance is found to be 0.8 Ω. It takes 30 A current from the supply system and develops an output power of 5560 W. The current through the field windings is 3 A and the flux per pole is 25 mWb. Find the speed and the shaft torque.

Solution

The value of the armature current is determined as,

$$I_a = 30 - 3 = 27\,\text{A} \tag{5.71}$$

The back emf can be determined as,

$$E_b = V_t - I_a R_a = 230 - 27 \times 0.8 = 208.4\,\text{V} \tag{5.72}$$

The value of the speed is calculated as,

$$N = \frac{60E_b A}{PZ\phi} = \frac{60\times 6\times 208.4}{6\times 450\times 25\times 10^{-3}} = 1111.5 \text{ rpm} \tag{5.73}$$

The shaft torque is determined as,

$$T_{sh} = 9.55\frac{P_o}{N} = 9.55\times\frac{5560}{1111.5} = 47.8 \text{ N-m} \tag{5.74}$$

Practice Problem 5.4

A shunt motor takes a current of 45 A from the 220 V DC supply and runs at a speed of 1500 rpm. Find the torque developed by the armature, if the armature and the shunt field resistances are 0.5 Ω and 65 Ω, respectively.

Practice Problem 5.5

A 6-pole shunt motor is energized by the 230 V DC supply. The motor has 500 conductors connected in a lap wound. The armature resistance is found to be 0.4 Ω. It takes 40 A from the supply system and develops an output power of 5.9 kW. The current through the field windings is 4 A and the flux per pole is 0.04 Wb. Calculate the shaft torque.

5.9 Speed of a DC Motor

The speed of a DC motor changes slightly with the increase of the load. The expression of speed can be derived from the basic knowledge of back emf. According to the generator action of the motor, the speed of the motor from equation (5.4) is written as,

$$N = \frac{60A}{P\phi Z}E_b \tag{5.75}$$

Substituting equation (5.42) into equation (5.75) yields,

$$N = \frac{60A}{PZ}\left(\frac{V_t - I_a R_a}{\phi}\right) \tag{5.76}$$

For a particular machine P, Z and A are constant. Therefore, equation (5.75) is modified as,

$$N = k\left(\frac{V_t - I_a R_a}{\phi}\right) \tag{5.77}$$

$$N = k\frac{E_b}{\phi} \tag{5.78}$$

where the expression of the constant is,

$$k = \frac{60A}{PZ} \tag{5.79}$$

$$N \propto \frac{E_b}{\phi} \tag{5.80}$$

From equation (5.80), it is seen that the speed of a DC motor is directly proportional to the back emf and inversely proportional to the flux per pole.

Let us consider that the initial values of speed, flux per pole and back emf of a DC motor are N_1, ϕ_1 and E_{b1}, respectively. The final values of those terms are N_2, ϕ_2 and E_{b2}, respectively. The equations of the speeds for those conditions are written as,

$$N_1 \propto \frac{E_{b1}}{\phi_1} \tag{5.81}$$

$$N_2 \propto \frac{E_{b2}}{\phi_2} \tag{5.82}$$

Dividing equation (5.81) by equation (5.82) yields,

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1} \tag{5.83}$$

Practically, the flux in the shunt motor is always constant with the change in applied voltage. Therefore, the initial flux is equal to the final flux, i.e., $\phi_1 = \phi_2$ and equation (5.83) becomes,

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \tag{5.84}$$

For a series motor, flux is directly proportional to the armature current before the saturation point. Equation (5.83) can be modified as,

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}} \tag{5.85}$$

where,

Ia_1 is the initial armature current,

Ia_2 is the final armature current.

The percentage change in flux can be expressed as,

$$\phi_{PR} = \frac{\phi_1 - \phi_2}{\phi_1} \times 100 \tag{5.86}$$

Example 5.6

A DC shunt motor takes a current of 5 A at a no-load with a terminal voltage of 230 V and runs at a speed of 1000 rpm. The armature and field resistances are 0.2 Ω and 230 Ω respectively. Under load conditions, the motor takes a current of 30 A. Determine the motor speed in this condition.

Solution

No-load condition:

The shunt field current is determined as,

$$I_{sh} = \frac{230}{230} = 1\,\text{A} \tag{5.87}$$

The value of the armature current is calculated as,

$$I_{a1} = I_L - I_{sh} = 5 - 1 = 4\,\mathrm{A} \tag{5.88}$$

The initial back emf can be determined as,

$$E_{b1} = V_t - I_{a1}R_a = 230 - 4 \times 0.2 = 229.2\,\mathrm{V} \tag{5.89}$$

The initial speed is given as,

$$N_1 = 1000\ \mathrm{rpm} \tag{5.90}$$

Under load condition:

The value of the armature current is calculated as,

$$I_{a2} = I_L - I_{sh} = 30 - 1 = 29\,\mathrm{A} \tag{5.91}$$

The final value of the back emf is determined as,

$$E_{b2} = V_t - I_{a2}R_a = 230 - 29 \times 0.2 = 224.2\,\mathrm{V} \tag{5.92}$$

The motor speed under load conditions can be determined as,

$$N_2 = \frac{E_{b2}}{E_{b1}} N_1 = \frac{224.2}{229.2} \times 1000 = 978.2\,\mathrm{rpm} \tag{5.93}$$

Example 5.7

A DC series motor takes a current of 56 A from the 230 V supply and runs at a speed of 900 rpm. The armature and field resistances are 0.2 Ω and 0.25 Ω respectively. Find the speed of the motor when the line current is 15 A. Consider the flux for 15 A current is 40% of the flux for the line current of 56 A.

Solution

The initial back emf is determined as,

$$E_{b1} = V_t - I_{a1}(R_a + R_{sh}) = 230 - 65 \times (0.2 + 0.25) = 200.8\,\mathrm{V} \tag{5.94}$$

The initial speed is given as,

$$N_1 = 900\ \mathrm{rpm} \tag{5.95}$$

The final back emf is calculated as,

$$E_{b2} = V_5 - I_{a2}(R_a + R_{sh}) = 230 - 15 \times (0.2 + 0.25) = 223.3\,\text{V} \tag{5.96}$$

The value of the final flux is given as,

$$\phi_2 = 0.4\phi_1 \tag{5.97}$$

The speed of the motor is determined as,

$$N_2 = N_1 \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} = 2.5 \times \frac{223.3}{200.8} \times 900 = 2502\,\text{rpm} \tag{5.98}$$

Practice Problem 5.6

A DC shunt motor takes a 3 A current at no load with a terminal voltage of 240 V and runs at a speed of 1200 rpm. The armature and field resistances are 0.25 Ω and 135 Ω, respectively. Under load conditions, the motor takes a current of 45 A. Determine the motor speed under load conditions.

Practice Problem 5.7

A 220 V DC series motor takes a current of 120 A and runs at a speed of 1000 rpm. The armature and field resistances are 0.15 Ω and 0.25 Ω, respectively. Find the speed of the motor when the line current is 35 A. Consider the flux for 35 A current is 70% of the flux for the line current of 120 A.

5.10 Speed Regulation of a DC Motor

The armature current of the DC motor increases with the increase of the load. As a result, the back emf decreases and the speed of the motor also decreases slightly. The speed regulation (*SR*) of a DC motor is defined as the change in speed from full load to no load and is expressed as a percentage of the speed at full load. The percentage speed regulation is expressed as,

$$SR = \frac{\text{No-load speed-Full load speed}}{\text{Full load speed}} \times 100 \tag{5.99}$$

$$SR = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100 \tag{5.100}$$

where,

SR is the speed regulation,

N_{nl} is the no-load speed,

N_{fl} is the full load speed.

5.11 Losses in a DC Motor

The armature circuit and shunt field winding of a DC motor contain small resistances. As a result, a small amount of power will be wasted and the remaining power will be converted into mechanical power. A small part of the developed power will be wasted again due to rotational loss. Finally, the remaining power will be available at the shaft of the motor for doing useful work. Higher losses decrease the efficiency of the motor and lower losses increase the efficiency and performance. The total losses are classified as copper loss, iron loss and mechanical loss. The copper losses include armature copper loss ($I_a^2R_a$), shunt field copper loss ($I_{sh}^2R_{sh}$) and brush copper loss (2_{vbIa}). Similarly, compensating winding copper loss and interpole winding loss are also included. The mechanical losses include air friction of the rotating armature (windage loss) and friction between the commutator and brush (frictional loss). The losses that occur in the iron parts of the motor are known as iron loss or core loss. The iron loss is again classified as an eddy current loss and hysteresis loss. When the flux leaks the iron part or core with respect to time induces the voltage. Due to this induced voltage, a small current circulates through the core. The power loss due to this eddy current is known as eddy current loss. The eddy current loss (P_e) is expressed as,

$$(5.101)\, P_e = k_e f^2 B_m{}^2 t^2 V$$

where,

f is the frequency in Hz,

B_m is the magnetic flux density in T,

t is the thickness in m,

V is the volume in m^3.

When the current increases, magnetic field intensity increases and the magnetic flux density also increases up to the saturation point of the iron material. After the saturation point, the magnetic flux density does not increase with increasing the current. Afterwards, decreasing the current, the magnetic flux density decreases but follows a different path known as the retentivity property of the material. Due to this property, the hysteresis loss occurs. The hysteresis loss (Ph) is expressed as,

$$(5.102)\, P_h = k_h f B_m^{1.6} V$$

Different stages of the power flow diagram of the DC motor are shown in Fig. 5.11. The expression of the input power is written as,

$$(5.103)\, P_{in} = V_t I_L$$

The expression of the copper loss of the armature and shunt field is written as,

$$(5.104)\, P_{cu} = I_a^2 R_a + I_{sh}^2 R_{sh}$$

The mechanical power developed in the armature is expressed as,

$$(5.105)\, P_{md} = P_{in} - P_{cu} = E_b I_a$$

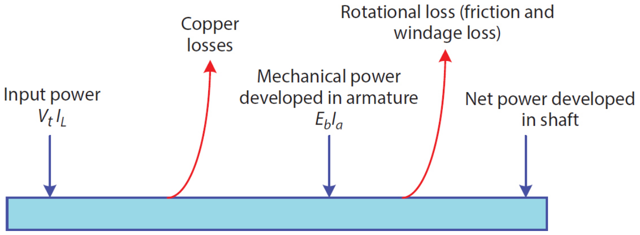

Copper losses
Rotational loss (friction and windage loss)
Mechanical power developed in armature $E_b I_a$
Input power $V_t I_L$
Net power developed in shaft

Fig. 5.11 Different stages of power flow diagram.

The net power at the shaft is expressed as,

$$(5.106)\, P_{net} = P_{md} - P_{rotational}$$

5.12 Efficiency of a DC Motor

The efficiency of any machine can be defined as the ratio of the output power to the input power. Mathematically, it can be expressed as,

$$\eta = \frac{P_o}{P_{in}} \times 100 \tag{5.107}$$

$$\eta = \frac{P_o - P_{loss}}{P_{in}} \times 100 \tag{5.108}$$

The copper loss due to the armature resistance is,

$$P_{\mathrm{cu}} = I_a^{\,2} R_a \tag{5.109}$$

The expression of the constant loss is written as,

$$P_{\mathrm{const}} = P_{\mathrm{iron}} + P_{\mathrm{friction}} \tag{5.110}$$

The total loss is determined as,

$$P_{\mathrm{loss}} = P_{\mathrm{cu}} + P_{\mathrm{const}} \tag{5.111}$$

Substituting equation (5.109) into equation (5.111) yields,

$$P_{\mathrm{loss}} = I_a^{\,2} R_a + P_{\mathrm{const}} \tag{5.112}$$

Substituting equations (5.103) and (5.112) into equation (5.108) yields,

$$\eta = \frac{V_t I_L - I_a^{\,2} R_a - P_{\mathrm{const}}}{V_t I_L} \tag{5.113}$$

The shunt field current is very small compared to the armature current. Therefore, $I_a = I_L$ and equation (5.113) can be modified as,

$$\eta = \frac{V_t I_a - I_a^{\,2} R_a - P_{\mathrm{const}}}{V_t I_a} \tag{5.114}$$

Differentiating equation (5.78) with respect to I_a yields,

$$\frac{d\eta}{dI_a} = \frac{V_t I_a (V_t I_a - 2I_a R_a) - V_t (V_t I_a - I_a^2 R_a - P_{\text{const}})}{(V_t I_a)^2} \tag{5.115}$$

For maximum efficiency, setting $d\eta/dI_a = 0$, equation (5.115) becomes,

$$\frac{d\eta}{dI_a} = \frac{V_t I_a (V_t - 2I_a R_a) - V_t (V_t I_a - I_a^2 R_a - P_{\text{const}})}{(V_t I_a)^2} = 0 \tag{5.116}$$

$$V_t I_a (V_t - 2I_a R_a) - V_t (V_t I_a - I_a^2 R_a - P_{\text{const}}) = 0 \tag{5.117}$$

$$V_t I_a - 2I_a^2 R_a - V_t I_a + I_a^2 R_a + P_{\text{const}} = 0 \tag{5.118}$$

$$P_{\text{const}} = I_a^2 R_a \tag{5.119}$$

From equation (5.119), it is seen that the efficiency of a DC motor will be maximum when the variable loss is equal to the constant loss.

Example 5.8

At no-load condition, a shunt motor takes 5 A current from a 230 V supply. The resistances of the armature and the field circuit are 0.25 Ω and 115 Ω, respectively. Under load conditions, the motor can carry 40 A current. Calculate the iron and friction losses and the efficiency.

Solution

At no-load condition: The input power is calculated as,

$$P_{in} = V_t I_L = 230 \times 5 = 1150 \text{ W} \tag{5.120}$$

The value of the shunt field current is determined as,

$$I_{sh} = \frac{230}{115} = 2 \text{ A} \tag{5.121}$$

The value of the armature current is calculated as, (5.122) $$I_a = 5 - 2 = 3\,\mathrm{A}$$

The armature copper loss is determined as,

(5.123) $$P_{acu} = I_a{}^2 R_a = 3^2 \times 0.25 = 2.25\,\mathrm{W}$$

The shunt field copper loss is calculated as,

(5.124) $$P_{shcu} = I_{sh}{}^2 R_{sh} = 2^2 \times 115 = 460\,\mathrm{W}$$

The iron and friction losses are determined as,

(5.125) $$P_{iron-friction} = 1150 - (2.25 + 460) = 687.75\,\mathrm{W}$$

Under load condition: The armature current is calculated as,

(5.126) $$I_a = 40 - 2 = 38\,\mathrm{A}$$

The armature cu loss is determined as,

(5.127) $$P_{acu} = I_a{}^2 R_a = (38)^2 \times 0.25 = 361\,\mathrm{W}$$

The total loss is determined as,

(5.128) $$P_{loss} = 687.75 + 460 + 361 = 1508.75\,\mathrm{W}$$

The input power is calculated as,

(5.129) $$P_{in} = 230 \times 40 = 9200\,\mathrm{W}$$

The output power is determined as,

(5.130) $$P_o = P_{in} - P_{loss} = 9200 - 1508.75 = 7691.25\,\mathrm{W}$$

The value of the efficiency is calculated as,

(5.131) $$\eta = \frac{P_o}{P_{in}} = \frac{7691.25}{9200} = 84\%$$

Example 5.9

A shunt motor takes a current of 80 A from the 220 V supply and runs at 800 rpm. The shunt field and armature resistances are 50 Ω and 0.1 Ω, respectively. The iron and friction losses are 1600 W.

Determine the copper losses, armature torque, shaft torque and efficiency.

Solution

The value of the shunt field current is calculated as,

$$I_{sh} = \frac{220}{50} = 4.4\text{ A} \tag{5.132}$$

The value of the armature current is determined as,

$$I_a = I_L - I_{sh} = 80 - 4.4 = 75.6\text{ A} \tag{5.133}$$

The back emf is calculated as,

$$E_b = V_t - I_a R_a = 220 - 75.6 \times 0.1 = 212.44\text{ V} \tag{5.134}$$

The input power is determined as,

$$P_{in} = V_t I_L = 220 \times 80 = 17600\text{ W} \tag{5.135}$$

The mechanical power developed in the armature is calculated as,

$$P_{md} = E_b I_a = 212.44 \times 75.6 = 16060.46\text{ W} \tag{5.136}$$

The value of the copper loss is determined as,

$$P_{cu} = P_{in} - P_{md} = 17600 - 16060.46 = 1539.54\text{ W} \tag{5.137}$$

The armature torque can be determined as,

$$T_a = 9.55\frac{E_b I_a}{N} = 9.55\frac{212.44 \times 75.6}{800} = 191.72\text{ N-m} \tag{5.138}$$

The output power is calculated as,

$$P_o = P_{md} - P_{i\&f} = 16060.46 - 1600 = 14460.46\text{ W} \tag{5.139}$$

The value of the shaft torque is determined as,

$$T_{sh} = 9.55\frac{P_0}{N} = 9.55 \times \frac{14460.46}{800} = 172.62\text{ N-m} \tag{5.140}$$

The value of the efficiency is determined as,

$$\eta = \frac{P_o}{P_{in}} = \frac{14460.46}{17600} = 82\% \quad (5.141)$$

Practice Problem 5.8

At no-load condition, a shunt motor takes 4 A current from the 220 V supply. The resistances of the armature and the field circuit are 0.15 Ω and 155Ω, respectively. Under load conditions, the motor can carry 36 A current. Calculate the iron and friction losses and the efficiency of the motor.

Practice Problem 5.9

A shunt motor takes a current of 75 A from the 220 V supply and runs at a speed of 1100 rpm. The shunt field and armature resistances are 65 Ω and 0.2 Ω, respectively. The iron and friction losses are 900 W. Find the copper losses, armature torque, shaft torque and efficiency.

5.13 Characteristics of a DC Motor

The motor performance of the motor identifies by the curve known as the motor characteristics curve. There are three characteristic curves that are used to study the motor performance, and they are discussed here.

1. Torque-current characteristic: The curve that is obtained from the variation of torque with respect to the armature current is known as the torque-current characteristic. This characteristic is also known as the electrical characteristic.
2. Speed-current characteristic: The curve that is obtained from the variation of speed with respect to the armature current is known as the speed-current characteristic. This is also known as the speed characteristics curve.
3. Speed-torque characteristics: The curve that is obtained from the variation of speed with respect to the torque is known as the speed-torque characteristic. It is often called mechanical characteristics.

5.14 Characteristics of a Shunt Motor

In a shunt motor, the field winding is connected in parallel with the armature circuit. The field winding consists of many turns with fine wire. The general torque equation of the shunt motor is expressed as,

$$(5.142)\quad T_{sh} \propto \phi I_a$$

Since the shunt field is connected across the constant supply voltage, therefore, the flux remains constant. Then equation (5.141) is modified as,

$$(5.143)\quad T_{sh} \propto I_a$$

From equation (5.142), it is seen that the torque is directly proportional to the armature current. At zero armature current, the torque is zero. The torque will increase with the increase of the armature current and vice versa. The shunt motor needs more armature current to start when it is connected to a heavy load. Therefore, the torque versus armature current is a straight line that will pass through the origin, which is shown in Fig. 5.12.

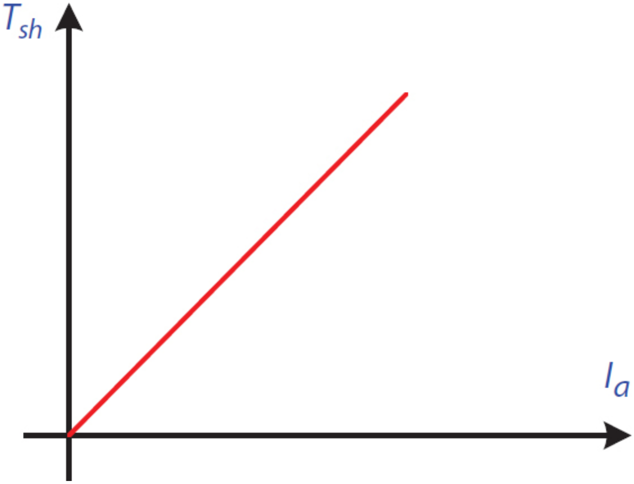


Fig. 5.12 Torque-current characteristics.

The speed of the shunt motor can be expressed as,

$$N_{sh} \propto \frac{E_b}{\phi} \tag{5.144}$$

Substituting equation (5.42) into equation (5.144) yields,

$$N_{sh} \propto \frac{V_t - I_a R_a}{\phi} \tag{5.145}$$

Assume that the shunt motor is operating at no-load condition with its rated speed. In this condition, the flux and the back emf are constant. Therefore, the speed is constant with the armature current as shown in Fig. 5.13.

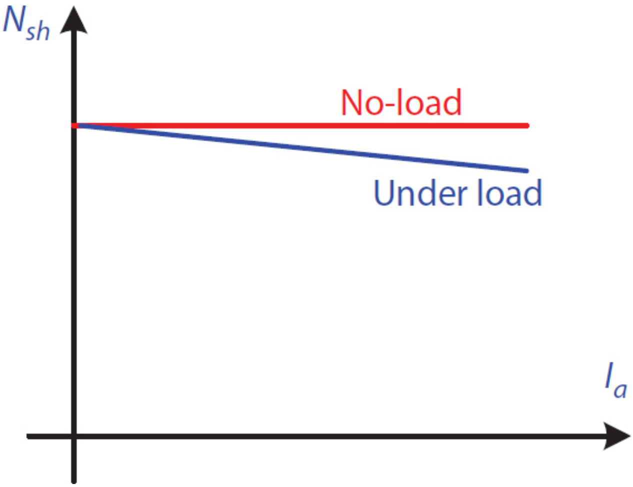


Fig. 5.13 Speed-current characteristics.

When the load across the motor increases, armature current will increase and back emf will decrease. As a result, the speed of the motor will decrease slightly as shown in Fig. 5.13. The armature current increases when the connected load increases. As a result, the torque increases and the back emf decreases. Therefore, the speed of the motor decreases with the variation of the torque, which is shown in Fig. 5.14.

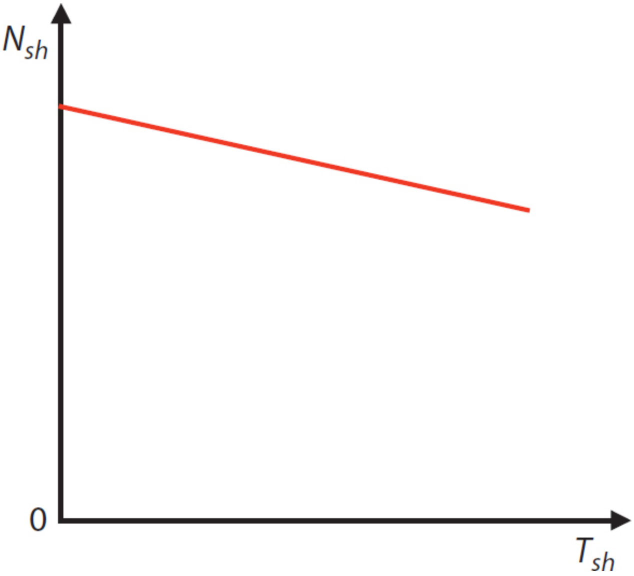


Fig. 5.14 Speed-torque characteristic.

5.15 Characteristics of a Series Motor

In a series motor, the field winding is connected in series with the armature. Therefore, the rated current flows through the field winding. The general torque equation of the motor is,

$$T_{se} \propto \phi I_a \tag{5.146}$$

Since the series field current is the same as the armature current, therefore, the flux is directly proportional to the armature current. This relation is,

$$\phi \propto I_a \tag{5.147}$$

Substituting equation (5.147) into equation (5.146) yields the final expression of torque,

$$T_{se} \propto {I_a}^2 \tag{5.148}$$

Based on equation (5.148), the torque versus armature current characteristic is shown in Fig. 5.15. From equation (5.148), it is seen that the torque will increase with the square of the armature current up to the saturation point of the magnetic material. After saturation, the flux will remain constant and the equation of the torque will be in the following form,

$$T_{se} \propto I_a \tag{5.149}$$

The basic speed equation of the series motor is expressed as,

$$N_{se} \propto \frac{E_b}{\phi} \tag{5.150}$$

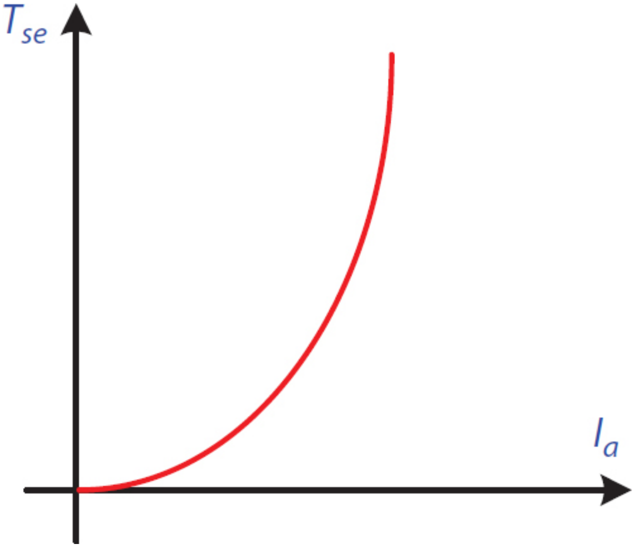


Fig. 5.15 Torque-current characteristic.

For the series motor, replace $E_b = V_t - I_a(R_a + R_{se})$ in equation (5.150) yields,

$$N_{se} \propto \frac{V_t - I_a(R_a + R_{se})}{\phi} \tag{5.151}$$

Since, the flux is proportional to the armature current, equation (5.151) becomes,

$$N_{se} \propto \frac{V_t - I_a(R_a + R_{se})}{I_a} \quad (5.152)$$

Under the no-load condition, the value of the armature current is small. Therefore, the numerator of equation (5.152) is large and the denominator is small. Under load conditions, the armature current is high which makes the numerator small and the denominator large. Therefore, the speed of the series motor decreases with a higher value of the armature current, which is shown in Fig. 5.16.

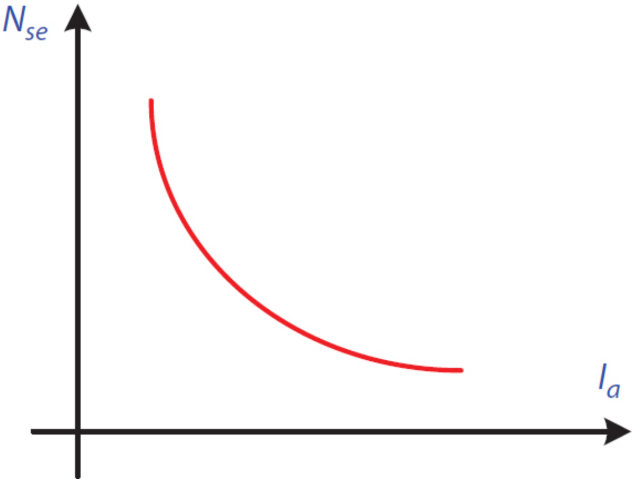


Fig. 5.16 Speed-current characteristic.

The armature current increases with the increase of the connected load. As a result, the torque increases based on equation (5.149). This increased value of the armature current also decreases the field flux. As a result of the increased flux, the speed of the motor will decrease based on equations (5.151) or (5.152). The speed versus torque characteristic of a series motor is shown in Fig. 5.17. From Fig. 5.17, it is seen that the speed of the series motor decreases exponentially with increasing the torque.

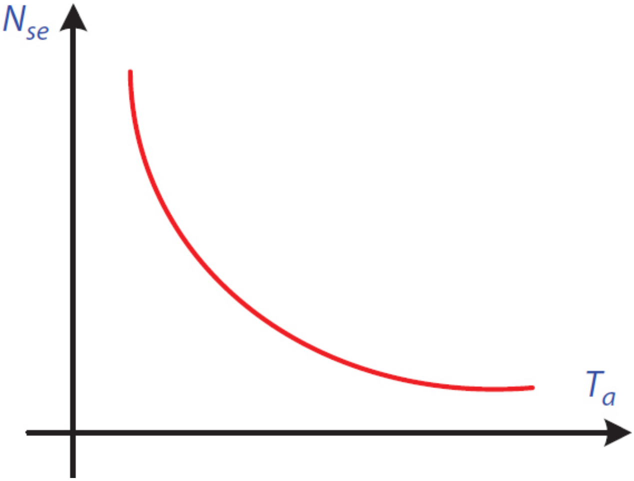


Fig. 5.17 Speed-torque characteristic.

5.16 Characteristics of a Cumulative Compound Motor

This motor can be classified as a short shunt and a long shunt. In a long shunt cumulative compound motor, the flux due to the series field will aid the shunt field flux. As a result, the total flux will be increased. The general torque equation of this motor is,

$$T_{cum} \propto (\phi_f + \phi_{se}) I_a \quad (5.153)$$

As the load increases, the armature current increases which increases the torque. The torque of the cumulative motor increases more than the shunt motor. In a differential compound motor, series field flux opposes the shunt field flux, which decreases the total flux. Therefore, the torque equation of this motor is,

$$T_{diff} \propto (\phi_f - \phi_{se}) I_a \quad (5.154)$$

As the load increases, the armature current increases, which decreases the torque. Therefore, the torque of the differential compound motor decreases more than the shunt motor. The torque-current characteristics of the cumulative and the differential compound motor are shown in Fig. 5.18.

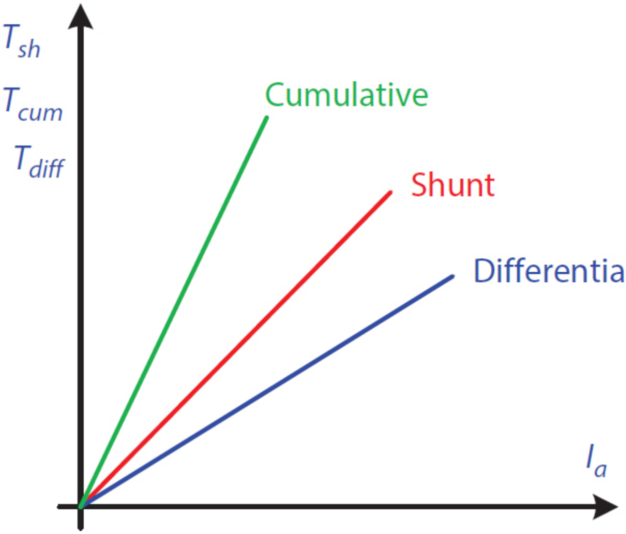


Fig. 5.18 Torque-current characteristic.

The basic speed equation of the cumulative motor is,

$$N_{cum} \propto \frac{V_t - I_a(R_a + R_{se})}{\phi_f + \phi_{se}} \tag{5.155}$$

As the load increases, the armature current increases. Thus, the flux produced by the series field also increases while the back emf decreases. The denominator of the speed equation (5.154) increases while the numerator decreases proportionately more in comparison to the shunt motor. Therefore, the speed of the cumulative motor will drop at a faster rate than the shunt motor. The basic speed equation of the differential compound motor is,

$$N_{diff} \propto \frac{V_t - I_a(R_a + R_{se})}{\phi_f - \phi_{se}} \tag{5.156}$$

From equation (5.156), it is observed that the total flux will be reduced due to an increase in armature current. Hence, the speed of the differential compound motor will increase and it will be higher than the shunt motor. The speed versus armature current of the cumulative and the differential compound motor are shown in Fig. 5.19.

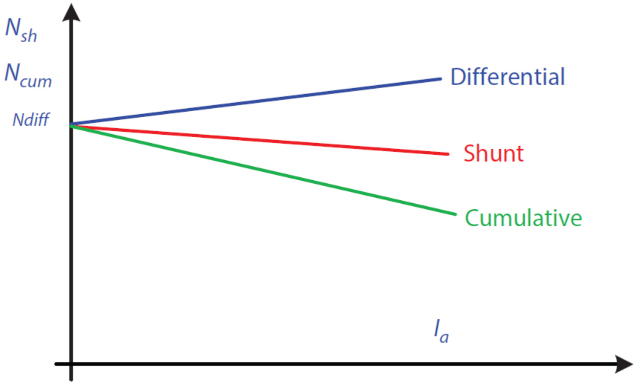


Fig. 5.19 Speed-current characteristic.

5.17 Comparison Between Generator and Motor

The comparison between the generator and the motor is mentioned in the following ways:

- The generator is driven by a mechanical machine and generates voltage. As a result of this voltage, a current will flow through the electrical circuit. The motor is connected across the supply. The armature of the motor takes a current from the supply, which in turn produces a force. This force produces a driving torque, which rotates the motor.
- In the generator, the direction of the generated voltage is determined by Fleming’s right-hand rule. In the motor, the direction of the armature current is determined by Fleming’s left-hand rule.
- In the generator, the relationship among the generated voltage, current and speed are considered as main characteristics; whereas in the motor, the relationship is considered among the speed, torque and current.
- In the generator, the generated voltage and the armature current are in the same direction; whereas in the DC motor, these are in opposite directions.

5.18 Application of DC Motors

Shunt motor: The characteristics of a shunt motor are that it has an approximately constant speed and a medium starting torque.

This motor is used in the lathe machine, drill machine, shaper machine, blower, etc.

Series motors: This motor is known as a variable speed motor and it has a high starting torque. This motor is used in trolley cars, cranes, electric traction, elevators, air compressors, vacuum cleaners, hair dryers, conveyor machines, etc.

Compound motors: The differential compound motor has poor torque characteristics. Because of poor characteristics, this motor is rarely used. However, the cumulative compound motor has a high starting torque and it is used in printing presses, ice machines, air compressors, rolling mills, etc.

Exercise Problems

5.1 The armature resistance of a DC motor is 0.3 Ω and takes a current of 115 A from the 415 V supply. Determine the back emf of the motor.

5.2 The armature current of a 4-pole DC motor is 140 A. The motor is connected across the 440 V supply. Calculate the back emf if the armature circuit resistance is 0.2 Ω.

5.3 The back emf of a 230 V DC motor is 210 V. The armature of the motor draws a current of 115 A from the supply. Find the armature resistance.

5.4 A DC motor is connected across the 240 V supply. The back emf of the motor is 220 V. Find the armature current if the armature circuit resistance is 0.2 Ω.

5.5 A shunt motor takes a total current of 18 A from the 220 V supply. The shunt field resistance of the motor is 150 Ω and the armature resistance is 0.4 Ω. Determine the back emf.

5.6 The field resistance of the shunt motor is 140 Ω and the field current is 2 A. The motor takes a line current of 32 A from the supply. Determine the armature resistance if the back emf of a shunt motor is 260 V.

5.7 A DC shunt motor takes a line current of 60 A from the supply. The shunt field current is 10 percent of the line current. The armature resistance of the motor is 0.5 Ω. Determine the shunt field resistance if the back emf of the motor is 213 V.

5.8 The back emf of a 230 V DC shunt motor is 209 V. The armature and shunt field resistances of the motor are 0.3 Ω and 75 Ω, respectively. Determine the line current.

5.9 A shunt motor takes a current of 35 A from the supply. The shunt field resistance of the motor is 120 Ω and it carries a current of 1.5 A. Calculate the armature resistance if the counter emf of the motor is 170 V. Also, calculate the armature current if the motor is stationary.

5.10 The field resistance of a 400 V DC shunt motor is 120 Ω. The armature circuit resistance is 0.2% of the shunt field resistance. Find the line current drawn by the motor if the back emf is 380 V.

5.11 The armature current of a 4-pole wave-connected DC motor is 40 A. The resistance of the armature circuit is 0.23 Ω and the motor is connected across the 415 V supply. The number of conductors of the armature circuit is 360 and the flux per pole is 0.05 Wb. Determine the speed of the motor.

5.12 A 4-pole DC motor has an armature resistance of 0.5 Ω. The motor is connected across the 440 V supply and takes an armature current of 75 A. The armature is lap wound by 500 conductors. Determine the speed of the motor if the useful flux per pole is 0.05 Wb.

5.13 A 6-pole 230 V lap wound DC motor has an armature resistance of 0.4 Ω. The armature is wound by 600 conductors and runs at a speed of 700 rpm. Determine the useful flux per pole if the armature current is 25 A.

5.14 A 4-pole lap wound DC motor is connected across the 400 V supply. The back emf of the motor is 90% of the supply voltage. The motor runs at a speed of 1000 rpm and the useful flux per pole is 0.05 Wb. Determine the number of conductors.

5.15 The armature circuit resistance of a DC motor is 0.5 Ω and is connected across the 220 V supply. The full-load armature current is 25 A and the no-load armature current is 5 A. Determine the change in back emf.

5.16 A 230 V DC motor has an armature resistance of 0.6 Ω. The full load armature current is 35 A. The change in back emf from no-load to full load is 20 V. Find the no-load armature current.

5.17 The armature resistance of a 250 V DC motor is 0.3 Ω. The no-load armature current is 3 A. The change in back emf from no-load to full load is 10% of the supply voltage. Find the full load armature current.

5.18 A shunt motor is connected across the 230 V supply and takes a current of 45 A. The motor runs at a speed of 1000 rpm. Find the torque developed by the armature, if the armature and the shunt field resistances are 0.4 Ω and 55 Ω, respectively.

5.19 A shunt motor takes a line current of 55 A from the 230 V supply. The motor runs at a speed of 1200 rpm and its field circuit resistance is 60 Ω. Determine the armature resistance if the torque developed by the armature is 80 N-m.

5.20 The back emf of a DC shunt motor is 205 V. The motor draws a line current of 40 A from the supply. The motor runs at a speed of 900 rpm and produces an armature torque of 75 N-m. Determine the shunt field current.

5.21 A 6-pole shunt motor takes a line current of 35 A from the 230 V DC supply. The motor is lap wound by 450 conductors and the armature circuit resistance is 0.6 Ω. The motor develops an output power of 5.7 kW. The current through the field windings is 4 A and the flux per pole is 0.05 Wb. Calculate the speed and the shaft torque.

5.22 The armature and field circuit resistance of the 400 V DC series motor is 0.6 Ω and takes a current of 44 A of current while running at 650 rpm. Determine the motor speed for a line current of 35 A.

5.23 A 440 V DC series motor, having an armature and a field circuit resistance of 0.5 Ω. It takes a 50 A line current when running at 1000 rpm. Calculate the speed of the motor if the line current is 60 percent of the initial line current.

5.24 A DC series motor takes a line current of 50 A from the 230 V supply while running at a speed of 800 rpm. The armature and field resistances of the motor are 0.2 Ω and 0.25 Ω, respectively. Find the speed of the motor when the line current is 25 A. Consider the flux for 25 A current is 60% of the flux for the line current of 50 A.

5.25 A DC series motor runs at a speed of 1000 rpm and draws a current of 110 A from the 230 V supply. The total resistance of the armature and the field is 0.3 Ω. Calculate the speed of the motor when it develops a torque that is half of the initial torque. Assume the magnetic circuit is unsaturated.

5.26 The speed of a DC series motor is 900 rpm and draws a current of 100 A from the 220 V supply. The total resistance of the armature and field is 0.35 Ω. Calculate the speed of the motor when it develops a torque, which is equal to 40% of the initial torque. Assume the magnetic circuit is unsaturated.

5.27 A DC series motor takes a line current of 45 A from the 420 V supply and runs at a speed of 700 rpm. Calculate the percentage change of torque and the speed of the motor if it takes a current of 35 A when the total resistance of the armature and the field is 0.35Ω.

5.28 A 400 V DC series motor takes a line current of 50 A and runs at a speed of 1000 rpm. The ratio of initial to final torque is 5. Determine the speed of the motor if the total resistance of the armature and field is 0.45 Ω.

5.29 A 240 V DC series motor takes a line current of 40 A. The load of the motor is reduced so that the motor is taking 30 A line current. Determine the percentage change in torque.

5.30 The percentage change in torque of the DC series motor is 45. Determine the ratio of initial to final armature current.

5.31 A 400 V DC series motor takes a line current of 32 A while running at a speed of 1200 rpm and develops a torque. The motor has an armature resistance of 0.3 Ω and a field resistance of 0.2 Ω. Determine the speed of the motor to develop the torque $0.5T$.

5.32 The armature of the shunt motor takes a current of 45 A from the 220 V supply while running at a speed of 900 rpm. The motor armature circuit has a resistance of 0.3 Ω. The flux is reduced to 15% by the field regulator. Determine the speed if the load torque is unchanged.

5.33 The armature circuit resistance of a DC shunt motor is 0.5 Ω. The armature takes a current of 25 A from the 415 V supply while it runs at a speed of 600 rpm. Determine the speed of the motor if the flux is reduced to 30% and the torque developed by the armature increases to 45%.

5.34 A 250 V DC shunt motor has a field resistance of 50 Ω and an armature resistance of 0.05 Ω. The motor line current is 127 A while running at a speed of 1250 rpm. Find the speed of the motor if the field resistance is raised to 60 Ω. Assume the line current is constant.

5.35 A shunt motor takes a line current of 6 A from the 220 V supply at no-load condition. The armature and shunt field resistances of the motor are 0.5 Ω and 110 Ω, respectively. Under load conditions, the motor can carry a current of 42 A. Determine the iron and friction losses and efficiency.

5.36 The line current of the 12 HP 230 V shunt motor is 42 A. The armature and field resistances are 0.2 Ω and 230 Ω respectively. The total brush drop is 3 V. Determine the full load efficiency.

5.37 A DC shunt motor draws a line current of 25 A from a 240 V supply. The armature resistance is 0.4 Ω and the shunt field resistance is 100 Ω. Determine the power input to the motor, power loss in the armature and shunt field and electrical power developed in the armature.

6

Control and Starting of DC Motors

6.1 Introduction

DC motors are widely used in different applications where adjustable speed, good speed regulation, frequent starting, braking and reversing are necessary. In some industrial applications, a variable-speed motor is required to operate the mechanical machines smoothly. This type of motor is normally used in elevators, paper mills, rolling mills, machine tools and railroads. In these types of applications, the motor gets to rest for some time. This kind of electrical braking increases the life of the motor and reduces the wear of the mechanical parts of the machine as well as the motor. The armature resistance of this motor is very small and it draws a large current if the motor is connected directly to the supply. This high magnitude of current might damage the motor itself. Therefore, an additional device or arrangement is necessary to start the DC motor. This special starting device is known as a starter. In this chapter, the control and starting of a DC motor will be discussed.

6.2 Speed Controlling Parameters

There are a few parameters which play an important role in controlling the speed of a DC motor. These parameters can be identified by considering the expression of the back emf. The general equation of the speed of a motor can be expressed as,

$$(6.1)\quad E_b = \frac{PZ\phi N}{60A} = V_t - I_a R_a$$

$$N = \frac{60A}{PZ} \frac{V_t - I_a R_a}{\phi} \qquad (6.2)$$

$$N = K \frac{V_t - I_a R_a}{\phi} \qquad (6.3)$$

Where the value of the constant is expressed as,

$$K = \frac{60A}{PZ} \qquad (6.4)$$

Equation (6.3) can be modified as,

$$N \propto \frac{V_t - I_a R_a}{\phi} \qquad (6.5)$$

From equation (6.5), it is observed that the speed of the motor can be controlled by controlling the flux, armature resistance and applied voltage. Depending on these parameters, the methods of controlling are:

- Flux per pole, known as the flux control method.
- Armature circuit resistance, namely, rheostatic control.
- Applied voltage, known as the voltage control method.

6.3 Flux Control Method

In the flux control method, the supply voltage *Vt* and the armature resistance *Ra* are always fixed. In this case, the speed is controlled by varying only the field current. Fig. 6.1 shows a connection diagram of a shunt motor with a variable field resistance. The field current can be expressed as,

$$I_f = I_{sh} = \frac{V_t}{R_f} \qquad (6.6)$$

The magnitude of the field current will be decreased for a higher value of the field resistance. This relation can be expressed as,

$$I_f \downarrow = I_{sh} \downarrow = \frac{V_t}{R_f \uparrow} \tag{6.7}$$

The magnitude of the field current will be increased for a lower value of the field resistance. This relation is written as,

$$I_f \uparrow = I_{sh} \uparrow = \frac{V_t}{R_f \downarrow} \tag{6.8}$$

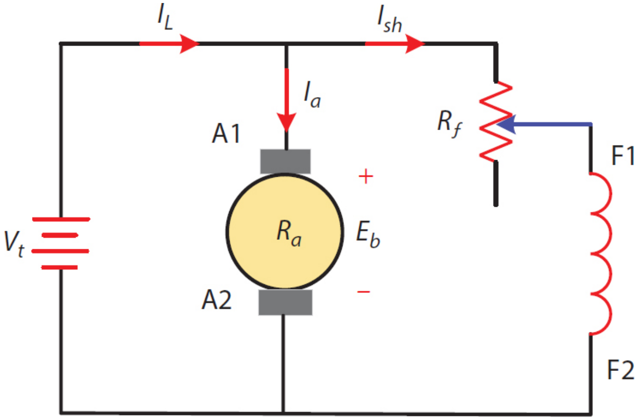


Fig. 6.1 Shunt motor with variable field resistance.

The flux ϕ decreases as the field current decreases. This reduced value of the flux decreases the value of the back emf as shown here,

$$E_b \downarrow = kN\phi \downarrow \tag{6.9}$$

Due to this reduced back emf, the current through the armature increases and it can be expressed as,

$$I_a \uparrow = \frac{V_t - E_b \downarrow}{R_a} \tag{6.10}$$

This increased armature current increases the torque and it can be written as,

$$T_{sh} \uparrow = kI_a \uparrow \tag{6.11}$$

This increasing torque increases the speed. As a result, the back emf increases as,

$$E_b \uparrow = kN \uparrow \phi \tag{6.12}$$

This increased value of back emf decreases the armature current as,

$$I_a \downarrow = \frac{V_t - E_b \uparrow}{R_a} \tag{6.13}$$

This decreased value of the armature current decreases the torque,

$$T_{sh} \downarrow = kI_a \downarrow \tag{6.14}$$

This torque decreases until it is equal to the full load torque at a higher speed. The no-load speed of the motor increases with the decrease of the flux. This method of speed control is very simple and permits remote control of the speed.

6.4 Armature Control Method

In the armature control method, the voltage across the armature terminals V_t and the shunt field current I_{sh} are kept constant at the rated voltage. Here, variable resistance is connected in series with the armature, which is shown in Fig. 6.2. The current through the variable resistance produces more voltage drop. As a result, a small voltage will appear across the armature terminals. Ultimately, the speed of the motor will be decreased as the speed is directly proportional to the applied voltage. In this method, a wide range of speed can be obtained and the motor can develop any desired torque over this wide range of speed. This method is normally recommended for a smaller size of the motor. The current in the variable resistance produces a large magnitude of power (loss) as well as heat. This loss reduces the overall efficiency of the motor. The following parameters can be defined as,

Ia_1 is the initial current of the armature without variable resistance,

Ia_2 is the final of the armature with variable resistance,

N_1 is the initial speed,

N_2 is the final speed.

The expression of the initial speed can be written as,

$$N_1 \propto V_t - I_{a1}R_a \tag{6.15}$$

Consider a variable resistance R, which is connected in series with the armature to reduce the speed N_2. At that time, the total armature circuit resistance is $R_a + R$. The expression of final speed is written as,

$$N_2 \propto V_t - I_{a2}(R_a + R) \tag{6.16}$$

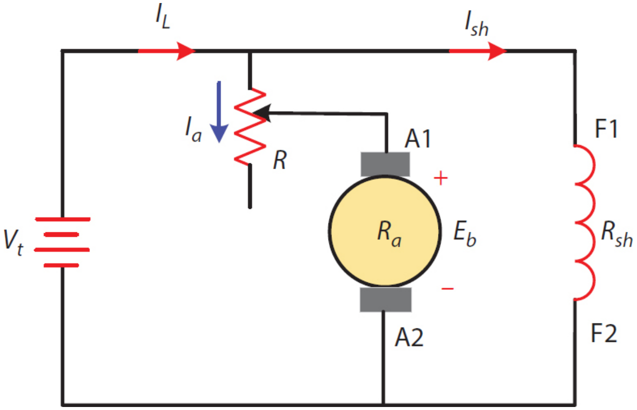


Fig. 6.2 Shunt motor with a variable resistance.

Dividing the equation (6.11) by the equation (6.10), yields,

$$\frac{N_2}{N_1} = \frac{V_t - I_{a2}(R_a + R)}{V_t - I_{a1}R_a} \tag{6.17}$$

This method of speed control is used where a speed lower than the rated speed is required for a short period. This method is also used where the load drops off suddenly with an increase in speed.

Example 6.1

A 220 V DC shunt motor takes an armature current of 10 A and runs at a speed of 900 rpm. Additional resistance is required in series with 1 Ω armature resistance to reduce the speed to 500 rpm. For the same armature current, find the value of the additional resistance.

Solution

The initial back emf is determined as,

$$E_{b1} = V_t - I_{a1} R_a = 220 - 10 \times 1 = 210\,\mathrm{V} \tag{6.18}$$

The final back emf is calculated as,

$$E_{b2} = V_t - I_{a1}(R_a + R) = 220 - 10 \times (R + 1) = 210 - 10R\ \mathrm{V} \tag{6.19}$$

Use the following relation to calculate *R*,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \tag{6.20}$$

Substituting the values of initial and final back emf to equation (6.20) yields,

$$\frac{500}{900} = \frac{210 - R}{210} \tag{6.21}$$

$$R = 9.3\Omega \tag{6.22}$$

Example 6.2

A shunt motor takes an armature current of 15 A from the 230 V supply. The armature circuit resistance is 0.4 Ω and runs at a speed of 650 rpm at full load. A variable resistance of 1 Ω is connected in series with the armature. Find the speed at the full load and half load torque.

Solution

The value of the initial back emf is calculated as,

$$E_{b1} = V_t - I_{a1}R_a = 230 - 15 \times 0.4 = 224\,\text{V} \tag{6.23}$$

At full load, the following relations are written,

$$T_2 = T_1 \tag{6.24}$$

$$I_{a2} = I_{a1} = 15\,\text{A} \tag{6.25}$$

The value of the final back emf is calculated as,

$$E_{b2} = 230 - 15 \times (1 + 0.4) = 209\,\text{V} \tag{6.26}$$

Use the following relation to find the speed yields,

$$N_2 = \frac{N_1 E_{b2}}{E_{b1}} = \frac{650 \times 204}{224} = 606.5\,\text{rpm} \tag{6.27}$$

From the torque-current relationship, at half load ($T_2 = T_1/2$), equation (6.28) becomes,

$$\frac{\frac{T_1}{2}}{T_1} = \frac{I_{a2}}{I_{a1}} \tag{6.28}$$

$$I_{a2} = \frac{15}{2} = 7.5\,\text{A} \tag{6.29}$$

The value of the back emf is calculated as,

$$E_{b2} = 230 - 7.5 \times 1.4 = 219.5\,\text{V} \tag{6.30}$$

The final speed can be determined as,

$$N_2 = \frac{N_1 E_{b2}}{E_{b1}} = \frac{650 \times 219.5}{224} = 636.9\,\text{rpm} \tag{6.31}$$

Practice Problem 6.1

A 240 V DC shunt motor takes an armature current of 10 A and runs at a speed of 1000 rpm. Some additional resistance is connected in series with a 0.6 Ω armature resistance to reduce the speed to 800 rpm. For the 15 A armature current, find the value of the additional resistance.

Practice Problem 6.2

A shunt motor takes an armature current of 10 A from the 220 V DC supply. The armature circuit resistance is 0.2 Ω and runs at a speed of 950 rpm at full load. A variable resistance of 1.5 Ω is connected in series with the armature. Find the speed at the full load torque.

6.5 Armature Voltage Control Method

In the armature voltage control method, the voltage across the armature terminals is varied by changing the speed of the motor. Here, the armature circuit resistance and the shunt field current are kept constant. A variable voltage is supplied to the motor armature by a suitable variable transformer and switchgear as shown in Fig. 6.3.

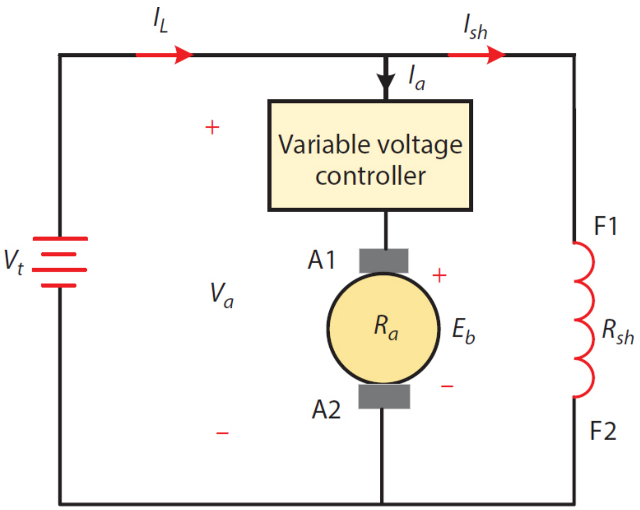


Fig. 6.3 Connection diagram of armature voltage control method.

The speed of the motor is varied with the variable supply voltage to the armature. The speed of the motor will be approximately proportional to the applied voltage. If the armature voltage of a separately excited motor is reduced in such a way, then it becomes less than the back emf. In this case, the direction of the armature current will be reversed and the motor will act as a generator. This type of operation will be continued until that speed where the back emf is equal to the applied voltage.

The voltage across the armature terminals should be changed carefully during the increase and decrease of the speed. Otherwise, a sudden change in armature voltage will cause a high current, which may damage the commutator. This method of speed control is an expensive approach, as it requires a variable DC voltage supply for the armature circuit. Therefore, this method is rarely used to control the speed.

6.6 Speed Control of a Series Motor

The speed of a DC series is controlled based on the expressions of $N \propto E_b/\phi$ or $N \propto (V_t - I_aR_a - I_aR_{se})/\phi$. Based on these expressions, the speed of the series motor is controlled by controlling the flux or field-controlled method and the armature resistance-controlled method. In a field-controlled method, a variable resistance called a diverter is connected in parallel with the series field winding as shown in Fig. 6.4. In this method, a portion of the current of the series field winding is shunted by the diverter resistance.

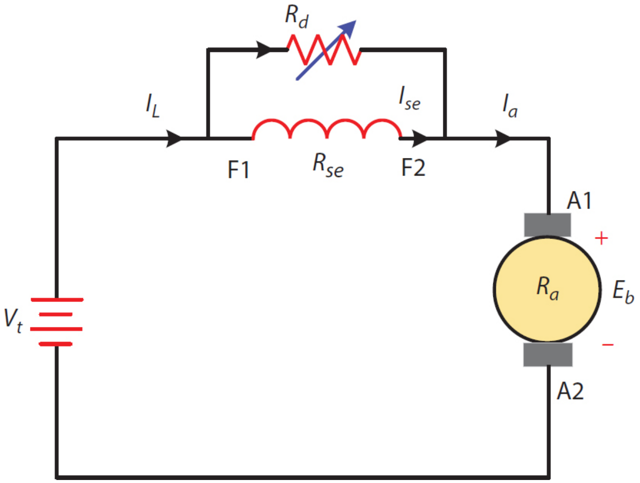


Fig. 6.4 Series field with diverter resistance.

Therefore, less current will flow through the series field winding, which will produce less flux. This reduced flux will increase the speed of the series motor; it can be explained in the following way:

$$N \propto \frac{1}{\phi} \tag{6.32}$$

The field flux will also be increased if the current through the diverter is zero. Therefore, the speed of the motor will decrease. The series field diverter is normally used for heavy-duty purposes. The speed of the series motor may also be decreased by connecting a resistance in series with the series field, which is shown in Fig. 6.5. As a result, total resistance will increase. Therefore, the supply voltage across the armature terminals will be reduced due to total resistive drop (IR). Due to this reduced armature voltage, the speed of the series motor is decreased. Therefore, different values of series resistance help to obtain different speeds of the series motor under the rated speed. The armature series resistance control method is widely used to control the speed of the series DC motor. Another method is also used to reduce the speed of the series motor. This method is known as the armature diverter method. In this method, the diverter resistance is connected in parallel to the armature, which is shown in Fig. 6.6. For a given load, the diverter resistance reduces the armature current. This reduced armature current increases the flux, which will decrease the speed of the series motor.

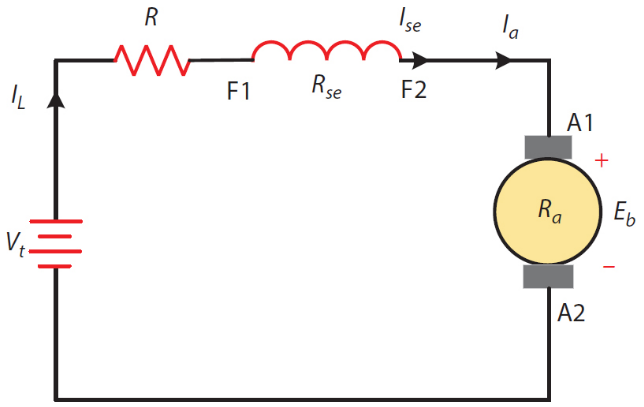


Fig. 6.5 Series field with series resistance.

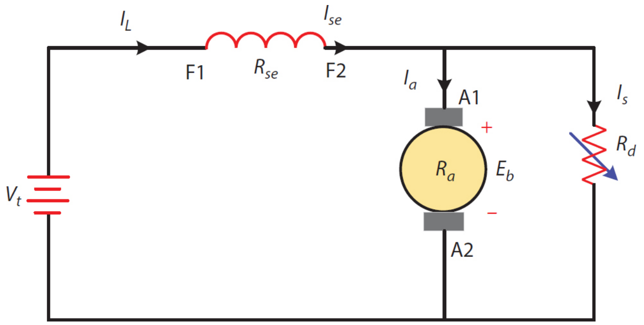


Fig. 6.6 Armature with diverter resistance.

Example 6.3

Variable resistance *R* is connected in series with the armature of the 230 V series motor to adjust its speed. The total resistance of the armature and the field is 0.2 Ω. For a certain load, the value of the resistance is zero (*R* = 0), the armature current is 10 A and the motor speed is 1000 rpm. For other loads, the armature current is found to be 8 A when R = 2 Ω. Find the new speed when the flux in the second case is 80% of the flux in the first case.

Solution

First case: The values of the given parameters are,

$$(6.33)\ I_{a1} = 10\,\text{A}$$

$$(6.34)\ R_a = 0.2 + 0 = 0.2\,\Omega$$

The value of the back emf is calculated as,

$$(6.35)\ E_{b1} = V_t - I_{a1}R_a = 230 - 10 \times 0.2 = 228\,\text{V}$$

Second case:

The values of the given parameters are,

$$(6.36)\ I_{a2} = 8\,\text{A}$$

$$R_a = 0.2 + 2 = 2.2\,\Omega \tag{6.37}$$

The value of the back emf is calculated as,

$$E_{b2} = V_t - I_{a2}R_a = 230 - 8 \times 2.2 = 212.4\,\text{V} \tag{6.38}$$

The value of the flux in the second case is given by,

$$\phi_2 = 0.8\phi_1 \tag{6.39}$$

The value of the new speed can be calculated as,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \tag{6.40}$$

$$N_2 = \frac{212.4}{228} \times \frac{\phi_1}{0.8\phi_1} \times 1000 = 1164.5\,\text{rpm} \tag{6.41}$$

Example 6.4

A 4-pole DC series motor runs at a speed of 1100 rpm and draws 15 A current from the 220 V DC supply. It has four parallel paths, which are connected in different ways. The value of series field resistance is 0.6 Ω and the armature resistance is 0.9 Ω. In the first case, all four series field resistors are connected in a series as shown in Fig. 6.7. The new line current is found to be 25 A when the equivalent coil of two series field coils is connected in parallel with the equivalent coil of the other two series field coils. The flux for 25 A current is 80% of the flux for 15 A current. Determine the speed for the second case.

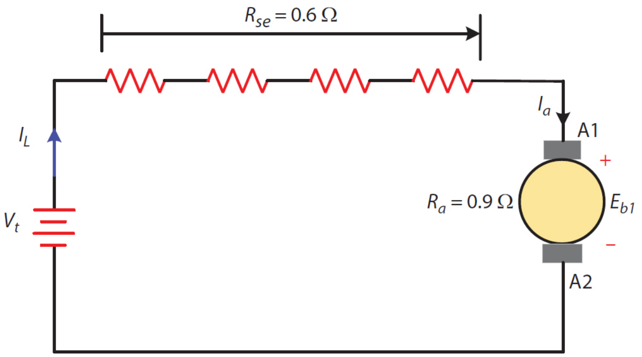


Fig. 6.7 Four series field resistors are in series.

Solution

First case:

Consider the circuit as shown in Fig. 6.7. The total series field resistance is given by,

$$R_{se} = 0.6\Omega \tag{6.42}$$

The value of the back emf is calculated as,

$$E_{b1} = 220 - 15 \times (0.6 + 0.9) = 197.5\,\text{V} \tag{6.43}$$

Second case:

Consider the circuit as shown in Fig. 6.8. The value of the resistance per path is calculated as,

$$R_{se} = \frac{0.6}{4} = 0.15\Omega \tag{6.44}$$

In this case, the value of the back emf is calculated as,

$$E_{b2} = 220 - 25 \times \left(0.9 + \frac{0.3}{2} \right) = 193.7\,\text{V} \tag{6.45}$$

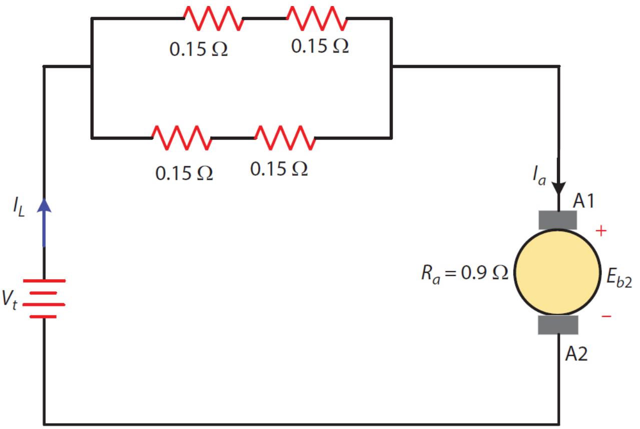


Fig. 6.8 Two series field resistors are in parallel.

The value of the flux is given by,

$$\phi_2 = 0.8\phi_1 \tag{6.46}$$

The new speed can be determined as,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \tag{6.47}$$

$$N_2 = \frac{193.7}{197.5} \times \frac{\phi_1}{0.8\phi_1} \times 1100 \tag{6.48}$$

$$N_2 = 1348.9\,\text{rpm} = 1349\,\text{rpm} \tag{6.49}$$

Practice Problem 6.3

A variable resistance R is connected in series with the 220 V series motor armature to adjust its speed. The total resistance of the armature and the field is 0.03 Ω. For a certain load, the value of the resistance is $R = 0.2$ Ω, the armature current is 15 A and the motor speed is 1100 rpm. For other loads, the armature current is found to be 10 A, when $R = 4$ Ω. Find the new speed when the flux in the second case is 90% of the flux in the first case.

Practice Problem 6.4

A 2-pole DC series motor runs at a speed of 900 rpm and draws 10 A current from the 250 V DC supply. The value of the series field resistance is 0.4 Ω and the armature resistance is 0.7 Ω. The new line current is found to be 20 A when one series field coil is connected in parallel with another series field coil. The flux for 20 A current is 95% of the flux for 10 A current. Calculate the new speed of the motor.

6.7 Ward Leonard System

In this system, the motor-generator (M-G) set is used to control the speed of the DC motor. Here, a three-phase induction motor is connected with the separately excited DC generator as shown in Fig. 6.9. A three-phase induction motor is used to run the DC generator. The generator armature is directly connected to the motor armature. The generator field and the motor field are connected across the DC voltage source.

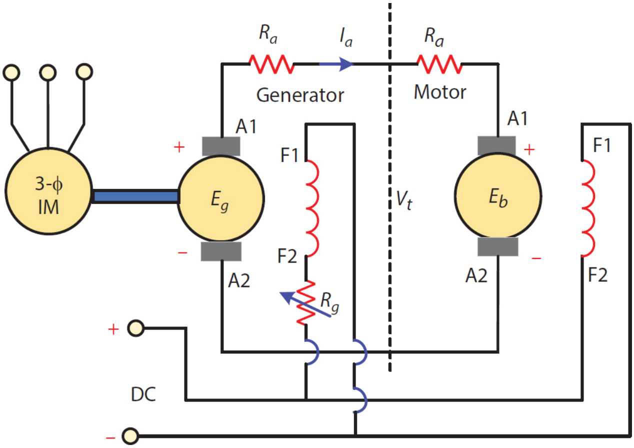
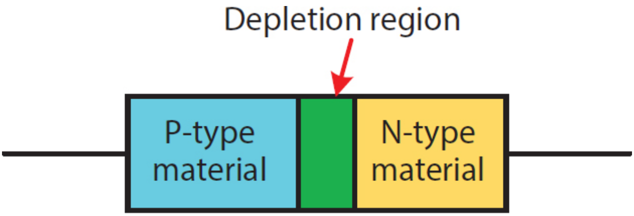


Fig. 6.9 Connection diagram of Ward Leonard system.

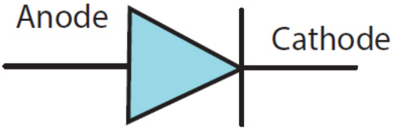
The variable resistance *Rg* is connected in series with the generator field winding. When the variable resistance decreases, the current in the generator field winding increases, which increases the field flux. As a result, the generated voltage *Eg* of the generator increases, which is equal to the motor terminal voltage. Due to this higher terminal voltage, the DC motor will rotate faster. Again, when the variable resistance increases, the current in the generator field winding decreases, which decreases the field flux. As a result, the generated voltage *Eg* of the generator decreases, which is equal to the motor terminal voltage. Due to this lower terminal voltage, the DC motor will rotate slower. Finally, when the variable resistance is gradually brought out as the generator picks up the speed and the motor begins to rotate. Therefore, the output of the generator is changed by varying the generator field resistance. The generator output is connected to the motor armature, thereby controlling the speed of the motor. This system is more flexible and expensive and permits fine adjustment of the speed.

6.8 Semiconductor Devices

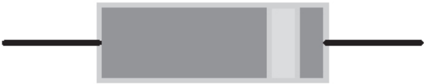
Semiconductor devices such as diodes and thyristors are used to control the speed of the DC motor by controlling the armature voltage. This armature voltage is controlled by AC-DC converters. However, the AC-DC converters affect the linearity of the torque-speed characteristics. Therefore, the armature voltage of the DC motor is controlled by different topologies of power semiconductor devices such as controlled and uncontrolled rectifiers using diodes and thyristors. Diodes and thyristors are discussed here to get fundamental knowledge about them. The two-terminal semiconductor device is known as a diode. It consists of p-type and n-type semiconductor materials. The basic structure and the symbol of the diode are shown in Fig. 6.10. If the arrow of the diode is connected to the positive terminal of the battery, then the diode is known as forward bias. In this mode of operation, the depletion region collapses and the diode acts as a closed switch which conducts current. Here, the voltage across the diode is zero ($V_d = 0$) and the current in the diode is more than zero ($i_d > 0$).



(**a**) P-N Structure



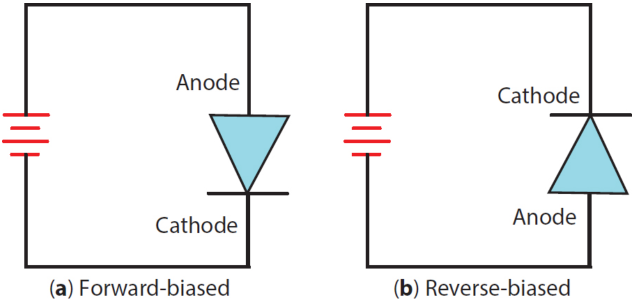
(**b**) Diode symbol



(**c**) Practical appearance

Fig. 6.10 Basic structure and symbol of a diode.

If the anode of the diode is connected to the negative terminal of the battery, then the diode is known as reverse bias. In the reverse bias, the depletion region increases and the diode acts as an open circuit that does not conduct current. Here, the voltage across the diode is less than zero ($V_d < 0$) and the current in the diode is zero ($i_d = 0$). The connection diagrams of the forward-biased and reverse-biased diode are shown in Fig. 6.11.



(**a**) Forward-biased (**b**) Reverse-biased

Fig. 6.11 Connection diagram.

The forward-biased voltage for a germanium diode is 0.3V and for a silicon diode is 0.7V. The diode initially produces a pulsating direct component of current when it is connected in series with the alternating voltage source and the load. Later on, a filter circuit is used to get a pure DC which is used in many applications. A few applications of the diode are the conversion of alternating voltage to direct voltage, speed control of AC and DC motors, acting as a switch, etc. The id-Vd characteristic of a diode is shown in Fig. 6.12. In the forward-bias region, the value of the voltage at which the conduction current starts to increase sharply is known as the knee voltage. In the reverse-bias region, the value of the voltage at which the current starts to increase rapidly to the opposite direction is called the breakdown voltage of the diode. The process to increase the reverse current in the diode is known as diode avalanche and this high magnitude of reverse current can destroy the diode.

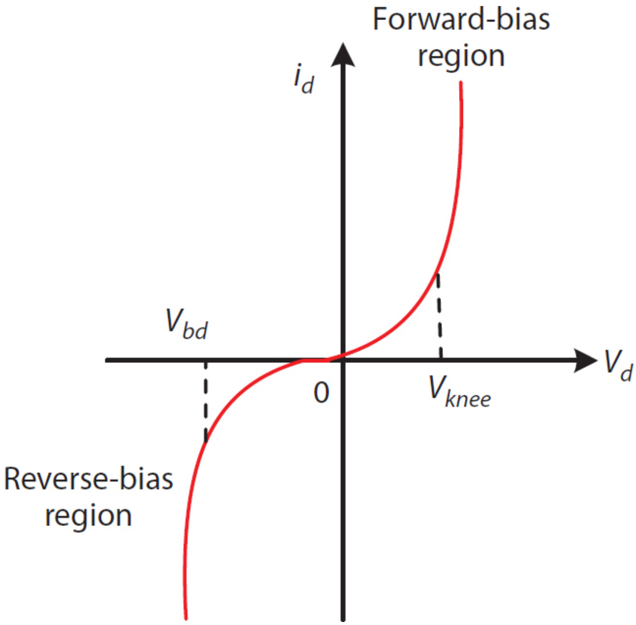


Fig. 6.12 Current-voltage characteristic of a diode.

The rectifier circuit is divided into types, namely, half-wave rectification and full-wave rectification circuits.

6.9 Half-Wave Rectifier

A single diode is connected in series with the source for a half-wave rectifier circuit as shown in Fig. 6.13. At the positive half cycle (0 to π) of the input voltage, the diode is forward-biased which conducts current. During the negative half cycle (π to 2π) of the input voltage, the diode is reverse-biased and it does not conduct current. Since there is no current in the load resistance, the voltage across the load resistance is zero. The input and output voltage waveforms are shown in Fig. 6.14.

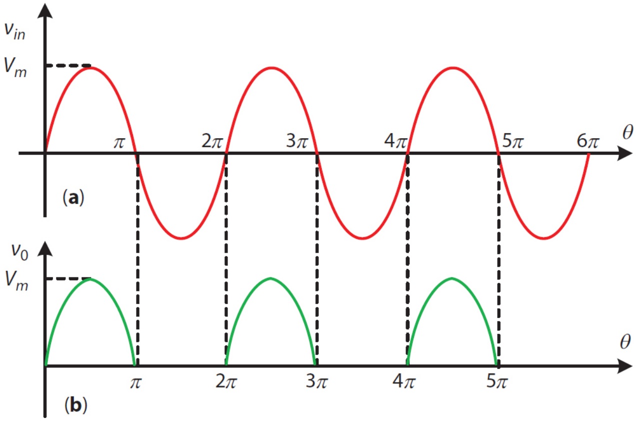


Fig. 6.14 Input and output voltage waveforms.

Consider the expression of the input voltage as,

$$v_{in} = V_m \sin\theta \tag{6.50}$$

The expression of the current is written as,

$$i = \frac{V_m \sin\theta}{R} \tag{6.51}$$

$$i = I_m \sin\theta \tag{6.52}$$

Where the maximum value of the current is expressed as,

$$I_m = \frac{V_m}{R} \tag{6.53}$$

The average value of the current for one cycle is calculated as,

$$I_{dc} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta \, d\theta \tag{6.54}$$

$$I_{dc} = \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} \tag{6.55}$$

$$I_{dc} = \frac{I_m}{\pi} \tag{6.56}$$

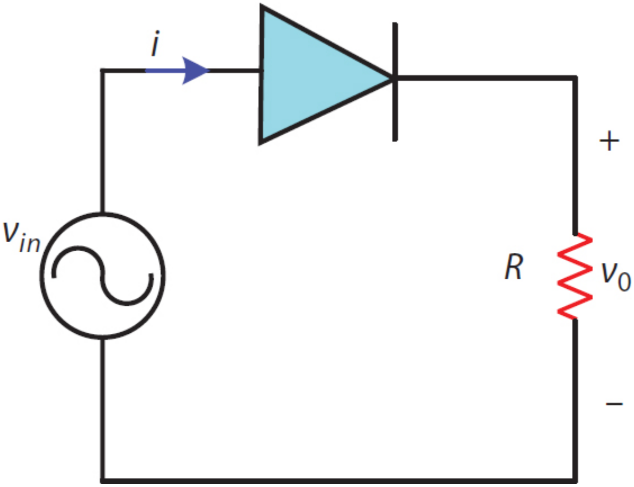


Fig. 6.13 Connection diagram of a half-wave rectifier.

The expression of the rms value of the current can be derived as,

$$I_{rms}{}^2 = \frac{1}{2\pi}\int_0^{\pi} (I_m \sin\theta)^2 \, d\theta \tag{6.57}$$

$$I_{rms}{}^2 = \frac{I_m{}^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} \, d\theta \tag{6.58}$$

$$I_{rms}{}^2 = \frac{I_m{}^2}{2\pi}\left[\frac{\theta}{2}\right]_0^{\pi} - 0 \tag{6.59}$$

$$I_{rms}{}^2 = \frac{I_m{}^2}{4} \tag{6.60}$$

$$I_{rms} = \frac{I_m}{2} \tag{6.61}$$

The expression of the input power is written as,

$$P_{in} = P_{ac} = I_{rms}{}^2 (R + r_d) \tag{6.62}$$

Substituting equation (6.61) into equation (6.62) yields,

$$P_{ac} = \left(\frac{I_m}{2}\right)^2 (R + r_d) \tag{6.63}$$

The expression of output power is written as,

$$P_o = P_{dc} = I_{dc}{}^2 R \tag{6.64}$$

Substituting equation (6.56) into equation (6.64) yields,

$$P_{dc} = \left(\frac{I_m}{\pi}\right)^2 R \tag{6.65}$$

The efficiency of a half-wave rectifier can be determined as,

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100 \tag{6.66}$$

Substituting equations (6.63) and (6.65) into equation (6.66) yields,

$$\eta = \frac{\left(\frac{I_m}{\pi}\right)^2 R}{\left(\frac{I_m}{2}\right)^2 (R + r_d)} \times 100 \tag{6.67}$$

$$\eta = \frac{40.5}{\left(1 + \frac{r_d}{R}\right)} \tag{6.68}$$

If the load resistance is higher than the diode resistance, then the term $r_d/R = 0$. Equation (6.68) can be modified as,

$$\eta = 40.5\% \tag{6.69}$$

From equation (6.69), it is seen that the efficiency of the half-wave rectifier circuit is 40.5%.

6.10 Full-Wave Rectifier

In the full-wave rectifier, both halves of the input voltages are used to produce the DC output. The full-wave rectifier circuit is classified as a center-tapped rectifier and bridge rectifier. In a center-tapped full-wave rectifier, two diodes are used with a center-tapped transformer. Whereas in a bridge full-wave rectifier, four diodes are used. A detailed analysis of these rectifiers is discussed here.

6.10.1 Centre Tap Full-Wave Rectifier

In a center-tapped full-wave rectifier, two diodes and a center-tapped transformer are used as shown in Fig. 6.15. At the positive half cycle of the input voltage, point A is positive and point B is negative. Hence, a positive terminal is connected to the anode of the diode D_1 which makes it forward-biased, whereas the negative terminal (point B) is connected to the diode D_2 which makes it reverse-biased. Therefore, diode D_1 will conduct current and diode D_2 will not conduct any current. During the negative half cycle of the input voltage, point B is positive and point A is negative. In this condition, the negative terminal (point A) is connected to the diode D_1 which makes it reverse-biased and does not conduct current. The positive terminal (point B) is connected to the anode of the diode D_2 which makes it forward-biased and conducts current. The input voltage and output voltage waveforms of the center-tapped full-wave rectifier are shown in Fig. 6.16.

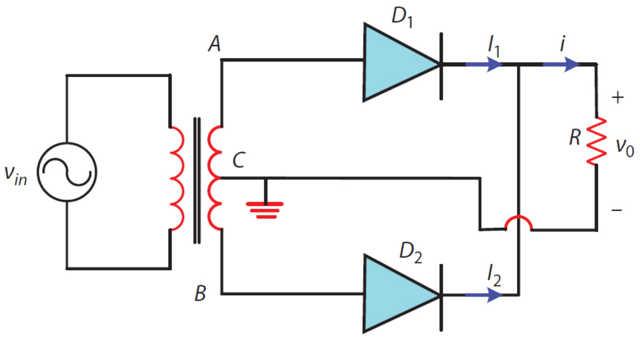


Fig. 6.15 Connection diagram of a center tap rectifier.

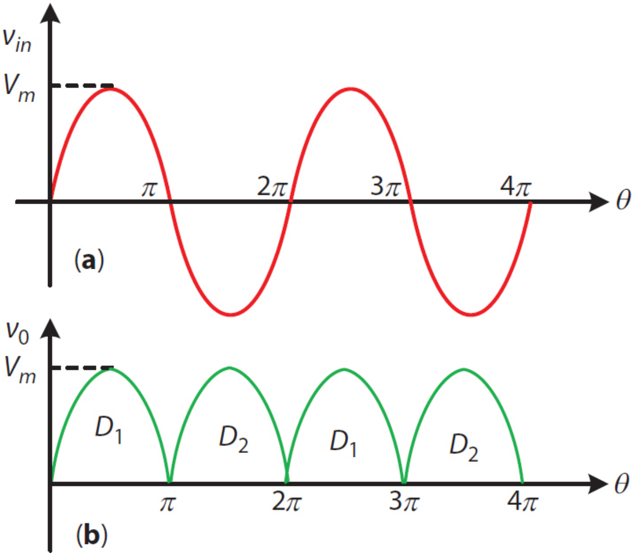


Fig. 6.16 Input and output voltage waveforms.

6.10.2 Bridge Type Full-Wave Rectifier

A connection diagram of a full-wave bridge rectifier is shown in Fig. 6.17. At the positive half cycle of the secondary voltage, point A is positive and point B is negative.

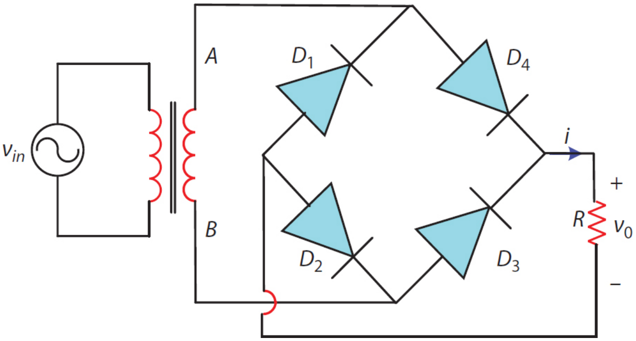


Fig. 6.17 Connection diagram of a bridge rectifier.

In this polarity, diodes D_4 and D_2 are forward-biased and conduct current, whereas the diodes D_1 and D_3 are reverse-biased, which makes them non-conducting. The current flows through D_4-R-D_2 as shown in Fig. 6.18.

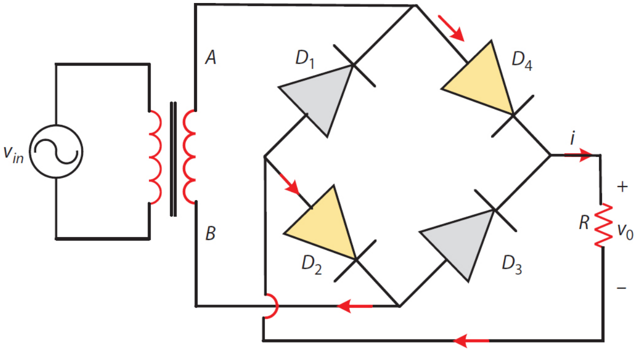


Fig. 6.18 Connection diagram of a bridge rectifier when point A is positive.

When point B is positive and point A is negative, diodes D_3 and D_1 are forward-biased that conduct current. In this condition, diodes D_2 and D_4 are reverse-biased and do not conduct current. In this case, the current through D_3-R-D_1 is shown in Fig. 6.19.

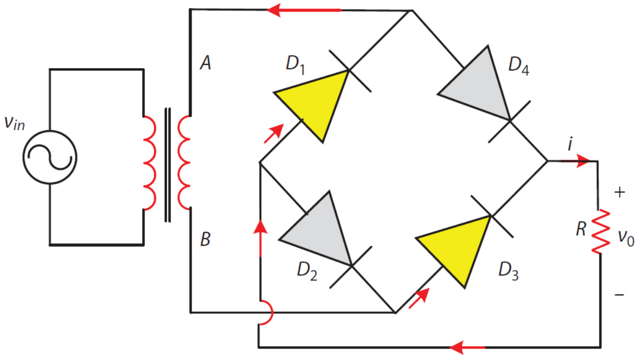


Fig. 6.19 Connection diagram of a bridge rectifier when point B is positive.

The input and output voltage waveforms are shown in Fig. 6.20. The average value of the current over one cycle is determined as,

$$I_{dc} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta \tag{6.70}$$

$$I_{dc} = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} \tag{6.71}$$

$$I_{dc} = \frac{2I_m}{\pi} \tag{6.72}$$

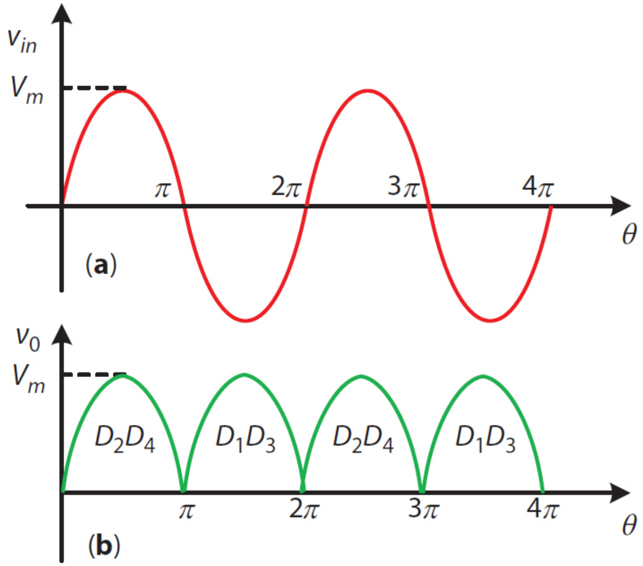


Fig. 6.20 Input and output voltage waveforms.

The expression of the rms value of the current can be derived as,

$$I_{rms}^{2} = \frac{1}{\pi}\int_0^{\pi}(I_m \sin\theta)^2\, d\theta \tag{6.73}$$

$$I_{rms}^{2} = \frac{I_m^{2}}{\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}\, d\theta \tag{6.74}$$

$$I_{rms}^{2} = \frac{I_m^{2}}{2\pi}\left[\int_0^{\pi} 1\, d\theta - \int_0^{\pi}\cos 2\theta\, d\theta\right] \tag{6.75}$$

$$I_{rms}^{2} = \frac{I_m^{2}}{2\pi}\left[\theta\right]_0^{\pi} - 0 \tag{6.76}$$

$$I_{rms}^{2} = \frac{I_m^{2}}{2} \tag{6.77}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \tag{6.78}$$

Substituting equation (6.78) into equation (6.62) and the expression of input power is written as,

$$P_{ac} = \left(\frac{I_m}{\sqrt{2}}\right)^2 (R + r_d) \tag{6.79}$$

Substituting equation (6.72) into equation (6.64) and the expression of output power is written as,

$$P_{dc} = \left(\frac{2I_m}{\pi}\right)^2 R \tag{6.80}$$

Substituting equations (6.79) and (6.80) into equation (6.66) yields the efficiency of a full-wave rectifier as,

$$\eta = \frac{\left(\frac{2I_m}{\pi}\right)^2 R}{\left(\frac{I_m}{\sqrt{2}}\right)^2 (R + r_d)} \times 100 \tag{6.81}$$

$$\eta = \frac{81.2}{\left(1 + \frac{r_d}{R}\right)} \tag{6.82}$$

The load resistance is higher than the diode resistance and the term $r_d/R = 0$. Equation (6.82) can be modified as,

$$\eta = 81.2\% \tag{6.83}$$

6.11 Speed Control by Thyristor and Diode

Silicon-controlled rectifier (SCR) is the most widely used for rectification, regulation of power, inversion, etc. SCR has a four-layer (P-N-P-N) and three junctions J1, J2 and J3 as shown in Fig. 6.21(a). It has three terminals anode (P-type), cathode (N-type) and gate (P-type) as shown in Fig. 6.21(b).

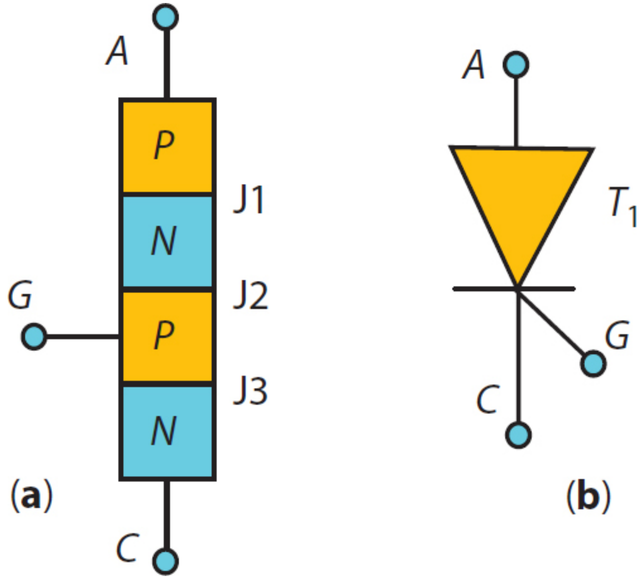


Fig. 6.21 Basic structure and symbol of SCR.

A single-phase half-wave rectifier with thyristor and diode is used to control the DC motor as shown in Fig. 6.22. During the positive half-cycle, the anode of the thyristor is positive with respect to the anode and the thyristor blocks the flow of the current until the proper gate-pulse is triggered. When the thyristor is fired by an angle α, the input voltage is converted to DC which is available across the DC motor armature.

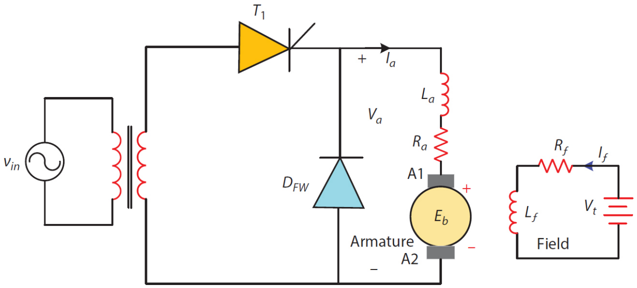


Fig. 6.22 Basic structure and symbol of SCR.

The average voltage across the armature terminals of the DC motor is determined as,

$$V_a = \frac{1}{2\pi}\int_\alpha^\pi V_m \sin\theta d\theta \tag{6.84}$$

$$V_a = \frac{V_m}{2\pi}\left[-\cos\theta\right]_\alpha^\pi = \frac{-V_m}{2\pi}\left(-1-\cos\alpha\right) \tag{6.85}$$

$$V_a = \frac{V_m}{2\pi}\left(1+\cos\alpha\right) \tag{6.86}$$

From equation (6.86), it is seen that the armature voltage is controlled by controlling the firing angle of the thyristor. Therefore, the speed of the DC motor is controlled by adjusting the thyristor firing angle from 0° to 180°. This method is suitable for the rating of the DC motor is about 0.5kW. The free-wheeler diode is used in this circuit to protect the thyristor from getting damaged by the reverse voltage generated by the inductive load of the motor.

Half-Controlled Bridge Rectifier

The half-controlled bridge rectifier is also known as the semi-converter which is used to control the speed of the DC motor. The semi-converter with a symmetrical configuration is shown in Fig. 6.23.

In the positive half-cycle of the input AC voltage, thyristor T1 and D2 are forward-biased. If the firing angle α of the thyristor is set up between 0° to 180°, then the current will flow and the DC voltage will appear across the motor armature terminals. Similarly, in the negative half-cycle of the input AC voltage, thyristor T2 and D1 are forward-biased. Due to a suitable firing angle α of the thyristor, the DC voltage will appear across the motor armature terminals. The average voltage across the armature terminals of the DC motor is determined as,

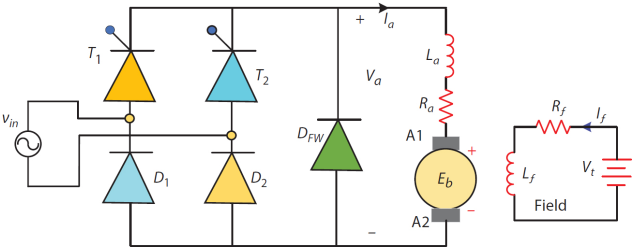


Fig. 6.23 Semi-converter with a symmetrical configuration.

$$V_a = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\theta d\theta \tag{6.87}$$

$$V_a = \frac{V_m}{\pi}\left[-\cos\theta\right]_{\alpha}^{\pi} = \frac{-V_m}{\pi}\left(-1-\cos\alpha\right) \tag{6.88}$$

$$V_a = \frac{V_m}{\pi}\left(1+\cos\alpha\right) \tag{6.89}$$

From equation (6.89), the speed of the DC motor is controlled by adjusting the thyristor firing angle from 0° to 180°.

6.12 Braking of a DC Motor

Apart from speed control, most of the users demand frequent starts and stops of a DC motor. In certain applications like sewing machines and dust control machines, it is necessary to stop the motor quickly. The motor may be stopped with the help of mechanical braking. However, it has some drawbacks. Mechanical braking depends on the skill of the operator and the condition of the braking surface. As an alternative way, the DC motor can be stopped by three commonly employed methods. These methods are plugging, dynamic braking and regenerative braking. Brief descriptions of those methods are given below.

6.12.1 Plugging

Plugging is required to brake the motor or exert very severe braking. This method is used to stop the motor by reversing the armature current. This method of braking is often known as reverse braking. The armature current can be reversed by changing the armature terminals. Fig. 6.24 shows the connection diagram, where the armature is in series with the variable resistance. Under normal conditions, the current through the armature is,

$$I_a = \frac{V_t - E_b}{R_a} \quad (6.90)$$

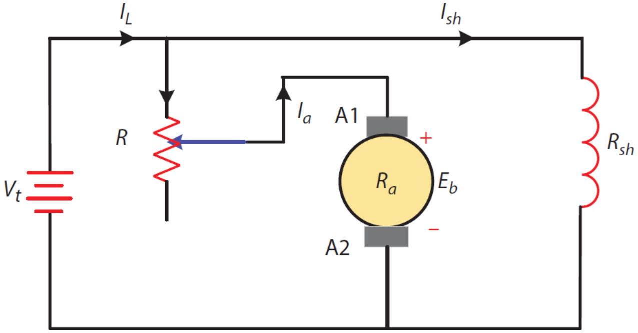


Fig. 6.24 Connection diagram of plugging.

If we suddenly change the terminals of the source voltage, then the back emf aids the supply voltage instead of counteraction. In this condition, the motor acts as a generator. Therefore, the expression of armature current under generator action is written as,

$$I_a = \frac{V_t + E_b}{R_a + R} \quad (6.91)$$

$$I_a = \frac{V_t}{R_a + R} + \frac{E_b}{R_a + R} \quad (6.92)$$

However, the general expression of back emf is,

$$E_b = \frac{PZ\phi N}{60A} \tag{6.93}$$

Substituting equation (6.93) into equation (6.92) yields,

$$I_a = \frac{V_t}{R_a + R} + \frac{PZ\phi N}{60A(R_a + R)} \tag{6.94}$$

Based on the general expression of torque, the braking torque is defined as,

$$T_b = \frac{PZ\phi I_a}{2\pi A} \tag{6.95}$$

Substituting equation (6.95) into equation (6.94) and the expression of braking torque is,

$$T_b = \frac{PZ\phi}{2\pi A}\left[\frac{V_t}{R_a + R} + \frac{PZ\phi N}{60A(R_a + R)}\right] \tag{6.96}$$

$$T_b = \frac{PZV_t\phi}{2\pi A(R_a + R)} + \left(\frac{PZ}{A}\right)^2 \frac{\phi^2 N}{2\pi(R_a + R)60} \tag{6.97}$$

$$T_b = k_1\phi + k_2\phi^2 N \tag{6.98}$$

Where the expression of constants are,

$$k_1 = \frac{PZV_t}{2\pi A(R_a + R)} \tag{6.99}$$

$$k_2 = \left(\frac{PZ}{A}\right)^2 \frac{1}{2\pi(R_a + R)60} \tag{6.100}$$

From equation (6.98), it is seen that there is torque even if the speed of the motor is zero ($N = 0$). This remaining torque is used to accelerate the motor in the reverse direction.

6.12.2 Dynamic Braking

In this method, the armature winding is disconnected from the supply and it is connected in series with a variable resistance R as shown in Fig. 6.25. Dynamic braking is often known as rheostatic braking. The motor now becomes a generator, which supplies energy to the resistor. The inertia energy is then converted into electrical energy and is dissipated as heat in the resistor. As a main driving force, the back emf will produce the armature current in the reverse direction. Because of the reverse direction of current in the armature, a torque will be produced in the armature winding. This torque opposes the rotation of the motor and the motor will stop. From Fig. 6.25, the expression of armature current is,

$$I_a = \frac{E_b}{R_a + R} \tag{6.101}$$

Substitute equation (6.93) into equation (6.101) yields,

$$I_a = \frac{PZ\phi N}{60A(R_a + R)} \tag{6.102}$$

Substitute equation (6.102) into equation (6.58) and the expression of braking torque is,

$$T_b = \frac{PZ\phi}{2\pi A}\left(\frac{PZ\phi N}{60A(R_a + R)}\right) \tag{6.103}$$

$$T_b = \frac{1}{2\pi}\left(\frac{PZ}{A}\right)^2 \frac{\phi^2 N}{60(R_a + R)} \tag{6.104}$$

$$T_b = k_3\phi^2 N \tag{6.105}$$

Where the expression of the constant is,

$$k_3 = \frac{1}{2\pi}\left(\frac{PZ}{A}\right)^2 \frac{1}{60(R_a + R)} \tag{6.106}$$

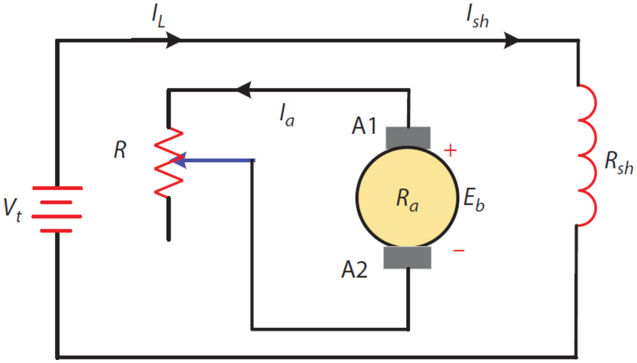


Fig. 6.25 Connection diagram of dynamic braking.

Finally, the expression of braking torque can be written as,

$$T_b \propto N \tag{6.107}$$

From equation (6.107), it is concluded that the braking torque decreases as the motor slows down. The braking torque will be zero when the speed is zero. This method has lower braking torque compared to reverse braking.

6.12.3 Regenerative Braking

This type of braking is used when the speed of the motor increases from its rated speed. These applications are elevators, cranes, electric trains, etc. These types of loads drive the DC motor faster than its rated speed. As a result, back emf becomes greater than the terminal voltage. Due to higher back emf, the armature current flows in the reverse direction, which forces the motor to act like a generator. This armature current produces a torque, which acts in the opposite direction to the motor speed. The motor speed falls until the back emf in the motor is less than the applied voltage. The power produced by regenerative action is normally returned to the source, which may be used elsewhere instead of wasted. Regenerative braking action with the direction of the currents is shown in Fig. 6.26. In Fig. 6.26, the back emf E_b is greater than the terminal voltage, i.e., $E_b > V_t$.

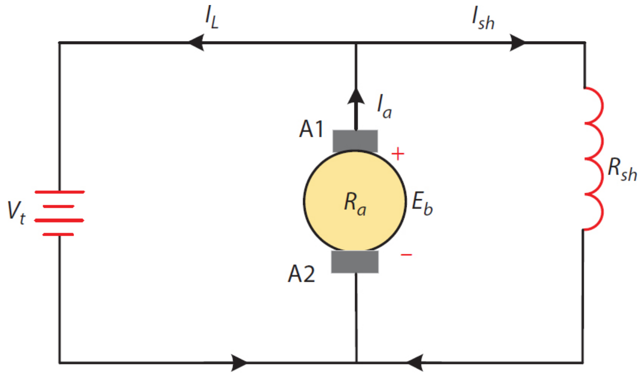


Fig. 6.26 Connection diagram of regenerative braking.

6.13 DC Motor Starter

Just at the time of starting the motor, the value of the back emf is zero. Therefore, the current in the armature circuit will be increased, and it can be explained in the following way,

$$I_a = \frac{V_t - 0}{R_a} \uparrow \tag{6.108}$$

For a smaller value of the armature circuit resistance, the armature current will be increased sharply. This high value of the starting current creates the following problems:

- The high current produces excessive heating, which may burn the armature partially.
- The high current produces a series of sparking, which damages the commutator and brushes.
- High current causes overloading of the connecting wires of the motor.
- Causes snapping off the shaft of the motor.
- Causes damage to the driven mechanical equipment.

An additional device is used to overcome these types of problems and to run the DC motor safely. This device is known as a starter; it is a variable resistance connected in series with the armature to limit the high starting current. The value of the variable resistance is gradually reduced as the motor gains its rated speed. Nowadays, an electronic system is often used to control the speed of the motor as well.

6.14 Three-Point Starter

The connection diagram of a three-point starter is shown in Fig. 6.27. The name of the three-point starter is due to the three points within the dotted lines. The positive terminal of the DC supply is connected to the starting resistance through the contact arm. This contact arm maintains the proper connection with the starting resistance by a suitable spring pressure.

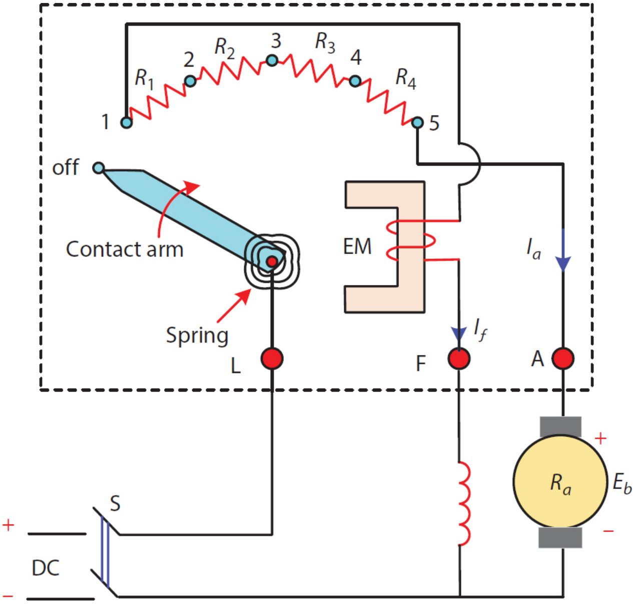


Fig. 6.27 Connection diagram of a three-point starter.

At the time of starting, the main switch is required to close and the contact arm will be moved slowly to the right. When the arm is moved to position 1, the field circuit is connected directly across the supply and all the starting resistances are connected in series with the armature. As a result, the large magnitude of armature circuit resistance limits the higher value of the starting current. When the arm moves gradually to positions 2, 3 and 4, the speed of the motor will be increased due to a decrease in the total resistance. The motor will run at its rated speed when the contact arm is at the final position (position 5). In this connection diagram, the function of the electromagnetic coil is to hold the contact arm to the final position when the motor is in normal operation. Sometimes, the electromagnetic coil does not create enough force to hold the contact arm in the right position. This is the main disadvantage of the three-point starter.

6.15 Four-Point Starter

The three-point starter has some drawbacks, which are discussed in section 6.14. To overcome the drawbacks of the three-point starter, a four-point starter is introduced, which is shown in Fig. 6.28.

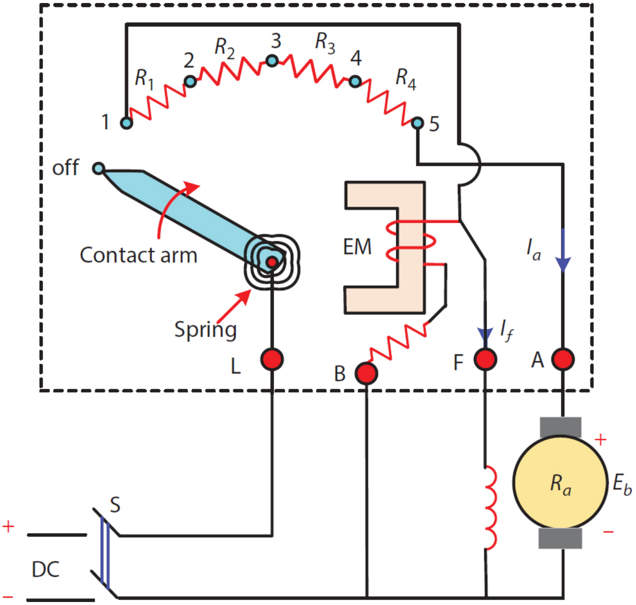


Fig. 6.28 Connection diagram of a four-point starter.

Here, the holding coil is in series with the current limiting resistance and it is connected in parallel with the shunt field as well with the armature circuit. In this arrangement, any change of shunt field current does not affect the electromagnetic force, which is created by the electromagnetic coil; meaning that the force created by the electromagnetic (EM) coil is enough to overcome the tension of the spring of the contact arm. The contact arm goes back to the off position if the supply voltage of the machine is disconnected.

6.16 Grading of Starting Resistance

Consider a shunt motor where extra starting resistance is connected in series with the armature, which is shown in Fig. 6.29. These starting resistances can be cut out from the circuit in segments by closing the contacts 1, 2, 3 and 4, respectively. When the contact arm is at position 1, then the maximum value of the starting current is calculated as,

$$I_1 = \frac{V_t}{R_1} \tag{6.109}$$

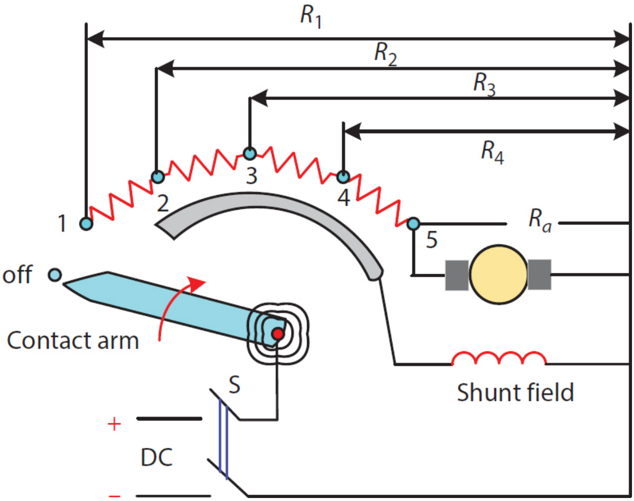


Fig. 6.29 Connection diagram of starting resistance.

The back emf will be developed at the time of leaving position 1, which will decrease the starting current. In this condition, the minimum value of the starting current is determined as,

$$I_2 = \frac{V_t - E_{b1}}{R_1} \tag{6.110}$$

When the contact arm is at position 2 by cutting out the resistance R_1, the value of the starting current must jump to its previous value I_1 due to reduced circuit resistance. The value of the current at position 2 is expressed as,

$$I_1 = \frac{V_t - E_{b1}}{R_2} \tag{6.111}$$

From equations (6.111) and (6.110), the ratio of two currents can be written as,

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} \tag{6.112}$$

Similarly, at positions 3 and 4, the values of maximum and minimum starting currents can be written in the following ways:

At the position 3, the expressions of currents are,

$$I_2 = \frac{V_t - E_{b2}}{R_2} \tag{6.113}$$

$$I_1 = \frac{V_t - E_{b2}}{R_3} \tag{6.114}$$

From equations (6.113) and (6.114), the ratio of currents can be written as,

$$\frac{I_1}{I_2} = \frac{R_2}{R_3} \tag{6.115}$$

At position 4, the expressions of the currents are,

$$I_2 = \frac{V_t - E_{b3}}{R_3} \quad (6.116)$$

$$I_1 = \frac{V_t - E_{b3}}{R_4} \quad (6.117)$$

The ratio of maximum to minimum currents from equations (6.116) and (6.117) is written as,

$$\frac{I_1}{I_2} = \frac{R_3}{R_4} \quad (6.118)$$

Similarly, for position 5, this ratio is written as,

$$\frac{I_1}{I_2} = \frac{R_4}{R_a} \quad (6.119)$$

From equations (6.112), (6.115), (6.118) and (6.119), the following relations are written as,

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \frac{R_4}{R_a} = k(\text{Let}) \quad (6.120)$$

$$R_4 = kR_a \quad (6.121)$$

$$R_3 = kR_4 \quad (6.122)$$

Substituting equation (6.121) into equation (6.122) yields,

$$R_3 = k^2 R_a \quad (6.123)$$

Again, the following relation is written as,

$$R_2 = kR_3 \quad (6.124)$$

Substituting equation (6.123) into equation (6.124) yields,

$$R_2 = k^3 R_a \quad (6.125)$$

Similarly, the initial value of the resistance is written as,

$$R_1 = kR_2 \tag{6.126}$$

Substituting equation (6.125) into equation (6.126) yields,

$$R_1 = k^4 R_a \tag{6.127}$$

Therefore, for the n^{th} step of the starting resistance, the relation becomes,

$$R_1 = k^n R_a \tag{6.128}$$

Taking natural logarithm in both sides of equation (6.128) for solving the expression of n yields,

$$\ln R_1 = \ln k^n + \ln R_a \tag{6.129}$$

$$\ln R_1 = n \ln k + \ln R_a \tag{6.130}$$

$$n = \frac{\ln\left(\dfrac{R_1}{R_a}\right)}{\ln k} \tag{6.131}$$

$$n = \frac{\ln\left(\dfrac{R_1}{R_a}\right)}{\ln\left(\dfrac{I_1}{I_2}\right)} \tag{6.132}$$

Example 6.5

The armature resistance of a 230 V shunt motor is 0.4 Ω. At the time of starting, the armature current does not exceed 30 A current. A three-step four-stud starter is used to run the motor. Calculate the values of the resistances that are used in the starter steps.

Solution

The value of the maximum resistance is,

$$R_1 = \frac{230}{30} = 7.67\,\Omega \quad (6.133)$$

The number of steps is given by,

$$n = 3 \quad (6.134)$$

The value of k can be determined as,

$$R_1 = k^n R_a \quad (6.135)$$

$$k^3 = \frac{7.67}{0.4} \quad (6.136)$$

$$k = 2.67 \quad (6.137)$$

Other values of resistance can be determined as,

$$\frac{R_1}{R_2} = k \quad (6.138)$$

$$R_2 = \frac{7.67}{2.67} = 2.87\,\Omega \quad (6.139)$$

$$\frac{R_2}{R_3} = k \quad (6.140)$$

$$R_3 = \frac{2.87}{2.67} = 1.07\,\Omega \quad (6.141)$$

$$\frac{R_3}{R_4} = k \quad (6.142)$$

$$R_4 = \frac{1.07}{2.67} = 0.4\,\Omega \tag{6.143}$$

The resistance of the first step is calculated as,

$$R_{1step} = R_1 - R_2 = 7.67 - 2.87 = 4.8\,\Omega \tag{6.144}$$

The resistance of the second step is determined as,

$$R_{2step} = R_2 - R_3 = 2.87 - 1.07 = 1.8\,\Omega \tag{6.145}$$

The resistance of the third step is calculated as,

$$R_{3step} = R_3 - R_4 = 1.07 - 0.4 = 0.67\,\Omega \tag{6.146}$$

Example 6.6

The output of a 220 V shunt motor is 3550 W and the efficiency is 85%. The starting current of the motor should not exceed twice the full load current. Find the values of resistances in a four-step five-stud starter. Assume the copper loss is half of the total loss.

Solution

The output power is given by,

$$P_o = 3550\,\text{W} \tag{6.147}$$

The input power is calculated as,

$$P_{in} = \frac{P_o}{\eta} = \frac{3550}{0.85} = 4176.5\,\text{W} \tag{6.148}$$

The total loss is calculated as,

$$P_{tloss} = P_{in} - P_0 = 4176.5 - 3550 = 626.5\,\text{W} \tag{6.149}$$

If we neglect the field current, then the value of the armature current will be equal to the line current. The value of the armature current is calculated as,

$$I_a = \frac{P_{in}}{V_t} = \frac{4176.5}{220} = 19\,\text{A} \tag{6.150}$$

The copper loss is determined as,

$$P_{cu} = I_a{}^2 R_a = \frac{1}{2} \times \text{Total loss} \tag{6.151}$$

$$P_{cu} = \frac{1}{2} \times 626.5 = 313.25\,\text{W} \tag{6.152}$$

The armature resistance is determined as,

$$R_a = \frac{313.25}{19^2} = 0.86\,\Omega \tag{6.153}$$

The maximum value of the starting current is calculated as,

$$I_1 = 2 \times 19 = 38\,\text{A} \tag{6.154}$$

The value of the maximum resistance is determined as,

$$R_1 = \frac{220}{38} = 5.8\,\Omega \tag{6.155}$$

The value of the constant is determined as,

$$R_1 = k^n R_a \tag{6.156}$$

$$k^4 = \frac{5.8}{0.86} \tag{6.157}$$

$$k = 1.61 \tag{6.158}$$

Other values of resistance are determined as,

$$\frac{R_1}{R_2} = k \tag{6.159}$$

$$R_2 = \frac{5.8}{1.61} = 3.6\Omega \tag{6.160}$$

$$R_3 = \frac{3.6}{1.61} = 2.23\Omega \tag{6.161}$$

$$R_4 = \frac{2.23}{1.61} = 1.39\Omega \tag{6.162}$$

$$R_5 = \frac{1.39}{1.61} = 0.86\Omega \tag{6.163}$$

The resistance of the first step is determined as,

$$R_{1step} = R_1 - R_2 = 5.8 - 3.6 = 2.2\Omega \tag{6.164}$$

The resistance of the second step is calculated as,

$$R_{1step} = R_2 - R_3 = 3.6 - 2.23 = 1.37\Omega \tag{6.165}$$

The resistance of the third step is determined as,

$$R_{3step} = R_3 - R_4 = 2.23 - 1.39 = 0.84\Omega \tag{6.166}$$

The resistance of the fourth step is calculated as,

$$R_{4step} = R_4 - R_5 = 1.39 - 0.86 = 0.53\Omega \tag{6.167}$$

Practice Problem 6.5

The armature resistance of a 220 V shunt motor is 0.3 Ω. The armature current does not exceed 25 A during the starting time. A four-step five-stud starter is used to run the motor. Calculate the values of the resistances that are used in the starter steps.

Practice Problem 6.6

The output of a 240 V shunt motor is 4750 W and the efficiency is 88%. The starting current of the motor should not exceed 1.5 times the full load current. Find the values of resistances in a four-step five-stud starter. Assume the copper loss is one-third of the total loss.

Exercise Problems

6.1 The armature resistance of a 220 V DC shunt motor is 0.3 Ω. Determine the (i) value of the additional resistance in series with the armature to limit the starting current of 45 A, and (ii) generated emf when the additional resistance is in series with the armature and the armature current has fallen to 35 A.

6.2 A 240 V DC shunt motor has an armature resistance of 0.4 Ω. Determine the (i) value of an extra resistance necessary in series with the armature to reduce the starting current of 70 A, and (ii) generated voltage when the armature current has fallen to 60 A and the extra resistance is in series with the armature.

6.3 A 220 V DC shunt runs at a speed of 900 rpm while taking an armature current of 15 A. An additional resistance is connected in series with a 0.3 Ω armature resistance to reduce the speed to 600 rpm. Determine the value of the additional resistance if the armature current is 25 A.

6.4 The armature current of a 230 V DC shunt motor is 30 A while running at 800 rpm. An additional resistance is connected in series with the armature resistance of 0.4 Ω to reduce the speed of 500 rpm. In the second case, the armature current has fallen to 20 A.

6.5 The armature resistance of a 220 V DC shunt motor is 0.3 Ω. Determine the (i) value of an extra resistance connected in series with the armature to limit the armature current of 35 A, and (ii) generated emf when the motor is running at constant speed in series with an extra resistance and the armature current of 20 A.

6.6 The armature of a 230 V DC motor is taking a current of 15 A while running at a speed of 900 rpm. The armature resistance of the motor is 1.2 Ω. Determine the value of extra resistance needed to be inserted in series with the armature to reduce the speed to 600 rpm with the same armature current.

6.7 A DC shunt motor of 0.5 Ω armature resistance is taking an armature current of 25 A from the 220 V supply. Determine the additional resistance that is required in series with the armature to half the speed if the load torque is constant.

6.8 The armature current of a 230 V DC shunt motor is 20 A. The resistance of the armature circuit is 0.4 Ω. Determine the value of an extra resistance that is connected in series with the armature to half the speed if the load torque is proportional to the square of the speed.

6.9 The full load speed of a 240 V DC shunt motor is 1200 rpm and its armature resistance is 0.5 Ω. Determine the resistance, which is required in series with the armature to reduce the speed to 1000 rpm with full load torque. The full load armature current is 40 A.

6.10 The total resistance of a 420 V DC series motor is 0.3 Ω. It takes a current of 40 A while running at 700 rpm. An additional resistance is inserted in series with the motor to reduce the speed of 600 rpm while the torque is half of the previous value. Consider the flux is proportional to the field current.

6.11 The armature resistance of a 220 V shunt motor is 0.35 Ω. At full load condition, the armature takes a current of 10 A and runs at a speed of 1000 rpm. A variable resistance of 1.1 Ω is connected in series with the armature. Find the speed of the motor at full load torque.

6.12 The armature and shunt field resistances of a 230 V DC shunt motor are 0.5 Ω and 220 Ω respectively. The armature current is 20 A while running at a speed of 700 rpm. If the speed is raised to 900 rpm, which value of resistance should be inserted in the shunt field? Assume the motor is driving a load of constant torque and the magnetic circuit is unsaturated.

6.13 The full load efficiency of an 8 kW, 230 V, and 1000 rpm shunt motor is 90%. The motor has an armature resistance of 0.04 Ω and a shunt field current of 2.5 A. Determine the additional resistance that is connected in series with the armature if the speed is reduced to 500 rpm. Consider the load torque remains constant.

6.14 A 4-pole DC series motor runs at a speed of 1000 rpm and draws 15 A current from the 250 V DC supply. The value of series field resistance is 0.2 Ω and the armature resistance is 0.4 Ω. The new line current is found at 25 A when one series field coil is connected in parallel with another series field coil. The flux for 25 A current is 85% of the flux for 15 A current. Determine the new speed.

6.15 A DC series motor takes a current of 45 A from the 440 V supply with a certain speed and negligible resistance. Determine the value of the resistance that is connected in the motor to reduce the speed to 85% of its previous value, if the load torque varies as the cube of the speed.

6.16 A DC series motor is taking a current of 50 A from the 240 V supply with a certain speed and negligible resistance. Determine the value of the resistance that is connected in the motor to reduce the speed to 90% of its previous value, if the load torque varies as the square root of the speed.

6.17 A DC series motor runs at a certain speed and has negligible resistance while taking 25 A from the 220 V supply. The load torque is varied as the square of the speed. Calculate the value of the resistance, which is connected to the motor to reduce the speed to 70% of the previous value.

6.18 A variable resistance is connected in series with the armature of a 230 V series motor to adjust its speed. The total resistance of the armature and the field is 0.2 Ω. Consider zero value of variable resistance, armature current is 15 A and motor speed is 1100 rpm for a certain load. For other loads, the armature current is found at 10 A when the variable resistance is 3 Ω. Find the new speed when the flux in the second case is 95% of the flux in the first case.

6.19 A diode having an internal resistance of 15 Ω is used for half-wave rectification for the motor control. Determine the (i) maximum value, average value, rms value of the current, (ii) DC output voltage, and (iii) efficiency. Consider the expression of the input voltage $v(t) = 45 \sin \theta$ and the load resistance 900 Ω.

6.20 A full wave rectifier with two diodes is used in the field circuit of the DC motor. The internal resistance of each diode is 15 Ω. The rms value of the secondary voltage (center tap to each end) is 45 V. Determine the (i) maximum current, (ii) direct component of the current, and (iii) rms value of the current. Consider the load resistance 700 Ω.

6.21 A 240 V DC shunt has an armature resistance of 0.5 Ω. The armature current does not exceed 45 A during the starting time. A four-step five-stud starter is used to run the motor. Calculate the values of the resistances that are used in the starter steps.

6.22 The output of a 220 V DC shunt motor is 5000 W and the efficiency is 90%. The starting current of the motor should not exceed 2.5 times the full load current. Find the values of resistances in a four-step five-stud starter. Assume the copper loss is one-fourth of the total loss.

7
Three-Phase Induction Motor

7.1 Introduction

Many scientists contributed to developing induction motors. In 1879, Walter Baily demonstrated the first primitive induction motor by manually turning switches on and off. In 1885, Italian scientist Galileo Ferraris first invented a three-phase induction motor and Nikola Tesla, American scientist, invented the improved performance three-phase induction motor in 1887. In 1888, George Westinghouse was working on developing an alternating power system; he licensed Tesla's patent and purchased a US patent on Ferraris's induction motor to develop it further. In 1891, General Electric (GE) began developing a three-phase induction motor. Later on, Westinghouse and GE signed a joint agreement for designing the bar-winding rotor by 1896; afterwards, it became the squirrel cage rotor. The induction motor is often known as polyphase induction and it is commonly used as an AC motor in the industry. Therefore, it is considered a workhorse. The stator winding of the polyphase induction motor is connected to the three-phase supply and the rotor receives power from the stator by induction. Therefore, it is named an induction motor. A three-phase induction motor runs essentially at a constant speed from no-load to full load, which is less than the synchronous speed. A three-phase induction motor is also known as an asynchronous machine.

The induction motor has some advantages:

- Simple and rugged in construction.
- Cheaper and requires less maintenance.
- High efficiency and good power factor.
- Self-starting device and offers a fair degree of speed control.

- Requires no excitation other than AC lines.

There are many applications of polyphase induction motors. These applications are in the jute industry, carpet industry, spinning mill, pumping water, oil companies, ships, etc.

In this chapter, construction, basic working principle, rotating field, different types of torque, characteristics, equivalent circuit, and test of a polyphase induction motor are discussed.

7.2 Construction

There are two main components of a polyphase induction motor, namely, (i) the stationary part called the stator, and (ii) the rotating part called the rotor. However, there is a small clearance between the stator and the rotor called an air gap. This air gap helps the rotor to rotate smoothly. This air gap ranges from 0.4 mm to 4 mm depending on the power rating of the motor. The different components of an induction motor are shown in Fig. 7.1.

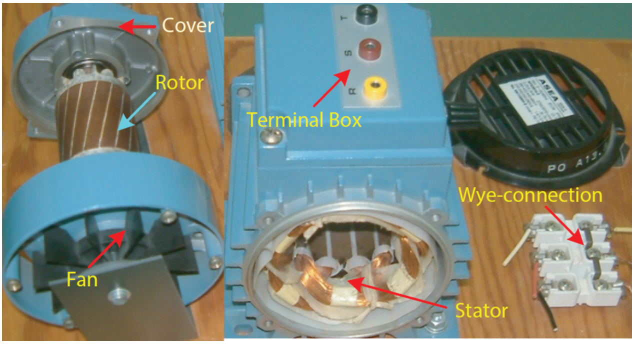


Fig. 7.1 Components of a three-phase induction motor.

Stator: The stator is the outer and stationary part of an induction motor, which is shown in Fig. 7.2. It consists of a steel frame that encloses a hollow cylindrical core made up of thin lamination of silicon steel. This thin lamination reduces the hysteresis loss and eddy current loss. The stator provides suitable slots in its inner periphery for winding purposes and it is wound for a definite number of poles. The windings in the slots are geometrically spaced 120 electrical degrees apart. A three-phase alternating supply is connected to the stator windings. As a result, an alternating current flows through the stator windings. This current produces a rotating constant magnetic field, which rotates at a synchronous speed.

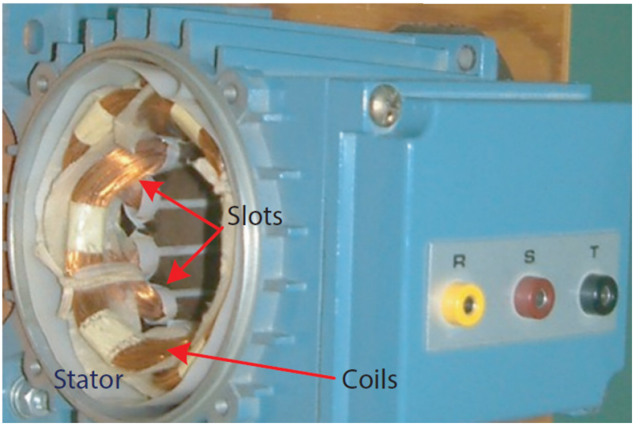


Fig. 7.2 Schematic of a stator.

Rotor: It is the inner part of the motor, which rotates during starting time as well as at running condition. The rotor receives power from the stator through a process of induction. There are two types of rotors used in an induction motor. One is called a squirrel cage rotor and the other is called a phase-wound rotor or simply wound rotor. Brief descriptions of these types of rotors are given.

Squirrel cage rotor: It consists of a highly laminated cylindrical core having parallel slots on its outer surface. Thick copper bars are placed on each slot and metal rings called end rings, short these copper bars. This type of rotor is used for low and medium-starting torque motors. The squirrel cage rotor is shown in Fig. 7.3.

Skew angle: The slots in the squirrel cage rotor are not always parallel to the shaft. These slots are twisted through an angle. This angle is known as a skew angle. The slots that are twisted with an angle are known as skewing. The skewing of the rotor removes the locking tendency of the rotor and magnetic hum, etc.

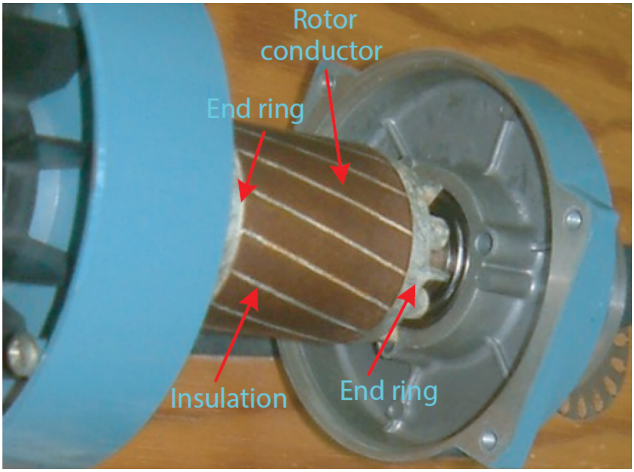

Rotor conductor
End ring
Insulation
End ring

Fig. 7.3 Squirrel cage type rotor.

Wound rotor: This type of rotor is used for high starting torque application. It consists of a laminated cylindrical core and carries three-phase double-layer distributed windings. The three terminals of these windings are connected to the slip rings, which are mounted on the shaft with brushes. The external resistances are connected in series to the rotor windings through the slip rings for starting purposes. Suitable switchgear equipment is used to vary the resistances. Finally, the resistances are short-circuited when the motor runs to its normal rated speed. Some holes are made for ventilation. The wound rotor with slip rings and the rheostat is shown in Fig. 7.4.

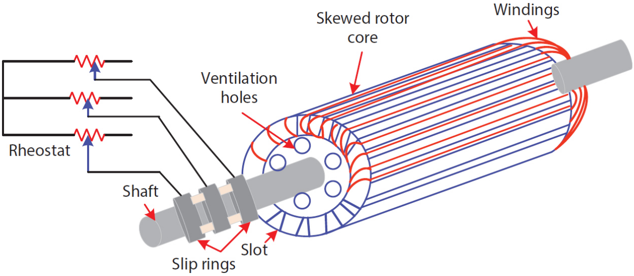

Skewed rotor core
Windings
Ventilation holes
Rheostat
Shaft
Slot
Slip rings

Fig. 7.4 Wound-type rotor.

7.3 Working Principle

Initially, the rotor is stationary before energizing stator conductors. When the three-phase stator winding of an induction motor is energized by a three-phase supply then current flows through the stator winding. This current produces a constant magnitude rotating magnetic field, which rotates at synchronous speed around the stator. This rotating magnetic field crosses the air gap and links or cuts the stationary rotor conductors as shown in Fig. 7.5. According to Faraday’s laws of electromagnetic induction, an emf is induced in the rotor conductors. Since the rotor conductors are short-circuited by the end rings, this emf produces the current in the rotor conductors. The direction of rotor emf or current can be determined by Fleming’s right-hand rule. The current in the rotor conductors produces its magnetic field. The magnetic field of the rotor conductors interacts with the rotating stator magnetic fields. As a result, it produces net force as well as torque, which tends to rotate the rotor.

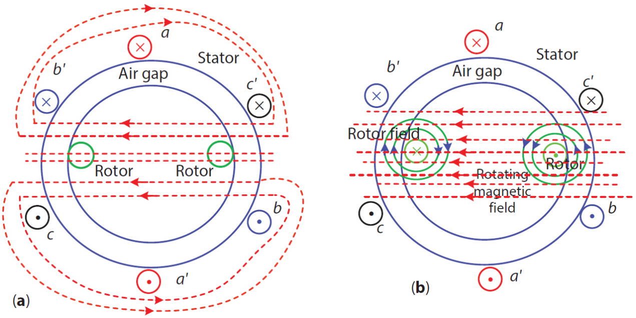


Fig. 7.5 Part of stator and rotor conductors.

The torque which develops during starting time is known as starting torque. If the load torque is less than the starting torque, the motor will rotate continuously.

7.4 Rotating Field

The elementary representation of a three-phase induction is shown in Fig. 7.6. In a stator, the three-phase windings (aa’, bb’, cc’) are placed 120° apart from each other. The winding aa’ represents the whole phase winding for phase a. Similarly, bb’ and cc’ represent the phase winding for phase b and phase c, respectively.

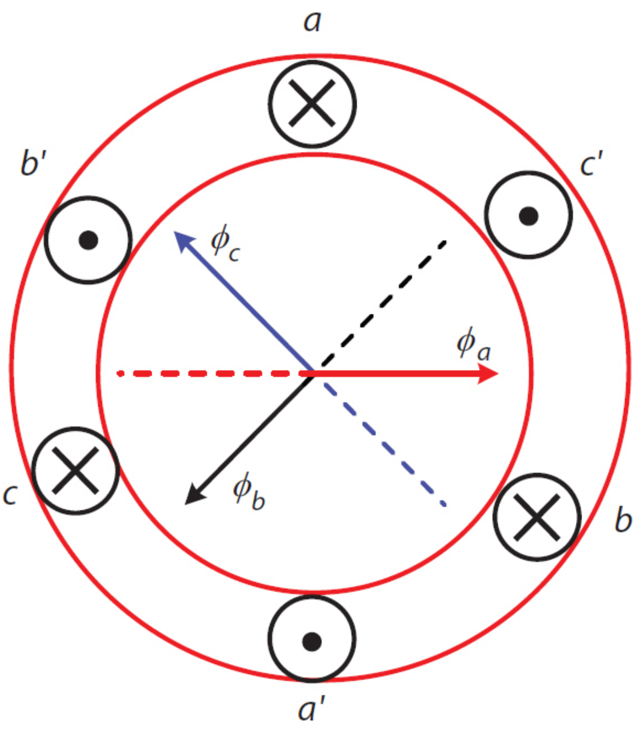


Fig. 7.6 Representation of the windings.

A rotating magnetic field of constant magnitude is produced when a three-phase voltage is applied to the stator windings. This rotating field is produced by the contributions of space-displaced phase windings, which carry appropriate time-displaced currents. The fluxes due to these currents, which are time displaced by 120 electrical degrees are given as,

$$\phi_a = \phi_m \sin \omega t \tag{7.1}$$

$$\phi_b = \phi_m \sin(\omega t - 120°) \tag{7.2}$$

$$\phi_c = \phi_m \sin(\omega t - 240°) \tag{7.3}$$

The sinusoidal waveforms of the fluxes are shown in Fig. 7.7, where phase *a* is considered as a reference. At point 1, the value of the angle is,

$$\theta = \omega t = 0^\circ \tag{7.4}$$

From equations (7.1), (7.2) and (7.3), the values of the fluxes are determined as,

$$\phi_a = \phi_m \sin \omega t = \phi_m \times 0 = 0 \tag{7.5}$$

$$\phi_b = \phi_m \sin(-120^\circ) = -0.86\phi_m \tag{7.6}$$

$$\phi_c = \phi_m \sin(-240^\circ) = 0.86\phi_m \tag{7.7}$$

The resultant flux is then calculated as,

$$\phi_{r1} = 2 \times 0.86\phi_m \times \cos 30^\circ = 1.5\phi_m \tag{7.8}$$

The phase angle of the resultant flux is written as,

$$\underline{\left|\phi_{r1}\right.} = 90^\circ \tag{7.9}$$

At point 2, the value of the angle is considered,

$$\theta = \omega t = 60^\circ \tag{7.10}$$

From equations (7.1), (7.2) and (7.3), the values of the fluxes are determined as,

$$\phi_a = \phi_m \sin 60^\circ = 0.86\phi_m \tag{7.11}$$

$$\phi_b = \phi_m \sin(-60^\circ) = -0.86\phi_m \tag{7.12}$$

$$\phi_c = \phi_m \sin(-180^\circ) = 0 \tag{7.13}$$

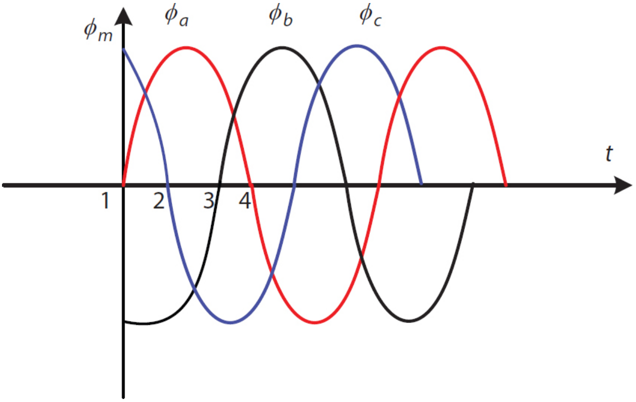


Fig. 7.7 Three-phase waveforms.

At point 2, the resultant flux is calculated as,

$$\phi_{r2} = 2 \times 0.86\phi_m \times \cos 30^\circ = 1.5\phi_m \tag{7.14}$$

The phase angle of the resultant flux is expressed as,

$$\underline{|\phi_{r2}} = 30^\circ \tag{7.15}$$

The phasor diagrams for different positions are shown in Fig. 7.8.

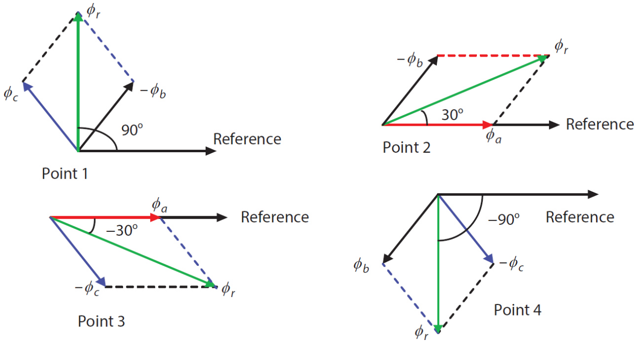

ϕ_r
ϕ_c
$-\phi_b$
90°
Reference
Point 1
$-\phi_b$
ϕ_r
30°
Reference
ϕ_a
Point 2
ϕ_a
Reference
−30°
$-\phi_c$
ϕ_r
Point 3
−90°
Reference
ϕ_b
$-\phi_c$
Point 4
ϕ_r

Fig. 7.8 Phasor diagrams of the fluxes at different positions.

At point 3, the value of the angle is,

$$\theta = \omega t = 120^\circ \tag{7.16}$$

From equations (7.1), (7.2) and (7.3), the values of the three fluxes are,

$$(7.17)\phi_a = \phi_m \sin 120^\circ = 0.86\phi_m$$

$$(7.18)\phi_b = \phi_m \sin(0^\circ) = 0$$

$$(7.19)\phi_c = \phi_m \sin(-120^\circ) = -0.86\phi_m$$

The resultant flux is written as,

$$(7.20)\phi_{r3} = 2 \times 0.86\phi_m \times \cos 30^\circ = 1.5\phi_m$$

The phase angle of the resultant flux is expressed as,

$$(7.21)\lvert\underline{\phi_{r3}} = -30^\circ$$

At point 4, the value of the angle is considered,

$$(7.22)\theta = \omega t = 180^\circ$$

From equations (7.1), (7.2) and (7.3), the values of the three fluxes are,

$$(7.23)\phi_a = \phi_m \sin 180^\circ = 0$$

$$(7.24)\phi_b = \phi_m \sin(60^\circ) = 0.86\phi_m$$

$$(7.25)\phi_c = \phi_m \sin(-60^\circ) = -0.86\phi_m$$

The resultant flux is calculated as,

$$(7.26)\phi_{r4} = 2 \times 0.86\phi_m \times \cos 30^\circ = 1.5\phi_m$$

The phase angle of the resultant flux is written as,

$$(7.27)\lvert\underline{\phi_{r4}} = -90^\circ$$

From the equations (7.8), (7.14), (7.20), and (7.26), the following points can be concluded:

- The magnitude of the resultant vector is constant throughout the time and its value is always equal to 1.5 times the maximum value of the flux.

- The resultant flux changes its position at an interval of 60º.
- The resultant flux produces the rotating field in the clockwise direction.

7.5 Synchronous Speed and Slip Speed

A rotating magnetic field is created when the stator windings are energized with three-phase AC voltage. This magnetic field rotates at synchronous speed through the air gap. The speed at which the revolving flux rotates in between the stator and rotor free space is called the synchronous speed. It is denoted by the letter *Ns*. The synchronous speed is directly proportional to the supply frequency and inversely proportional to the number of pairs of poles and it can be written as,

$$N_s \propto f \tag{7.28}$$

$$N_s \propto \frac{1}{\frac{P}{2}} \quad \text{rps} \tag{7.29}$$

Combining the equations (7.28) and (7.29) yields,

$$N_s \propto \frac{2f}{P} \quad \text{rps} \tag{7.30}$$

Now converting the speed rps into rpm of equation (7.30) yields,

$$N_s = \frac{120f}{P} \quad \text{rpm} \tag{7.31}$$

where,

P is the number of poles of the stator,

f is supply frequency in Hz.

The magnetic field produced by the rotor current rotates less than the speed of the magnetic field by the stator current. Therefore, there is a difference between the synchronous speed and rotor speed. Hence, the slip speed is defined as the difference between the synchronous speed (Ns) and the speed of the rotor (Nl) at any load. It is often called the relative speed of an induction motor. The slip speed or the relative speed can be expressed as,

$$N_{slip-speed} = N_{rel} = N_s - N_l \tag{7.32}$$

The slip is defined as the ratio of the slip speed to the synchronous speed. Mathematically, it can be expressed as,

$$s = \frac{N_{rel}}{N_s} \tag{7.33}$$

Substituting equation (7.32) into the equation (7.33) yields,

$$s = \frac{N_s - N_l}{N_s} \tag{7.34}$$

The slip is either expressed as a percentage (%) or a fractional number. The slip of an induction motor can not be zero and it varies from 1% to 5%. The rotor speed will be zero if the rotor blocks by any means, then N_l is zero. Then equation (7.34) can be modified as,

$$s = \frac{N_s - N_l}{N_s} = \frac{N_s - 0}{N_s} \tag{7.35}$$

$$s = 1 \tag{7.36}$$

From equation (7.34), the expression of the rotor speed can be derived as,

$$N_l = N_s(1 - s) \tag{7.37}$$

From equation (7.37), it is seen that the speed of the motor at any load can be calculated if the synchronous speed and slip are known.

7.6 Rotor Frequency and Speed

The frequency of the rotor circuit normally depends on the relative speed between the stator field and the rotor field. When the rotor is stationary ($N_l = 0$), the relative speed is $N_{rel} = N_s - N_l = N_s$. In this case, the rotor frequency is equal to the supply frequency. The expression of rotor frequency can be written as,

$$f_r = \frac{P \times N_{rel}}{120} \tag{7.38}$$

Substituting the expression of relative speed from equation (7.32) to equation (7.38) yields,

$$f_r = \frac{P \times (N_s - N_l)}{120} \tag{7.39}$$

$$f_r = \frac{P \times N_s \left(\dfrac{N_s - N_l}{N_s} \right)}{120} \tag{7.40}$$

Substitute equation (7.34) into equation (7.40) and the expression of rotor frequency becomes,

$$f_r = s\frac{P \times N_s}{120} \tag{7.41}$$

Under blocked rotor condition, the value of the slip and the rotor frequency are,

$$s = 1 \tag{7.42}$$

$$f_r = f_{BR} \tag{7.43}$$

Substituting equations (7.42) and (7.43) into equations (7.41) yields,

$$f_{BR} = \frac{PN_s}{120} \quad (7.44)$$

Again, substituting equation (7.44) into equation (7.41) yields,

$$f_r = s \times f_{BR} \quad (7.45)$$

where f_{BR} is the frequency under blocked rotor conditions.

The frequency under the blocked rotor condition is equal to the supply frequency (f). Therefore, the final expression of the rotor frequency is expressed as,

$$f_r = s \times f \quad (7.46)$$

From equation (7.46), it is seen that the rotor frequency is dependent on the slip. Therefore, the rotor frequency decreases with the decrease in slip and increases with the increase in slip.

The stator magnetic field is created when stator winding is energized by the three-phase supply and rotates at synchronous speed. This field is cut by the stationary rotor conductors that induce rotor current. This rotor current creates the rotating magnetic field in the rotor and the speed of this field is expressed as,

$$N_r = \frac{120 f_r}{P} \quad (7.47)$$

Substituting equation (7.46) into equation (7.47) yields,

$$N_r = \frac{120sf}{P} \quad (7.48)$$

Substituting equation (7.31) into equation (7.48) yields,

$$N_r = sN_s \quad (7.49)$$

Example 7.1

The full load speed of a three-phase, 230V, 4-pole, 50 Hz induction motor is 1445 rpm. Determine the synchronous speed, slip, and rotor frequency.

Solution

The synchronous speed is calculated as,

$$N_s = \frac{120f}{P} = \frac{120\times 50}{4} = 1500\,\text{rpm} \tag{7.50}$$

The value of the slip is determined as,

$$s = \frac{N_s - N_l}{N_s} = \frac{1500-1445}{1500} = 0.036 \tag{7.51}$$

The rotor frequency is calculated as,

$$f_r = s\times f = 0.036\times 50 = 1.8\text{ Hz} \tag{7.52}$$

Practice Problem 7.1

The rotor speed of a three-phase, 440 V, 8-pole 50 Hz induction motor is 690 rpm. Find the synchronous speed and rotor frequency.

7.7 Rotor Voltage and Rotor Reactance

A three-phase induction motor is normally compared with a two-winding transformer. The stator is compared with the primary winding and the rotor is compared with the secondary winding. Therefore, the expression of the rotor voltage compared to the transformer secondary voltage is,

$$E_r = 4.44 f_r N\phi_m \tag{7.53}$$

Substituting the expression of rotor frequency from equation (7.46) to equation (7.53) yields,

$$E_r = 4.44\times s\, f_{BR} N\phi_m \tag{7.54}$$

Substituting equations (7.42) and (7.43) into equation (7.54) yields,

$$E_{BR} = 4.44 f_{BR} N \phi_m \tag{7.55}$$

Substituting equation (7.55) into equation (7.54) yields the final expression of the rotor voltage as,

$$E_r = s\,E_{BR} \tag{7.56}$$

From equation (7.56), it is seen that the rotor voltage at any speed is equal to slip times the rotor blocked voltage.

The rotor reactance at the standstill is written as,

$$X_r = 2\pi f_r L \tag{7.57}$$

Again substituting equation (7.46) into equation (7.57) yields,

$$X_r = 2\pi s f_{BR} L \tag{7.58}$$

Applying the blocked rotor condition ($s = 1$, $X_r = X_{BR}$) to the equation (7.50), yields,

$$X_{BR} = 2\pi f_{BR} L \tag{7.59}$$

Substituting equations (7.42) and (7.43) into equation (7.59) yields the final expression of the rotor reactance as,

$$X_r = s\,X_{BR} \tag{7.60}$$

Example 7.2

A 4-pole, 50 Hz three-phase induction motor is running at 1450 rpm. Under blocked rotor conditions, the voltage is found to be 120 V. Calculate the synchronous speed, rotor frequency and rotor voltage.

Solution

The synchronous speed is calculated as,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\,\text{rpm} \tag{7.61}$$

The value of the slip is calculated as,

$$s = \frac{N_s - N_l}{N_s} = \frac{1500 - 1450}{1500} = 0.033 \quad (7.62)$$

The rotor frequency is determined as,

$$f_r = s \times f = 0.033 \times 50 = 1.66\ \text{Hz} \quad (7.63)$$

The rotor voltage is calculated as,

$$E_r = sE_{BR} = 0.033 \times 120 = 3.96\ \text{V} \quad (7.64)$$

Practice Problem 7.2

A 6-pole, 60 Hz, three-phase induction motor is running at 1100 rpm. Under blocked rotor conditions, the voltage is found to be 120 V. Find the rotor voltage.

7.8 Rotor Torque

The rotor parameters are represented with the suffix 2 or *r* and stator parameters are represented with the suffix 1 or *s*. In a three-phase induction motor, torque is proportional to the rotor current, flux per pole and power factor. It can be expressed as,

$$T \propto I_2 \quad (7.65)$$

$$T \propto \phi \quad (7.66)$$

$$T \propto \cos\theta_2 \quad (7.67)$$

Then combine equations (7.65), (7.66) and (7.67) yields,

$$T \propto \phi I_2 \cos\theta_2 \quad (7.68)$$

However, the rotor voltage depends on the flux and it can be expressed as,

$$E_2 \propto \phi \quad (7.69)$$

Substituting equation (7.69) into equation (7.54) and is modified as

$$T = kE_2I_2\cos\theta_2 \tag{7.70}$$

where,

I_2 is the rotor current at a standstill,

E_2 is the rotor induced emf at a standstill,

$\cos\phi_2$ is the rotor power factor at a standstill.

7.9 Starting Torque

The torque that is developed by the motor at the time of starting is known as staring torque. Just at the time of starting, the rotor reactance and rotor emf are considered at standstill conditions, i.e., without the slip as shown in Fig. 7.9. The expression of the starting torque is,

$$T_{st} = k_1E_2I_2\cos\theta_2 \tag{7.71}$$

At a standstill, the expression of the rotor impedance per phase is,

$$Z_2 = \sqrt{R_2^{\,2} + X_2^{\,2}} \tag{7.72}$$

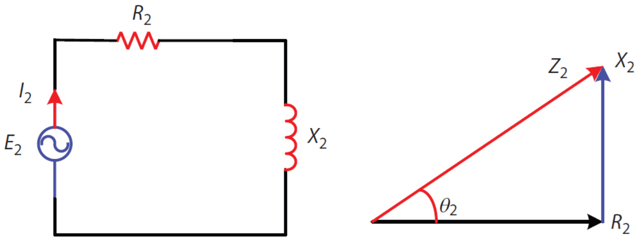


Fig. 7.9 Rotor circuit with the impedance triangle at a standstill.

At a standstill, the expression of the rotor current per phase is,

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^{\,2} + X_2^{\,2}}} \tag{7.73}$$

The expression of the rotor power factor is,

$$\cos\theta_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^{\ 2} + X_2^{\ 2}}} \tag{7.74}$$

Substituting equations (7.73) and (7.74) into equation (7.71) yields the expression of the starting torque at a standstill as,

$$T_{st} = k_1 E_2 \frac{E_2}{\sqrt{R_2^{\ 2} + X_2^{\ 2}}} \times \frac{R_2}{\sqrt{R_2^{\ 2} + X_2^{\ 2}}} \tag{7.75}$$

$$T_{st} = k_1 \frac{E_2^{\ 2} R_2}{R_2^{\ 2} + X_2^{\ 2}} \tag{7.76}$$

The flux is constant for a constant stator supply voltage. Hence, the rotor voltage E_2 is also a constant. Equation (7.76) is then modified as,

$$T_{st} = k_2 \frac{R_2}{R_2^{\ 2} + X_2^{\ 2}} \tag{7.77}$$

Differentiating equation (7.77) with respect to R_2 and it becomes,

$$\frac{dT_{st}}{dR_2} = k_2 \left[\frac{(R_2^{\ 2} + X_2^{\ 2})1 - R_2(2R_2)}{(R_2^{\ 2} + X_2^{\ 2})^2} \right] \tag{7.78}$$

The starting torque will be maximum if the derivative of the starting torque with respect to R_2 is equal to zero. Equation (7.78) can be modified as,

$$\frac{dT_{st}}{dR_2} = k_2 \left[\frac{(R_2^{\ 2} + X_2^{\ 2})1 - R_2(2R_2)}{(R_2^{\ 2} + X_2^{\ 2})^2} \right] = 0 \tag{7.79}$$

$$R_2^{\ 2} + X_2^{\ 2} = 2R_2^{\ 2} \tag{7.80}$$

$$R_2 = X_2 \tag{7.81}$$

From equation (7.81), it is seen that the starting torque will be maximum when the rotor resistance per phase is equal to the standstill rotor reactance per phase.

At a constant frequency and given supply voltage, the rotor resistance and reactance are constant. The rotor input voltage depends on the stator supply voltage per phase. Therefore, equation (7.76) can be rearranged as,

$$T_{st} \propto V_s^2 \tag{7.82}$$

where *Vs* is the stator supply voltage per phase. Again from equation (7.82), it is seen that the starting torque is directly proportional to the square of the per-phase stator supply voltage. Therefore, the expression of an original torque can be written as,

$$T_o \propto V_o^2 \tag{7.83}$$

Dividing equation (7.82) by equation (7.83) yields,

$$\frac{T_{st}}{T_o} = \left(\frac{V_s}{V_o}\right)^2 \tag{7.84}$$

According to equation (7.73), the starting current and original current can be expressed as,

$$I_{st} \propto V_s \tag{7.85}$$

$$I_o \propto V_o \tag{7.86}$$

Again, dividing equation (7.85) by equation (7.86) yields,

$$I_{st} = I_o \frac{V_s}{V_o} \tag{7.87}$$

From equation (7.87), it is seen that the starting current can be calculated if other parameters such as I_0, V_s and V_0 are given.

Example 7.3

The original torque of a 4-pole, 230 V, three-phase induction motor is 230 N-m and the starting current at rated voltage is 560 A. A reduced three-phase voltage of 150 V is applied to the motor terminals. Find the values of starting torque and current.

Solution

The value of the original torque and voltage are,

$$T_o = 230\,\text{N-m} \tag{7.88}$$

$$V_o = 230\,\text{V} \tag{7.89}$$

The value of the stator voltage is,

$$V_s = 150\,\text{V} \tag{7.90}$$

The starting torque can be determined as,

$$\frac{T_{st}}{230} = \left(\frac{150}{230}\right)^2 \tag{7.91}$$

$$T_{st} = 97.8\,\text{N-m} \tag{7.92}$$

From equation (7.87), the value of starting current can be calculated as,

$$I_{st} = 560\frac{150}{230} = 365.2\,\text{A} \tag{7.93}$$

Example 7.4

The per-phase rotor resistance and standstill rotor reactance of a three-phase induction motor are 0.15Ω and 0.6Ω, respectively. Find the rotor current and power factor if the per-phase induced voltage at the rotor circuit is 80V.

Solution

The value of the per phase rotor current is calculated as,

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} = \frac{80}{\sqrt{0.15^2 + 0.6^2}} = 129.35\,\text{A} \qquad (7.94)$$

The power factor is calculated as,

$$\cos\theta_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{0.15}{\sqrt{0.15^2 + 0.6^2}} = 0.24\,\text{Lag} \qquad (7.95)$$

Practice Problem 7.3

The original torque of a 6-pole, 208 V, three-phase, induction motor is 180 N-m and the original current at rated voltage is 300 A. A reduced three-phase voltage of 108 V is applied to the motor terminals. Determine the values of starting torque and current.

Practice Problem 7.4

The per-phase rotor resistance and standstill rotor reactance of a three-phase induction motor is 0.09Ω and 0.56Ω, respectively. Find the rotor current and power factor if the per-phase induced voltage at the rotor circuit is 65V.

7.10 Running Torque

The induction motor draws a high current during starting. However, the motor draws full load current under running conditions. The torque under the running condition is known as running torque. The initial expression of running torque is,

$$(7.96)\quad T_r \propto \phi I_{2r} \cos\theta_{2r}$$

where,

$\cos\phi_{2r}$ is the rotor power factor under running conditions,

T_r is the torque under running conditions,

I_{2r} is the rotor current per phase under running conditions.

Under running conditions, the slip is considered with the rotor standstill voltage and the rotor standstill reactance as shown in Fig. 7.10. Therefore, the rotor current per phase under running conditions at any slip s is,

$$I_r = \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad (7.97)$$

The expression of the rotor power factor is written as,

$$\cos\theta_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad (7.98)$$

Substituting equations (7.97) and (7.98) into equation (7.96) and the expression of running torque becomes,

$$T_r = k_3\phi \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad (7.99)$$

$$T_r = k_3\phi \frac{sE_2R_2}{R_2^2 + (sX_2)^2} \quad (7.100)$$

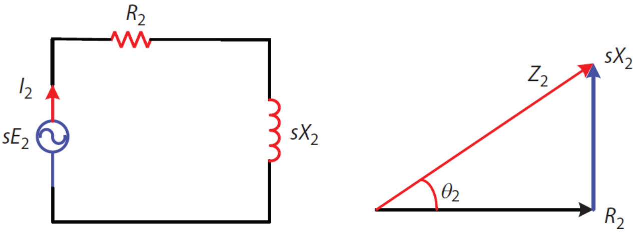


Fig. 7.10 Rotor circuit with the impedance triangle at any slip s.

However, the rotor induced voltage is directly proportional to the flux. Equation (7.100) can be modified as,

$$T_r = k_3 \frac{sE_2^2R_2}{R_2^2 + (sX_2)^2} \quad (7.101)$$

If the stator supply voltage of the motor is constant, then the stator flux as well as E_2 will be constant. The final expression of the running torque is expressed as,

$$T_r = k_4 \frac{sR_2}{R_2{}^2 + (sX_2)^2} \tag{7.102}$$

where k_4 is another constant.

Differentiate equation (7.102) with respect to slip s and it becomes,

$$\frac{dT_r}{ds} = k_4 \left[\frac{\{R_2{}^2 + (sX_2)^2\}R_2 - sR_2\{2sX_2\}X_2}{\{R_2{}^2 + (sX_2)^2\}^2} \right] \tag{7.103}$$

The running torque will be maximum if the differentiation of this torque with respect to slip s is equal to zero. Then equation (7.103) can be modified as,

$$\frac{dT_r}{ds} = k_4 \left[\frac{\{R_2{}^2 + (sX_2)^2\}R_2 - sR_2\{2sX_2\}X_2}{\{R_2{}^2 + (sX_2)^2\}^2} \right] = 0 \tag{7.104}$$

$$\{R_2{}^2 + (sX_2)^2\}R_2 - sR_2\{2sX_2\}X_2 = 0 \tag{7.105}$$

$$R_2{}^2 + (sX_2)^2 - 2(sX_2)^2 = 0 \tag{7.106}$$

$$R_2{}^2 = (sX_2)^2 \tag{7.107}$$

$$R_2 = sX_2 \tag{7.108}$$

From equation (7.108), it is seen that the running torque will be maximum when the rotor resistance per phase is equal to the product of the slip and the standstill rotor reactance per phase. Substituting equation (7.108) into equation (7.102) to find the maximum torque. The expression of the maximum torque is written as,

$$T_{mxm} = k_4 \frac{s^2 X_2}{(sX_2)^2 + (sX_2)^2} \tag{7.109}$$

$$T_{mxm} = k_4 \frac{s^2 X_2}{2(sX_2)^2} \tag{7.110}$$

$$T_{mxm} = k_4 \frac{1}{2X_2} \tag{7.111}$$

Example 7.5

The per-phase rotor resistance and standstill rotor reactance of a three-phase induction motor is 0.05Ω and 1.8Ω, respectively. Calculate the rotor current and power factor if the per-phase induced voltage at the rotor circuit is 100V and the slip is 4%.

Solution

The value of the per phase rotor current is calculated as,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{0.04 \times 100}{\sqrt{0.05^2 + (0.04 \times 1.8)^2}} = 45.63\,\text{A} \tag{7.112}$$

The power factor is calculated as,

$$\cos\theta_2 = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{0.05}{\sqrt{0.05^2 + (0.04 \times 1.8)^2}} = 0.57\,\text{Lag} \tag{7.113}$$

Practice Problem 7.5

The per-phase rotor resistance and standstill rotor reactance of a three-phase induction motor is 0.06Ω and 1.2Ω, respectively. Calculate the rotor current and power factor if the per-phase induced voltage at the rotor circuit is 90V and the slip is 5%.

7.11 Relationship Between Torques

The expressions of starting torque, maximum torque and full load torque are required to establish the relationship between them. These are written as,

$$T_{st} \propto \frac{R_2}{R_2^{\,2} + X_2^{\,2}} \tag{7.114}$$

$$T_{mxm} \propto \frac{1}{2X_2} \tag{7.115}$$

$$T_f \propto \frac{sR_2}{R_2^{\,2} + (sX_2)^2} \tag{7.116}$$

Dividing equation (7.114) by equation (7.115) yields,

$$\frac{T_{st}}{T_{mxm}} = \frac{\dfrac{R_2}{R_2^{\,2} + X_2^{\,2}}}{\dfrac{1}{2X_2}} \tag{7.117}$$

$$\frac{T_{st}}{T_{mxm}} = \frac{2X_2R_2}{R_2^{\,2} + X_2^{\,2}} \tag{7.118}$$

Dividing the right-hand side of equation (7.118) by X_2^2 yields,

$$\frac{T_{st}}{T_{mxm}} = \frac{\dfrac{2X_2R_2}{X_2^{\,2}}}{1 + \dfrac{R_2^{\,2}}{X_2^{\,2}}} \tag{7.119}$$

$$\frac{T_{st}}{T_{mxm}} = \frac{\dfrac{2R_2}{X_2}}{1 + \left(\dfrac{R_2}{X_2}\right)^2} \tag{7.120}$$

$$\frac{T_{st}}{T_{mxm}} = \frac{2a}{1+a^2} \quad (7.121)$$

where the expression of slip under maximum running torque is written as,

$$a = s_m = \frac{R_2}{X_2} \quad (7.122)$$

The relationship between the full load/running torque and the maximum torque can be derived by dividing equation (7.116) by equation (7.115) as,

$$\frac{T_f}{T_{mxm}} = \frac{\dfrac{sR_2}{R_2{}^2 + (sX_2)^2}}{\dfrac{1}{2X_2}} \quad (7.123)$$

$$\frac{T_f}{T_{mxm}} = \frac{2sX_2R_2}{R_2{}^2 + (sX_2)^2} \quad (7.124)$$

Dividing the right-hand side of the equation (7.124) by $X_2{}^2$ yields,

$$\frac{T_f}{T_{mxm}} = \frac{\dfrac{2sX_2R_2}{X_2{}^2}}{\dfrac{R_2{}^2}{X_2{}^2} + s^2} \quad (7.125)$$

$$\frac{T_f}{T_{mxm}} = \frac{2s\dfrac{R_2}{X_2}}{s^2 + \left(\dfrac{R_2}{X_2}\right)^2} \quad (7.126)$$

$$\frac{T_f}{T_{mxm}} = \frac{2as}{s^2 + a^2} \quad (7.127)$$

Equation (7.127) can be expressed in the following ways,

$$\frac{T_{mxm}}{T_f} = \frac{1}{2}\left(\frac{a}{s} + \frac{s}{a}\right) \quad (7.128)$$

$$\frac{T_f}{T_{mxm}} = \frac{2as}{s^2 + a^2} = \frac{2}{\frac{s^2 + a^2}{as}} \quad (7.129)$$

$$\frac{T_f}{T_{mxm}} = \frac{2}{\frac{s^2}{as} + \frac{a^2}{as}} \quad (7.130)$$

$$\frac{T_f}{T_{mxm}} = \frac{2}{\frac{s}{a} + \frac{a}{s}} \quad (7.131)$$

Example 7.6

The full load slip of a 50 Hz, 8-pole three-phase induction motor is 3%. The rotor resistance and standstill reactance per phase are 0.01Ω and 0.1Ω, respectively. Determine the speed at which maximum torque occurs and the ratio of the maximum torque to the full load torque.

Solution

The synchronous speed is determined as,

$$N_s = \frac{120f}{P} = 120\frac{50}{8} = 750\,\text{rpm} \quad (7.132)$$

The slip at which maximum torque occurs is calculated as,

$$s = \frac{R_2}{X_2} = \frac{0.01}{0.1} = 0.1 \quad (7.133)$$

The rotor speed at maximum torque is determined as,

$$N_l = (1 - s)N_s = (1 - 0.1)750 = 675\,\text{rpm} \quad (7.134)$$

The ratio of full load torque to maximum torque can be determined as,

$$\frac{T_f}{T_{mxm}} = \frac{2as}{s^2 + a^2} = \frac{2 \times 0.1 \times 0.03}{(0.1)^2 + (0.03)^2} \quad (7.135)$$

$$\frac{T_{mxm}}{T_f} = 1.82 \quad (7.136)$$

Practice Problem 7.6

The full load slip of a 50 Hz, 4-pole three-phase induction motor is 4%. The rotor resistance and standstill reactance per phase are 0.03Ω and 0.4Ω, respectively. Find the speed and the ratio of the maximum to the full load torque at which maximum torque occurs.

7.12 Equivalent Circuit

The voltage and current induced in the rotor circuit of an induction motor is essentially a transformer action. Therefore, the equivalent circuit of an induction motor is very similar to the equivalent circuit of a transformer. In a three-phase inductor motor, the rotor rotates and develops mechanical power at its shaft.

7.12.1 Transformer Model of an Induction Motor

Like a transformer, there is specific resistance and self-inductance in the primary (stator) windings, which can be represented in an equivalent circuit. This equivalent circuit is shown in Fig. 7.11. The following parameters are defined as,

V_1 is the applied voltage in the stator winding,

R_1 is the stator resistance per phase,

X_1 is the stator reactance per phase,

R_0 is the magnetizing circuit resistance,

X_0 is the magnetizing circuit reactance,

E_1 is the induced emf in the stator winding,

E_2 is the induced emf in the rotor winding,

R_2 is the rotor resistance per phase,

X_2 is the rotor reactance per phase.

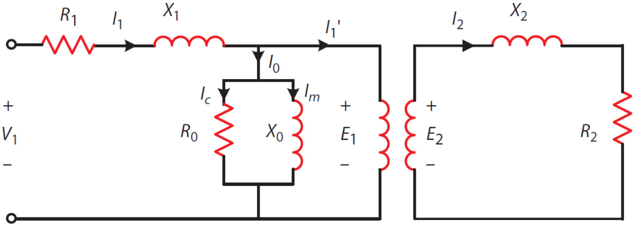


Fig. 7.11 Equivalent circuit of an induction motor.

The stator voltage can be calculated by the following equation,

$$V_1 = (R_1 + jX_1)I_1 + I_0 Z_0 \tag{7.137}$$

where the expression of no-load circuit or magnetizing circuit impedance is,

$$Z_0 = \frac{R_0(jX_0)}{R_0 + jX_0} \tag{7.138}$$

The core-loss component of the current is calculated as,

$$I_c = \frac{E_1}{R_0} \tag{7.139}$$

The magnetizing component of the current is calculated as,

$$I_m = \frac{E_1}{X_0} \tag{7.140}$$

7.12.2 Rotor Circuit Model

A three-phase voltage is supplied to stator windings that create the constant magnitude magnetic field. This magnetic field is cut by the rotor conductors which create the rotor voltage. Due to this voltage, the rotor current will flow as the rotor conductors form a closed circuit by the two end rings. The rotor equivalent circuit is shown in Fig. 7.12.

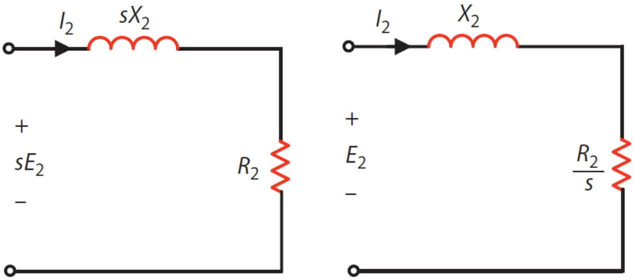


Fig. 7.12 Rotor equivalent circuit.

From the circuit in Fig. 7.12, the magnitude of the rotor circuit current at any slip can be written as,

$$I_2 = \frac{sE_2}{R_2 + jsX_2} \tag{7.141}$$

Equation (7.141) can be rearranged in the following way,

$$I_2 = \frac{\frac{sE_2}{s}}{\frac{R_2 + jsX_2}{s}} \tag{7.142}$$

$$I_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}} \quad (7.143)$$

The equivalent impedance from this point of view is written as,

$$Z_2 = \frac{R_2}{s} + jX_2 \quad (7.144)$$

In Fig. 7.12, the fixed value of the rotor reactance X_2 is connected in series with variable or hypothetical resistance R_2/s. This resistance can be represented in the following way,

$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right) \quad (7.145)$$

The modified rotor equivalent circuit is shown in Fig. 7.13, where those two resistances are included.

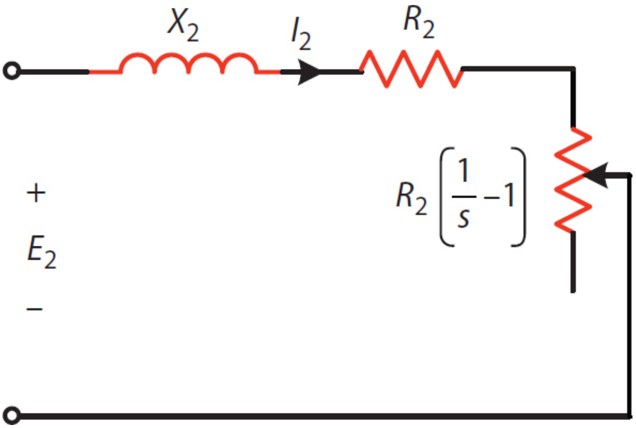


Fig. 7.13 Modified rotor equivalent circuit with load resistance.

In the right-hand side of equation (7.145), the first term represents the resistance of the rotor itself and the second term represents the load resistance. The value of the load resistance depends on the speed of the motor. The complete equivalent circuit referred to as the stator is shown in Fig. 7.14.

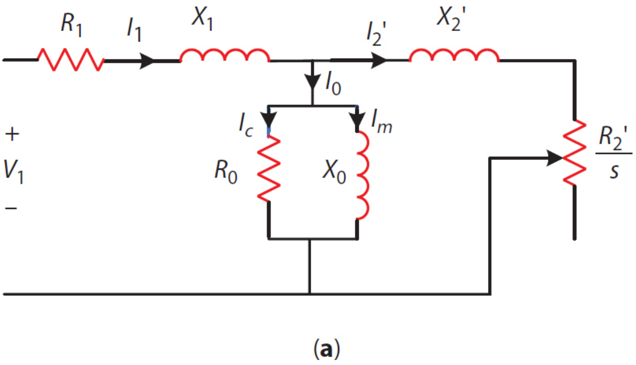

R_1
I_1
X_1
I_2'
X_2'
I_0
I_c
I_m
+
V_1
−
R_0
X_0
$\frac{R_2'}{s}$
(a)

Fig. 7.14(a) Complete equivalent circuit referred to stator.

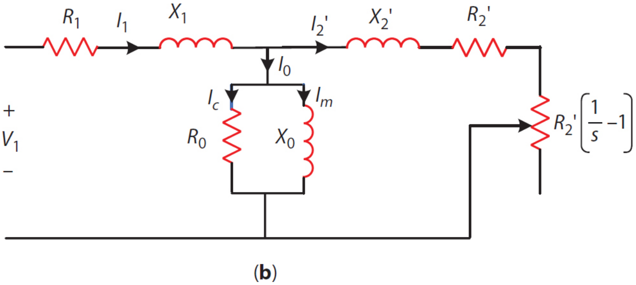

R_1
I_1
X_1
I_2'
X_2'
R_2'
I_0
I_c
I_m
+
V_1
−
R_0
X_0
$R_2'\left(\frac{1}{s}-1\right)$
(b)

Fig. 7.14(b) Complete equivalent circuit referred to stator.

7.12.3 IEEE Recommended Equivalent Circuit

Generally, the value of the magnetizing circuit resistance is higher than the value of the magnetizing circuit reactance. The value of the magnetizing circuit current is 30% to 40% of the full load current. From this point of view, IEEE recommended that the magnetizing circuit resistance be removed from the circuit. The proposed equivalent circuit by IEEE is shown in Fig. 7.15.

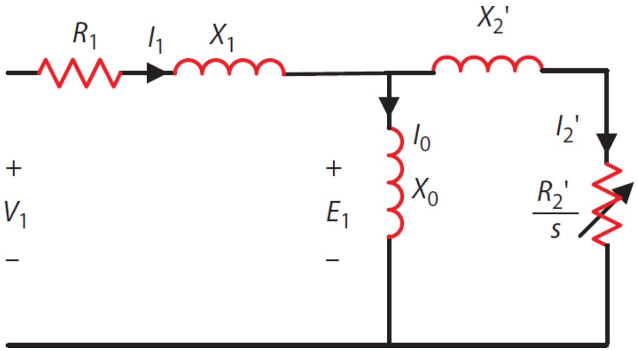


Fig. 7.15 IEEE recommended equivalent circuit.

7.12.4 Thevenin Equivalent Circuit

For simplifying computations, stator side parameters like V_1, R_1, X_1 and magnetizing circuit reactance X_0 can be replaced by Thevenin parameters. Thevenin impedance can be calculated as,

$$Z_{th} = (R_1 + jX_1) \parallel (jX_0) \tag{7.146}$$

$$Z_{th} = \frac{(R_1 + jX_1)(jX_0)}{R_1 + j(X_1 + X_0)} \tag{7.147}$$

$$Z_{th} = \frac{jR_1X_0 - X_1X_0}{R_1 + j(X_1 + X_0)} \tag{7.148}$$

On the right side of equation (7.148), multiply the numerator and the denominator by the term $R_1 - j(X_1 + X_0)$ and it becomes,

$$Z_{th} = \frac{(jR_1X_0 - X_1X_0)[R_1 - j(X_1 + X_0)]}{R_1^2 + (X_1 + X_0)^2} \tag{7.149}$$

$$Z_{th} = \frac{X_0^2}{R_1^2 + (X_1 + X_0)^2} R_1 + j\frac{X_0(R_1^2 + X_1^2 + X_0X_1)}{R_1^2 + (X_1 + X_0)^2} \tag{7.150}$$

$$Z_{th} = R_{th} + jX_{th} \tag{7.151}$$

where the expression of the Thevenin resistance and reactance are written as,

$$R_{th} = \frac{X_0^2}{R_1^2 + (X_1 + X_0)^2} R_1 \tag{7.152}$$

$$X_{th} = \frac{X_0(R_1^2 + X_1^2 + X_0 X_1)}{R_1^2 + (X_1 + X_0)^2} \tag{7.153}$$

The Thevenin equivalent circuit is shown in Fig. 7.16.

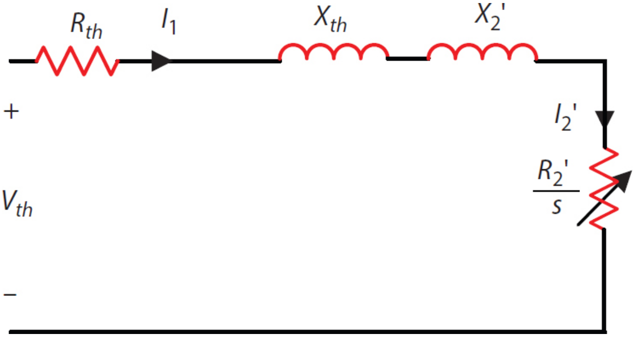


Fig. 7.16 Thevenin equivalent circuit.

If $R_1^2 < (X_1+X_0)^2$, then the expression of Thevenin resistance from equation (7.152) can be modified as,

$$R_{th} = \frac{X_0^2}{(X_1 + X_0)^2} R_1 \tag{7.154}$$

If $R_1^2 < (X_1+X_0)^2$ and $X_1 < X_0$ then equation (7.153) can be modified as,

$$X_{th} = \frac{X_0(0 + X_1^2 + X_0 X_1)}{0 + (X_1 + X_0)^2} \tag{7.155}$$

$$X_{th} = \frac{X_0 X_1 (X_1 + X_0)}{(0 + X_0)^2} \tag{7.156}$$

$$X_{th} = \frac{X_1 (0 + X_0)}{X_0} \tag{7.157}$$

$$X_{th} = X_1 \tag{7.158}$$

The expression of the Thevenin voltage is modified as,

$$V_{th} = \frac{X_0}{\sqrt{R_1{}^2 + (X_1 + X_0)^2}} V_1 \tag{7.159}$$

If $R_1{}^2 < (X_1 + X_0)^2$, then equation (7.159) can be modified as,

$$V_{th} = \frac{X_0}{\sqrt{0 + (X_1 + X_0)^2}} V_1 \tag{7.160}$$

$$V_{th} = \frac{X_0}{X_1 + X_0} V_1 \tag{7.161}$$

7.13 Power Relationships

A three-phase induction motor converts electrical power to mechanical power through its shaft. A suitable motor starter is used to run the induction motor. During starting, the stator of an induction motor receives power from the three-phase source. This input power can be expressed as,

$$P_{in} = 3 V_1 I_1 \cos\theta_1 \tag{7.162}$$

where,

V_1 is the phase voltage of the source,

I_1 is the phase current of the stator,

θ_1 is the phase angle of the current.

The electrical power transfers from the stator to the rotor through an air gap. Most of the air gap power is then converted into mechanical power and the remaining of the power is wasted as the rotor copper loss. Therefore, the air gap power can be written as,

$$P_{ag} = P_{mech} + P_{rcu} \tag{7.163}$$

The rotor copper loss can be written as,

$$P_{rcu} = 3I_2^{\ 2}R_2 \tag{7.164}$$

The variable resistance R_2/s normally consumes the air gap power and its expression can be written as,

$$P_{ag} = 3I_2^{\ 2}\frac{R_2}{s} \tag{7.165}$$

Substituting equation (7.164) into the equation (7.165) yields,

$$P_{ag} = \frac{P_{rcu}}{s} \tag{7.166}$$

Substituting equation (7.166) into equation (7.163) yields,

$$P_{ag} = P_{mech} + sP_{ag} \tag{7.167}$$

$$P_{mech} = (1-s)P_{ag} \tag{7.168}$$

Substituting equation (7.166) into equation (7.168) yields the mechanical power in terms of rotor copper loss as,

$$P_{mech} = \left(\frac{1-s}{s}\right)P_{rcu} \tag{7.169}$$

Air gap power: Mechanical power: Rotor cu loss is,

$$P_{ag} : P_{mech} : P_{rcu} = \frac{1}{s} : \frac{1-s}{s} : 1 = 1 : (1-s) : s \tag{7.170}$$

From equation (7.170), it is seen that the rotor input or air gap power is dissipated in the rotor circuit resistance as a fraction of s and the fraction of (1- s) is converted into mechanical power. From the basic definition of torque, it is expressed as,

$$T = \frac{P_{ag}}{\omega_s} \tag{7.171}$$

$$T = \frac{P_{ag}}{\dfrac{2\pi N_s}{60}} \tag{7.172}$$

From equation (7.172), the expression of rotor input can be written as,

$$P_{ag} = \frac{2\pi N_s}{60} \times T = k_r T \tag{7.173}$$

where the expression of the constant is expressed as,

$$k_r = \frac{2\pi N_s}{60} \tag{7.174}$$

Substituting equation (7.165) into equation (7.173) yields,

$$k_r T = 3{I_2}^2 \frac{R_2}{s} \tag{7.175}$$

$$T = \frac{1}{k_r} 3{I_2}^2 \frac{R_2}{s} \tag{7.176}$$

If the rotor resistance is constant, then the expression of torque becomes,

$$T = k_5 {I_2}^2 \frac{1}{s} \tag{7.177}$$

$$T \propto \frac{I_2^{\,2}}{s} \tag{7.178}$$

From equation (7.178), it is seen that the torque is directly proportional to the square of the rotor current and inversely proportional to the slip.

Example 7.7

The rotor input of a three-phase 4-pole, 440 V, 50 Hz three-phase induction motor is 1.5 kW. The copper loss of the rotor circuit is 250 W. Determine the slip, synchronous speed, shaft speed and mechanical power developed.

Solution

The slip can be calculated as,

$$s = \frac{P_{rcu}}{P_{ag}} = \frac{250}{1500} = 0.17 \tag{7.179}$$

The synchronous speed is calculated as,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\,\text{rpm} \tag{7.180}$$

The shaft speed or rotor speed is calculated as,

$$N_l = (1-s)N_s = (1-0.17)\times 1500 = 1245\,\text{rpm} \tag{7.181}$$

The mechanical power developed is,

$$P_{mech} = (1-s)P_{ag} = (1-0.17)\times 1500 = 1245\,\text{W} \tag{7.182}$$

Practice Problem 7.7

The rotor speed of a three-phase, 6-pole, 415 V, 60 Hz, 30 kW three-phase induction motor is 1150 rpm. The friction and windage losses of an induction motor are 550W. Calculate the mechanical power developed, slip, air gap power and rotor copper loss.

7.14 Approximate Equivalent Circuit

An approximate equivalent circuit is necessary for quick identification of the performance of a three-phase induction motor. It is drawn by shifting the no-load circuit across the supply voltage. This type of equivalent circuit is used to simplify different types of analysis in an induction motor. Fig. 7.17 shows an approximate equivalent circuit for an induction motor. The equivalent impedance is written as,

$$Z_{eqr} = R_1 + \frac{R_2}{s} + j(X_1 + X_2) \tag{7.183}$$

The magnetizing component of the current is calculated as,

$$I_m = \frac{V_1}{X_0} \tag{7.184}$$

The core loss component of the current is determined as,

$$I_w = \frac{V_1}{R_0} \tag{7.185}$$

The rotor circuit current is determined as,

$$I_2 = \frac{V_1}{Z_{eq}} \tag{7.186}$$

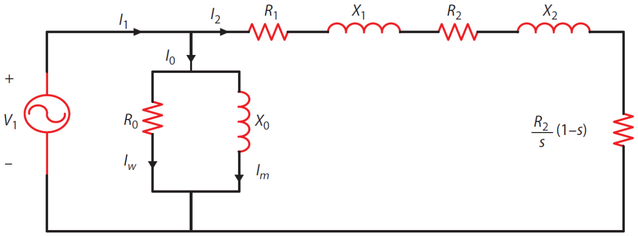


Fig. 7.17 Approximate equivalent circuit.

The per phase current supplied by the source can be calculated as,

$$I_1 = I_2 + I_w + I_m \tag{7.187}$$

The input power is then calculated as,

$$P_{in} = 3V_1 I_1 \cos\phi_1 \tag{7.188}$$

The stator copper loss is determined as,

$$P_{scu} = 3I_2^{\,2} R_1 \tag{7.189}$$

The rotor copper loss is calculated as,

$$P_{rcu} = 3I_2^{\,2} R_2 \tag{7.190}$$

The core loss can be determined as,

$$P_{core(iron)} = 3I_w^{\,2} R_0 \tag{7.191}$$

For a given windage and bearing friction losses (*Pw&f*), the available output power can be determined as,

$$P_0 = P_{in} - P_{scu} - P_{rcu} - P_{core} - P_{w\&f} \tag{7.192}$$

7.15 Condition for Maximum Mechanical Power

The expressions of rotor current and the impedance need to be known to find the condition for maximum mechanical power developed. From Fig. 7.17, the expression of rotor current can be written as,

$$I_2 = \frac{V_1}{R_1 + R_2 + j(X_1 + X_2) + R_2\left(\frac{1-s}{s}\right)} \tag{7.193}$$

$$I_2 = \frac{V_1}{R_{eq} + jX_{eq} + R_2\left(\frac{1-s}{s}\right)} \tag{7.194}$$

The magnitude of the rotor current is determined as,

$$I_2 = \frac{V_1}{\sqrt{\left[R_{eq} + R_2\left(\frac{1-s}{s}\right)\right]^2 + X_{eq}^{\ 2}}} \tag{7.195}$$

where the expression of equivalent resistance and reactance are written as,

$$R_{eq} = R_1 + R_2 \tag{7.196}$$

$$X_{eq} = X_1 + X_2 \tag{7.197}$$

Using equations (7.164) and (7.169) to write down the expression of mechanical power developed,

$$P_{mech} = 3I_2^{\ 2}R_2\left(\frac{1-s}{s}\right) \tag{7.198}$$

Substituting the expression of I_2 from equation (7.195) into equation (7.198) yields,

$$P_{mech} = \frac{3V_1^{\ 2}}{\left[R_{eq} + R_2\left(\frac{1-s}{s}\right)\right]^2 + X_{eq}^{\ 2}}R_2\left(\frac{1-s}{s}\right) \tag{7.199}$$

$$P_{mech} = \frac{3V_1^2}{R_{eq}^{\ 2} + 2R_{eq}R_2\left(\frac{1-s}{s}\right) + \left[R_2\left(\frac{1-s}{s}\right)\right]^2 + X_{eq}^{\ 2}}R_2\left(\frac{1-s}{s}\right) \tag{7.200}$$

$$P_{mech} = \frac{3V_1^{\ 2}R_2}{(R_{eq}^{\ 2} + X_{eq}^{\ 2})\frac{s}{1-s} + 2R_{eq}R_2 + R_2^{\ 2}\left(\frac{1-s}{s}\right)} \tag{7.201}$$

The mechanical power developed will be maximum if the denominator of equation (7.201) is minimum. To find the minimum value, differentiate the denominator of equation (7.201) with respect to slip and set equal to zero. The following relation can be written as,

$$\frac{d}{ds}\left\{(R_{eq}{}^2+X_{eq}{}^2)\frac{s}{1-s}+2R_{eq}R_2+R_2{}^2\left(\frac{1-s}{s}\right)\right\}=0 \tag{7.202}$$

$$(R_{eq}{}^2+X_{eq}{}^2)\frac{1-s+s}{(1-s)^2}+R_2{}^2\left(-\frac{1}{s^2}\right)=0 \tag{7.203}$$

$$(R_{eq}{}^2+X_{eq}{}^2)\frac{1}{(1-s)^2}=R_2{}^2\frac{1}{s^2} \tag{7.204}$$

$$(R_{eq}{}^2+X_{eq}{}^2)=R_2{}^2\left(\frac{1-s}{s}\right)^2 \tag{7.205}$$

$$Z_{eq}{}^2=R_2{}^2\left(\frac{1-s}{s}\right)^2 \tag{7.206}$$

Where the equivalent impedance is expressed as,

$$Z_{eq}{}^2=R_{eq}{}^2+X_{eq}{}^2 \tag{7.207}$$

Equation (7.206) can be rearranged as,

$$Z_{eq}=R_2\left(\frac{1-s}{s}\right) \tag{7.208}$$

$$\frac{Z_{eq}}{R_2}=\left(\frac{1-s}{s}\right) \tag{7.209}$$

$$\frac{R_2}{Z_{eq}}=\frac{s}{1-s} \tag{7.210}$$

Equation (7.208) can be re-arranged as,

$$(Z_{eq}+R_2)s=R_2 \tag{7.211}$$

$$s_{mp}=\frac{R_2}{(Z_{eq}+R_2)} \tag{7.212}$$

Substituting equations (7.209) and (7.210) into equation (7.201) and the expression of maximum mechanical power developed is,

$$P_{mech(mxm)}=\frac{3V_1^{\,2}R_2}{Z_{eq}^{\,2}\dfrac{R_2}{Z_{eq}}+2R_{eq}R_2+R_2^{\,2}\dfrac{Z_{eq}}{R_2}} \tag{7.213}$$

$$P_{mech(mxm)}=\frac{3V_1^{\,2}R_2}{Z_{eq}R_2+2R_{eq}R_2+Z_{eq}R_2} \tag{7.214}$$

$$P_{mech(mxm)}=\frac{3V_1^{\,2}}{2(Z_{eq}+R_{eq})} \tag{7.215}$$

From equation (7.215), it is seen that the maximum mechanical power is dependent on supply voltage, equivalent resistance and impedance.

Example 7.8

The per-phase standstill stator impedance and rotor impedance of a 150 V, 50 Hz, 4-pole delta-connected three-phase induction motor is $Z_1 = 0.12 + j0.16\ \Omega$, $Z_2 = 0.22 + j0.28\ \Omega$, respectively. Calculate the maximum mechanical power, maximum torque and the value of the slip at this condition.

Solution

The equivalent impedance is calculated as,

$$Z_{eq}=0.12+j0.16+0.22+j0.28=0.34+j0.44=0.55\angle 52.31^\circ\Omega \tag{7.216}$$

The maximum mechanical power developed is,

$$P_{mech(mxm)} = \frac{3V_1^2}{2(R_2 + Z_{eq})} = \frac{3 \times 150^2}{2(0.22 + 0.55)} = 37.92\,\text{kW} \tag{7.217}$$

The value of the slip is calculated as,

$$s_{mp} = \frac{R_2}{(Z_{eq} + R_2)} = \frac{0.22}{0.22 + 0.55} = 0.29 \tag{7.218}$$

The expression of synchronous speed can be determined as,

$$N_s = f / \frac{P}{2} = \frac{2f}{P} \tag{7.219}$$

$$\omega_s = 2\pi N_s \tag{7.220}$$

$$\omega_s = 2\pi \times \frac{2f}{P} = 4\pi \times \frac{50}{4} = 157.08 \quad \text{rad/s} \tag{7.221}$$

The speed of the rotor is calculated as,

$$\omega = (1 - s_{mp})\omega_s = (1 - 0.29) \times 157.08 = 111.53 \quad \text{rad/s} \tag{7.222}$$

The value of the maximum torque is determined as,

$$T_{mxm} = \frac{P_{mech(mxm)}}{\omega} = \frac{37.92 \times 1000}{111.53} = 340\,\text{N-m} \tag{7.223}$$

Practice Problem 7.8

The per-phase standstill stator impedance and rotor impedance of a 120 V, 60 Hz, 6-pole delta-connected three-phase induction motor is $Z_1 = 0.09 + j0.15\ \Omega$, $Z_2 = 0.18 + j0.25\ \Omega$, respectively.

Determine the maximum mechanical power, maximum torque and the value of the slip at this condition.

7.16 Power Stages

It is easier to understand how electrical power is converted into mechanical power with the help of some relevant stages. The real power P_{in} flows to the stator windings from the three-phase power supply. The stator slots have three-phase coils which provide the stator copper loss, P_{scu}. However, a portion of the input power P_{in} is dissipated as heat (P_{scu}) in the stator windings and a small portion of P_{in} is dissipated as heat (P_{iron}) in the stator core. The remaining real power P_2 or P_{ag} is available across the air gap, which is transferred to the rotor by electromagnetic induction. The rotor circuit has some conductors which provide rotor copper loss, P_{rcu}. Again, from the rotor input power, a portion of this power is dissipated as heat in the rotor circuit as P_{rcu} and the remaining portion of P_2 is available as mechanical power, P_{mech}. The shaft power or output power is obtained by subtracting windage and bearing friction losses from the mechanical power. The different types of losses along with other powers are mentioned in Fig. 7.18.

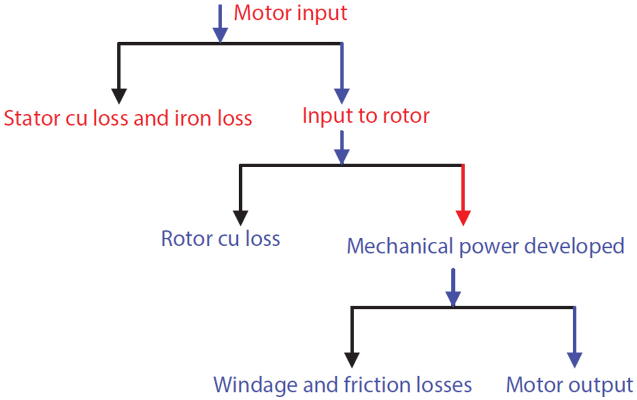

Motor input
Stator cu loss and iron loss
Input to rotor
Rotor cu loss
Mechanical power developed
Windage and friction losses
Motor output

Fig. 7.18 Power flow diagram.

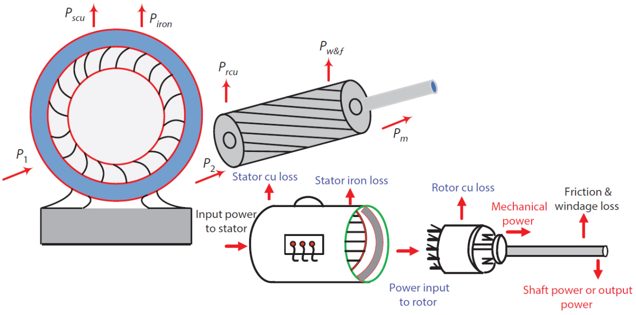

P_{scu}
P_{iron}
$P_{w\&f}$
P_{rcu}
P_1
P_2
P_m
Stator cu loss
Stator iron loss
Rotor cu loss
Friction & windage loss
Mechanical power
Input power to stator
Power input to rotor
Shaft power or output power

Fig. 7.19 Power flow diagram in a real induction motor.

However, it will be easier for the reader to understand and visualize different types of powers and associated losses with a suitable diagram of a three-phase induction motor. Fig. 7.19 is introduced for better visualization of different types of losses along with the input and output powers. The different types of losses are shown on the upper side and different power steps in the lower side.

Example 7.9

A three-phase 440 V, 6-pole, 50 Hz induction motor has an input power of 45 kW and it is running at a speed of 900 rpm. Total stator losses are measured to be 2 kW and the friction and windage losses are 1.5 kW. Determine the slip, rotor cu loss, shaft or output power and motor efficiency.

Solution

The synchronous speed is calculated as,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000\,\text{rpm} \tag{7.224}$$

The value of the slip is calculated as,

$$s = \frac{N_s - N_l}{N_s} = \frac{1000 - 900}{1000} = 0.1 \tag{7.225}$$

The rotor input or air gap power is calculated as,

$$P_{ag} = (45-2)\times 1000 = 43000\,\text{W} \tag{7.226}$$

The rotor cu loss is determined as,

$$P_{rcu} = sP_{ag} = 0.1\times 43000 = 4300\,\text{W} \tag{7.227}$$

The mechanical power is calculated as,

$$P_{mech} = 43000 - 4300 = 38700\,\text{W} \tag{7.228}$$

In an alternative way, the mechanical power can be determined as,

$$P_{mech} = (1-s)P_{ag} = (1-0.1)\times 43000 = 38700\,\text{W} \tag{7.229}$$

The output power is calculated as,

$$P_0 = 38700 - 1500 = 37200\,\text{W} \tag{7.230}$$

The efficiency is determined as,

$$\eta = \frac{P_0}{P_{in}} = \frac{37200}{45000} = 82.67\% \tag{7.231}$$

Practice Problem 7.9

A three-phase 415 V, 6-pole, delta-connected, 50 Hz induction motor consumes power of 72 kW with a line current of 71 A and runs at a slip of 2%. The stator iron loss is 1.5 kW, and friction and windage loss is 2 kW. If the stator resistance per phase is 0.12 Ω, determine the rotor input power, rotor cu loss, shaft or output power, and motor efficiency.

7.17 Torque-Slip Characteristics

The torque-slip characteristics are important to identify the performance of a three-phase induction motor. The torque slip characteristics curve is obtained with the variation of slip for a particular value of rotor resistance R_2. The torque under running conditions is written as,

$$T \propto \frac{sR_2}{R_2^2 + (sX_2)^2} \tag{7.232}$$

From equation (7.232), the torque-slip characteristic can be explained in the following ways:

i. The value of the torque is zero for a zero value of the slip. Hence, the torque-slip curve will start from the origin.

ii. The value of the slip is very small when the motor attains its normal rated speed. Under this condition, the term sX_2 is negligible as compared to R_2. Then equation (7.232) is modified as,

$$T \propto \frac{sR_2}{R_2^2} \tag{7.233}$$

$$T \propto \frac{s}{R_2} \tag{7.234}$$

The rotor resistance of a squirrel cage rotor is essentially constant from no-load to full load. Then equation (7.234) is modified as,

$$T \propto s \tag{7.235}$$

From equation (7.235), it is seen that the torque is directly proportional to the slip. Therefore, the torque will increase with increasing the slip until it reaches the maximum value.

iii. Torque increases as the slip increases and it will be maximum at the following value of the slip.

$$s = \frac{R_2}{X_2} \tag{7.236}$$

This maximum value of the torque of an induction motor is called pullout torque or breakdown torque.

iv. If the slip increases more, then the term $(sX_2)^2$ increases sharply. Therefore, the term R_2^2 may be neglected as compared to $(sX_2)^2$. Again, the expression of the torque becomes,

$$T \propto \frac{s}{(sX_2)^2} \tag{7.237}$$

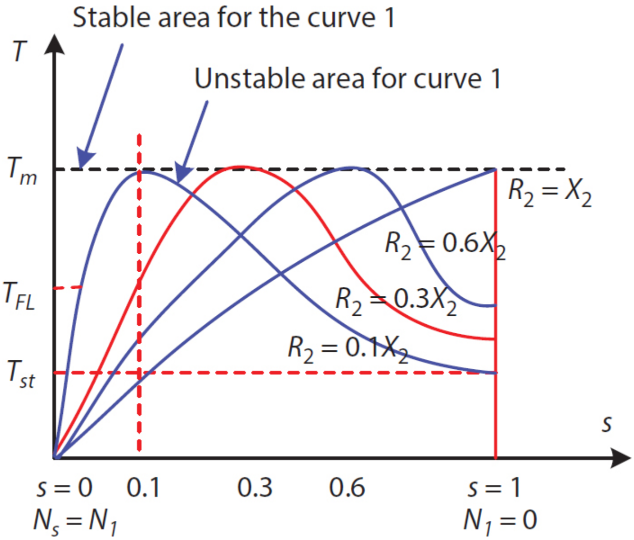


Fig. 7.20 Torque-slip characteristics curve.

If X_2 is constant, then equation (7.237) becomes,

$$T \propto \frac{1}{s} \tag{7.238}$$

From the equation (7.238), it is seen that the torque is inversely proportional to the slip. In this condition, the torque will decrease with increasing slip as shown in Fig. 7.20. At $s = 1$ and $N_l = 0$, the torque produced by the motor is known as the starting torque as mentioned in Fig. 7.20.

7.18 Linear Induction Motor

The motor converts electrical input power into linear motion instead of rotational motion as in the case of a three-phase induction motor known as linear induction motor (LIM). A linear induction motor has a stator and rotor. The stator serves as a primary where the three-phase supply is provided and the rotor serves as a secondary. The secondary of this motor is made of flat aluminum straight conductors with a ferromagnetic core. A linear induction motor is obtained if the induction motor is cut and laid in a flat position as shown in Fig. 7.21.

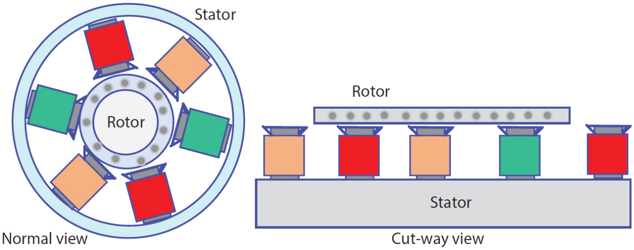

Stator
Rotor
Normal view
Rotor
Stator
Cut-way view

Fig. 7.21 Normal and cut-way view of linear induction motor.

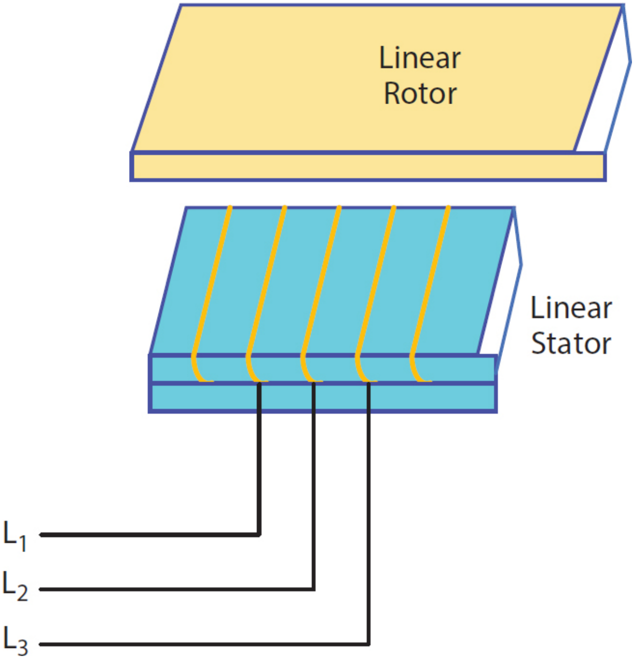

Linear
Rotor
Linear
Stator
L_1
L_2
L_3

Fig. 7.22 Schematic of linear induction motor.

The basic structure of the linear induction motor (LIM) is shown in Fig. 7.22. The secondary winding is designated as a reaction plate. When a three-phase voltage energizes the primary winding, a travelling magnetic field is generated which travels along the length of the primary conductor. The relative motion due to the travelling magnetic field and the reaction plate induces a current in the rotor which creates the magnetic field. The magnetic field in the rotor interacts with the travelling magnetic field of the stator which in turn creates the linear force. This magnetic field induces a current in the reaction plate, which in turn creates its magnetic field. The interaction of the two magnetic fields generates high force and direct linear motion. The force can be increased if two LIMs face each other. The speed of the travelling magnetic field can be expressed as,

$$v_s = 2wf \tag{7.239}$$

where,

v_s is the linear synchronous speed in m/s,

w is the width of one pole pitch in m,

f is the supply frequency in Hz.

The rotor speed (v_l) is always less than the synchronous speed (v_s) of the stator magnetic field. As a three-phase induction motor, the slip of a linear induction motor is defined as,

$$s = \frac{v_s - v_l}{v_s} \tag{7.240}$$

where,

s is the slip,

v_l is the speed of the rotor in m/s.

Equation (7.240) is modified as,

$$s v_s = v_s - v_l \tag{7.241}$$

$$v_l = v_s(1 - s) \tag{7.242}$$

The ratio of air gap power to the linear synchronous speed is known as the thurst or linear force. It is represented by F and it is expressed as,

$$F = \frac{\text{Air gap power}(P_2)}{v_s} \tag{7.243}$$

Like a three-phase induction motor, copper loss in the rotor circuit and mechanical power developed is calculated as,

$$P_{cur} = sP_2 \tag{7.244}$$

$$P_m = P_2 - P_{cur} \tag{7.245}$$

Example 7.10

The train moves at a speed of 120 km/h when the stator frequency is found to be 50 Hz. Find the length of the pole pitch of the linear induction motor.

Solution

The value of the synchronous speed in m/s is calculated as,

$$v_s = 120\,\text{km/h} = \frac{120 \times 1000}{60 \times 60} = 33.33\,\text{m/s} \tag{7.246}$$

The length of the pole-pitch is determined as,

$$w = \frac{v_s}{2f} = \frac{33.33}{2 \times 50} = 0.33\,\text{m} \tag{7.247}$$

Example 7.11

Two three-phase, 4-pole linear induction motors are attached to the rail. The pole pitch of the motor is 80 mm and is supplied by a three-phase supply with a variable frequency source. One of the motors was tested before operating the rail. The input power to the stator is 8 kW, the stator frequency is 25 Hz, the stator copper and iron loss is 1.6 kW, and the rail speed is 2.8 m/s. Calculate the synchronous speed, slip, power input to the rotor, rotor copper loss, mechanical power and linear force.

Solution

The value of the synchronous speed in m/s is calculated as,

$$v_s = 2wf = 2\times 0.08\times 25 = 4\,\text{m/s} \tag{7.248}$$

The value of the slip is calculated as,

$$s = \frac{v_s - v_l}{v_s} = \frac{4-2.8}{4} = 0.3 \tag{7.249}$$

The power to the rotor circuit is calculated as,

$$P_2 = 8 - 1.6 = 6.4\,\text{kW} \tag{7.250}$$

The rotor copper loss is calculated as,

$$P_{cur} = sP_2 = 0.3\times 6.4 = 1.92\,\text{kW} \tag{7.251}$$

The available mechanical power is calculated as,

$$P_m = P_2 - P_{cur} = 6.4 - 1.92 = 4.48\,\text{kW} \tag{7.252}$$

The value of the linear force is calculated as,

$$F = \frac{\text{Air gap power}(P_2)}{v_s} = \frac{6.4}{4} = 1.6\,\text{N} \tag{7.253}$$

Practice Problem 7.10

The train moves by a power from the 100 Hz source and the length of the pole-pitch is 200 mm. Determine the synchronous speed of the train.

Practice Problem 7.11

Two three-phase, 6-pole linear induction motors are attached to a manufacturing crane. The pole pitch of the motor is 40 mm and is supplied by a three-phase supply with a variable frequency source. One of the motors was tested before operating the rail. The input power to the stator is 10 kW, the stator frequency is 30 Hz, the stator copper and iron loss is 2.1 kW, and the rail speed is 1.6 m/s. Calculate the synchronous speed, slip, power input to the rotor, rotor copper loss, mechanical power and linear force.

7.19 Classification of Induction Motor by Properties

The National Electrical Manufacturers Association (NEMA) has developed a letter system in which each three-phase induction is designed and classified based on their design standard. These are class A, class B, class C and class D.

Class A: In this group, the speed of the motor is constant throughout time and it has a very good heat dissipation capacity. During the starting time, the current density is higher and is uniformly distributed over the rotor surface. It has a high locked rotor current and a high locked rotor torque. It has a slip ranging from 0.5% to 5%. The efficiency of this type of motor is high or medium. This type of motor is used in fans, blowers, centrifugal pumps, and motor-generator sets where relatively low starting is required. The motor in this group can start directly.

Class B: Most of the three-phase induction motors fall in this group. Sometimes, the motor in this group is called a general-purpose motor. This type of motor has a normal locked rotor current and torque, i.e., the starting current of this type of motor varies from 4 to 5 times the rated current. It has a slip ranging from 0.5% to 5% and this type of motor is used in fans, blowers, centrifugal pumps, and motor-generator sets where relatively low starting is required.

Class C: The motor of this group has high starting torque with a normal starting current and the slip varies from 1% to 5%. The starting torque is 2 to 2.5 times and the starting current is 3 to 5 times higher than the class A and class B types of motor. This type of motor has medium efficiency. This design is normally used where high starting torque is required, such as conveyors, stirring motors, agitators, reciprocating pumps, crushers and piston-type compressors.

Class D: The motor of this group has very high starting torque, low starting current and low full load speed. The slip of this motor is very high and varies between 5% to 13% and it has a high efficiency. This type of motor is used in punch presses, elevators, hoists, oil-well pumping, etc.

Exercise Problems

7.1 The full load speed of a 4-pole, 415 V, 60 Hz, three-phase induction motor is 1700 rpm. Determine the synchronous speed, slip and rotor frequency.

7.2 The synchronous speed of a 50 Hz, 425 V, three-phase induction motor is 1500 rpm and the rated speed is 1450 rpm. Find the number of poles, slip and rotor frequency.

7.3 The synchronous speed of a 50 Hz, 425 V, three-phase induction motor is 1500 rpm. Determine the number of poles, and rotor speed at 3% slip and rotor frequency.

7.4 A 4-pole three-phase induction motor operates by 420 V, 50 Hz supply. Calculate the synchronous speed, rotor speed at 5% slip and rotor frequency when the rotor runs at 700 rpm.

7.5 The blocked rotor voltage and running speed of a 4-pole, 50 Hz, 415 V, three-phase induction motor are 115 V and 950 rpm, respectively. Calculate the synchronous speed, slip, rotor frequency and rotor voltage.

7.6 The blocked rotor voltage of a three-phase 60 Hz induction motor is 98 V. When running at rated load, the shaft speed is 1770 rpm and the slip speed is 30 rpm. Calculate the number of poles, slip and rotor voltage.

7.7 A 4-pole three-phase induction motor is running at 1250 rpm from the 50 Hz, 220 V supply. A mechanical device blocks the rotor and the voltage is measured at 95 V. Find the synchronous speed, rotor frequency and rotor voltage.

7.8 The original starting torque of a three-phase, 220 V, 4-pole induction motor is 250 N-m and the starting current at rated voltage is 450 A. A reduced three-phase voltage of 115 V is applied to the motor terminals. Calculate the starting torque and starting current.

7.9 The original starting torque of a three-phase, 220 V, 4-

pole induction motor is 50% of 450 N-m and the starting current at rated voltage is 350 A. A reduced three-phase voltage of 110 V is applied to the motor terminals. Determine the final starting torque and starting current.

7.10 The ratio of the final starting torque to the original starting torque of a three-phase induction motor is 2. Calculate the final starting current of the induction motor if the original starting current is 300 A.

7.11. The original starting current of a three-phase induction motor is 1.5 times the final starting current. Determine the ratio of the final starting torque to the original starting torque.

7.12 The full load slip of a 4-pole, 50 Hz, three-phase induction motor is 4%. The rotor resistance and standstill reactance are 0.02 Ω and 0.2 Ω, respectively. Calculate the speed at which maximum torque occurs and the ratio of maximum torque to full load torque.

7.13 A 440 V three-phase induction motor has a rotor resistance of 0.03 Ω per phase and a standstill reactance of 0.1 Ω per phase. Additional resistance is connected in the rotor circuit to make the starting torque equal to the maximum torque of the motor. Find the value of additional resistance.

7.14 A 4-pole three-phase induction motor is running from the 50 Hz, 415 V supply. The rotor resistance and standstill reactance per phase are 0.02 Ω and 0.12 Ω, respectively. Determine the external value of the rotor resistance to obtain the ratio of starting to a maximum torque of 60%.

7.15 The synchronous speed and full load slip of a three-phase induction motor are 1000 rpm and 5%, respectively. The rotor resistance and standstill reactance per phase are 0.01 Ω and 0.12 Ω respectively. Determine the ratio of full load to maximum torque and speed at the maximum torque.

7.16 A three-phase, 4-pole, 50 Hz induction motor has a ratio of starting torque to full load torque of 1.5 and maximum torque to full load torque of 3. Find the full load speed and the speed at the maximum torque.

7.17 The effective rotor resistance per phase of an 8-pole, 50

Hz, three-phase induction motor is 0.05 Ω. If the rotor runs at a speed of 650 rpm, then find the rotor reactance per phase. Also, find the additional resistance that is included to get the maximum torque during starting.

7.18 A three-phase, 4-pole, 50 Hz induction motor is running at a reduced load and the input of the rotor is 1.6 kW. The rotor copper loss is 250 W and the combined friction, windage and stray power losses are 200 W. Calculate the slip, shaft or rotor speed and mechanical power developed.

7.19 A 15 kW, 230 V, 4-pole, 50 Hz three-phase induction motor is running at a speed of 1400 rpm. The rotor copper loss is 300 W and the friction and windage losses are 220 W. Determine the synchronous speed, slip, mechanical power developed and air gap power.

7.20 The input power of a 4-pole, 50 Hz, three-phase induction motor is 40 kW and this motor is running at 1400 rpm. The total stator losses are 1.5 kW and the combined friction and windage losses are 2 kW. Find the slip, rotor copper losses, rotor output and efficiency.

7.21 The output of a three-phase induction motor is 40 kW and it is running at a 30% slip. The total mechanical losses are 1.4 kW. Find the copper losses in the rotor circuit. Also, find the efficiency of the motor if the total stator losses are 5 kW.

7.22 The output of a 6-pole, 440 V, 50 Hz, three-phase induction motor is 38 kW and it is running at a speed of 950 rpm. The friction and windage losses are 1.7 kW. The stator losses are 1.9 kW and the power factor is 0.9. Calculate the rotor copper loss and line current.

7.23 An electric train receives power from the 60 Hz source and the length of the pole-pitch is 345 mm. Determine the synchronous speed of the train.

7.24 Two three-phase, 4-pole linear induction motors are attached to a crane of the pharmaceutical industry. The pole pitch of the motor is 50 mm and is supplied by a three-phase supply with a variable frequency source. Before the operation, one of the motors was tested before operating the rail. The input power to the stator is 8 kW, the stator frequency is 26 Hz, the stator copper and iron loss is 1.7 kW, and the rail speed is 2.1 m/s. Find the synchronous speed, slip, power input to the rotor, rotor copper loss, mechanical power and linear force.

8
Starting and Control of an Induction Motor

8.1 Introduction

A three-phase AC power supply is required to run a three-phase induction motor. During starting, a three-phase induction motor normally draws five to seven times its full load rated current and develops only 1.5 to 2.5 times the full load torque due to the inertia of the rotor when connected directly to the supply. This high magnitude of starting current may damage the stator windings, fuses and circuit breakers. Therefore, it is necessary to reduce the starting current using suitable starting methods. In addition to starting, there are some applications in the manufacturing industry where the reduced speed of an induction motor is essential. For these purposes, it is necessary to control the speed of the induction motor. The different methods of starting and controlling an induction motor are discussed in this chapter.

8.2 Direct Starting

Direct starting may be used for small motors where a high starting current is permissible. In this type of starting, a compact push button contactor is used. Here, the coil of the contactor controls a large magnitude of the current. The expression of the torque in terms of starting current and slip is derived in Chapter 7 and it is in the following form,

$$T \propto \frac{I^2}{s} \tag{8.1}$$

During the starting of the motor, the value of the slip is equal to one. Therefore, equation (8.1) can be modified as,

$$T_{st} \propto I_{st}^{2} \tag{8.2}$$

There will be a slip in the motor when running, which means from starting condition to full load condition. The expression of full load or running torque is expressed as,

$$T_f \propto \frac{I_f^{2}}{s_f} \tag{8.3}$$

Dividing equation (8.2) by equation (8.3) yields,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f \tag{8.4}$$

During starting when the motor is connected directly to the supply system, the starting current will be equal to the short circuit current. Under this condition, the starting current is expressed as,

$$I_{st} = I_{sc} \tag{8.5}$$

Substitute equation (8.5) into equation (8.4) and the torque ratio becomes,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 s_f \tag{8.6}$$

A magnetic contactor with a line voltage of 220V is considered to make a direct-on-line (DOL) starting method as shown in Fig. 8.1.

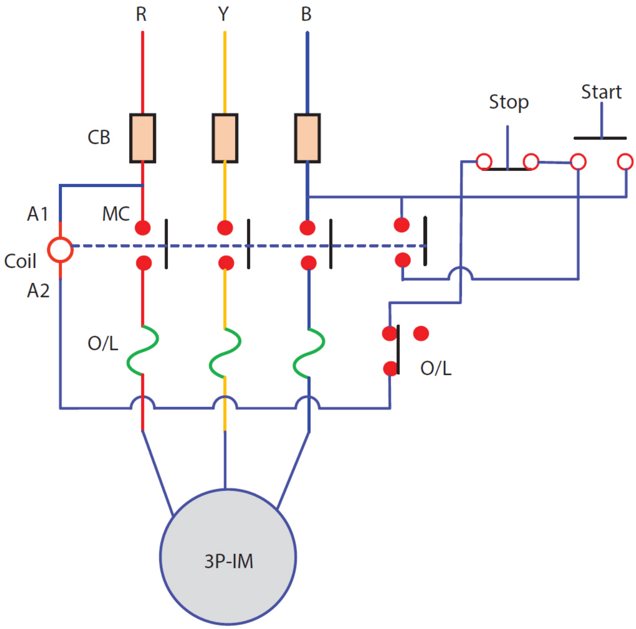


Fig. 8.1 Connection diagram of DOL starter.

When pressing the start pushbutton switch, current passes through the coil and energizes it. Due to this current, the Lorentz force ($F = BlI$) will be created which will pull the armature of the magnetic contactor. As a result, normally close (NC) contacts will be opened and normally open contacts will be closed. The motor will run continuously. When the stop pushbutton switch is energized, the path will be disconnected and no current pass through the magnetic contactor. As a result of the no current, the coil of the magnetic contactor will be de-energized and finally, the motor will be stopped.

Example 8.1

A three-phase induction motor is started from a three-phase supply directly. The starting torque is 1.5 times the full load torque. For a 3% full load slip, determine the value of the short circuit current in terms of full load current.

Solution

The value of the starting torque is given as,

$$T_{st} = 1.5T_f \tag{8.7}$$

From equation (8.6), the short circuit current is determined as,

$$\frac{1.5T_f}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times 0.03 \tag{8.8}$$

$$\frac{I_{sc}}{I_f} = \sqrt{\frac{1.5}{0.03}} \tag{8.9}$$

$$\frac{I_{sc}}{I_f} = 7.07 \tag{8.10}$$

$$I_{sc} = 7.07I_f \tag{8.11}$$

Practice Problem 8.1

A three-phase induction motor is designed to start from the three-phase supply directly with a 4% full load slip. The starting or short circuit current is 1.1 times the full load current. Determine the ratio of starting torque to full load torque.

8.3 Variable Resistance Starter

A three-phase supply is applied to the main switch through a three-phase circuit breaker. In this method, variable resistances are connected in series with the stator winding as shown in Fig. 8.2. This external variable resistance causes a voltage drop across it and only reduced voltage is applied to the motor terminals. As a result, the motor starts slowly and the external variable resistance will cut up gradually in steps when the motor picks up to its rated speed. Let the voltage be reduced to a fraction of x and per phase applied to the stator winding be xV_S. Therefore, the starting current per phase is,

$$I_{st} = xI_{sc} \tag{8.12}$$

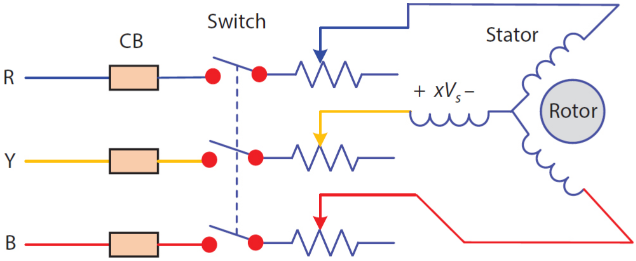


Fig. 8.2 Connection diagram for variable resistance starter.

Equation (8.4) is modified as,

$$\frac{T_{st}}{T_f} = \left(\frac{xI_{sc}}{I_f}\right)^2 s_f \tag{8.13}$$

$$T_{st} = x^2 \times T_f \left(\frac{I_{sc}}{I_f}\right)^2 s_f \tag{8.14}$$

From equation (8.14), it is seen that the starting torque is equal to x^2 times the direct starting torque.

8.4 Autotransformer Starter

An autotransformer is used to reduce the magnitude of the line voltage by tapping at a suitable point on it. An autotransformer is generally tapped at the 10, 20, 30, 50, 60 and 80% points to get the proper starting torque. An autotransformer starter may be either a manually or magnetically operated device. The reduced voltage is applied to the stator of an induction motor at the time of starting. If the motor is started at a reduced voltage of xV_s, then the corresponding starting current per phase is reduced to xIs_c. During starting, the expression of starting torque is directly proportional to the square of the reduced current and it is written as,

$$T_{st} \propto x^2 I_{sc}{}^2 \tag{8.15}$$

The motor will pick up its full load speed after applying the full supply voltage by changing the connections of an autotransformer. The connection diagram of an autotransformer starter is shown in Fig. 8.3.

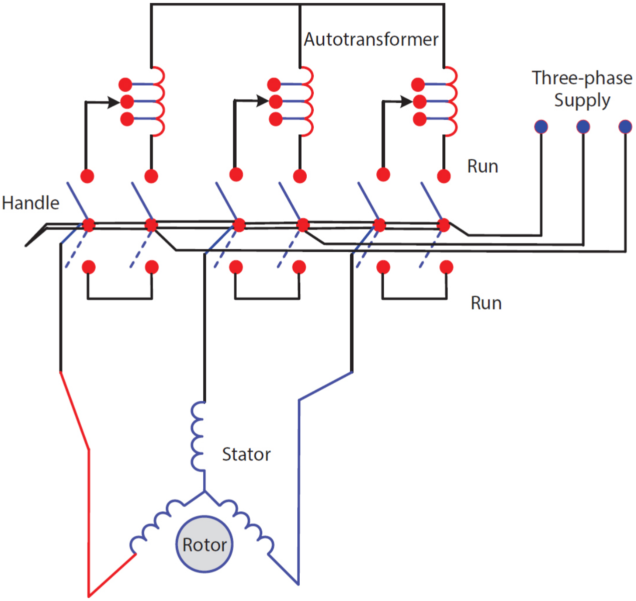

Autotransformer
Three-phase Supply
Run
Handle
Run
Stator
Rotor

Fig. 8.3 Connection diagram of an autotransformer starter.

In this starter, the ratio of starting torque to full load torque is,

$$\frac{T_{st}}{T_f} = \left(\frac{xI_{sc}}{I_f}\right)^2 s_f \tag{8.16}$$

From equation (8.16), the expression of x can be written as,

$$x = \frac{I_f}{I_{sc}}\sqrt{\frac{T_{st}}{T_f}\frac{1}{s_f}} \tag{8.17}$$

Example 8.2

An autotransformer is used to start the induction motor against 50% of full load torque. The full load slip is 3% and the short circuit current in normal voltage is 5 times the full load current. Calculate the percentage tapping required on the autotransformer.

Solution

The ratio of starting torque to full load torque is calculated as,

$$\frac{T_{st}}{T_f} = \frac{50}{100} = \frac{1}{2} \tag{8.18}$$

The ratio of short circuit current to full load current is given as,

$$\frac{I_{sc}}{I_f} = 5 \tag{8.19}$$

The full load slip is given as,

$$s_f = 0.03 \tag{8.20}$$

The percentage tapping required on the transformer is determined as,

$$x = \frac{I_f}{I_{sc}} \sqrt{\frac{T_{st}}{T_f} \frac{1}{s_f}} = \frac{1}{5} \sqrt{\frac{1}{0.03 \times 2}} = 0.816 \tag{8.21}$$

Practice Problem 8.2

An autotransformer is used to start the induction motor against 40% of full load torque. The full load slip is 4% and the percentage tapping on the autotransformer is 80%. Calculate the ratio of short circuit current to full load current.

8.5 Star-Delta Starter

This method of starting is widely used for high-rating three-phase induction motors; for example, submersible water pump motors, oil drilling motors, rice mill motors, etc. A three-phase induction motor with a star-delta starter and a three-pole double-through

(TPDP) switch is connected as shown in Fig. 8.4. When the switch TPDP is connected to the start position, then the motor is connected to the star connection and it starts to run slowly. When the motor picks up around 80% of its rated speed, then the switch connected delta connection and the motor runs its full load or run speed. The simple star-delta connection is shown in Fig. 8.5. In delta connection, the current per phase is calculated as,

$$I_{sc} = \frac{V_L}{Z} \tag{8.22}$$

The line current at the time of starting when stator windings are connected in the delta is,

$$I_L = \sqrt{3} I_{sc} \tag{8.23}$$

When the motor is connected to the star, the voltage per phase is,

$$V_P = \frac{1}{\sqrt{3}} V_L \tag{8.24}$$

The starting current per phase is calculated as,

$$I_{st} = \frac{V_P}{Z} \tag{8.25}$$

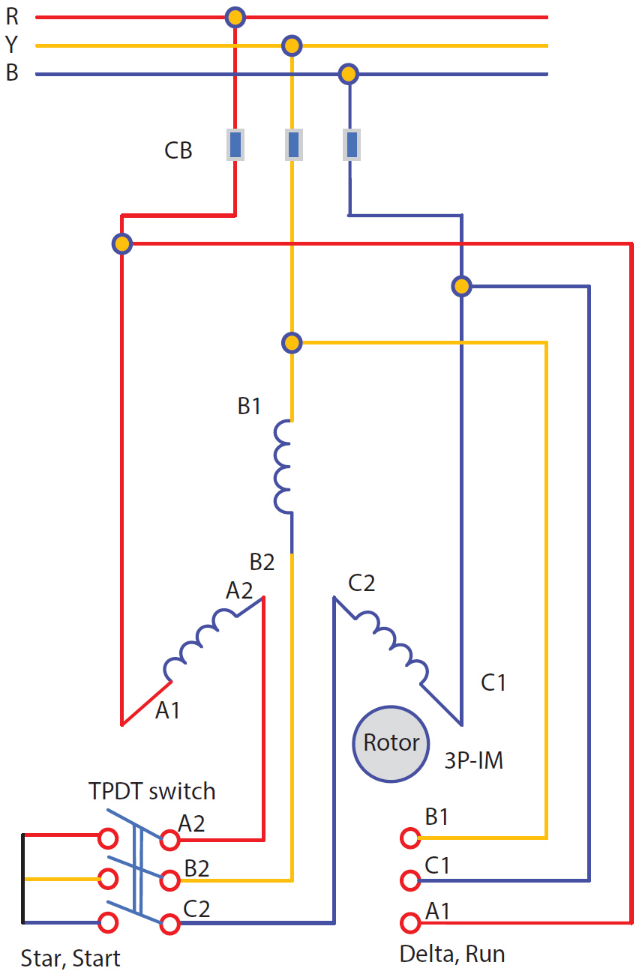

R
Y
B
CB
B1
B2
A2
C2
A1
C1
Rotor
3P-IM
TPDT switch
A2
B2
C2
B1
C1
A1
Star, Start
Delta, Run

Fig. 8.4 Connection diagram of star-delta starter.

Substituting equation (8.24) into equation (8.25) yields,

$$I_{st} = \frac{\frac{V_L}{\sqrt{3}}}{Z} \tag{8.26}$$

$$I_{st} = \frac{1}{\sqrt{3}} \frac{V_L}{Z} \tag{8.27}$$

Substituting equation (8.22) into equation (8.27) yields,

$$I_{st} = \frac{I_{sc}}{\sqrt{3}} \tag{8.28}$$

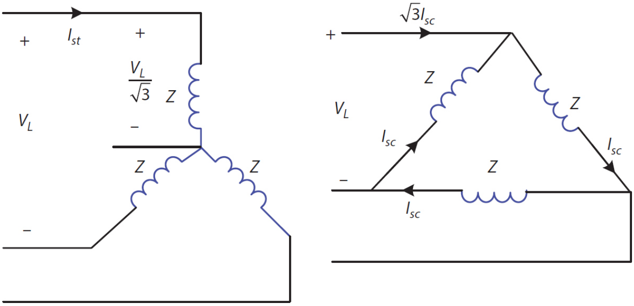


Fig. 8.5 Simple star-delta connection.

The ratio of the star-delta line current to the line current in the delta connection is,

$$\frac{\text{Line current with star-delta starting}}{\text{Line current with delta starting}} = \frac{\frac{I_{sc}}{\sqrt{3}}}{\sqrt{3}\, I_{sc}} \tag{8.29}$$

$$\frac{\text{Line current with star-delta starting}}{\text{Line current with delta starting}} = \frac{1}{3} \tag{8.30}$$

From equation (8.30), it is seen that the line current of the star-delta starting is reduced to one-third of the line current when the stator windings are connected directly with the delta. In this method, the relation between the starting and full load torque becomes,

$$\frac{T_{st}}{T_f} = \left(\frac{\frac{I_{sc}}{\sqrt{3}}}{I_f} \right)^2 s_f \tag{8.31}$$

$$\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f \quad (8.32)$$

Example 8.3

A star-delta starter starts a three-phase induction motor. At the time of starting, it takes 120% of full load line current and develops 25% of full load torque. Determine the full load slip.

Solution

At the time of starting, the line current is,

$$I_{sc} = 3 \times 1.2 \times I_f = 3.6 I_f \quad (8.33)$$

The full load slip can be determined as,

$$\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f \quad (8.34)$$

$$\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{3.6 I_f}{I_f}\right)^2 s_f \quad (8.35)$$

$$0.25 = 4.43 s_f \quad (8.36)$$

$$s_f = 0.06 \quad (8.37)$$

Practice Problem 8.3

The star-delta starter is used to start a three-phase induction motor. The short circuit current of the motor at normal voltage is 4 times the full load current and the full load slip is 3%. Determine the ratio of starting torque to full load torque.

8.6 Simple Magnetic Starter

A simple magnetic starter is used to start a small and medium-sized three-phase induction motor. The main devices used in the starter are a magnetic contactor, a push button on-off switch, a transformer (X-er) and an overload (O/L). The connection diagram of a simple magnetic starter is shown in Fig. 8.6.

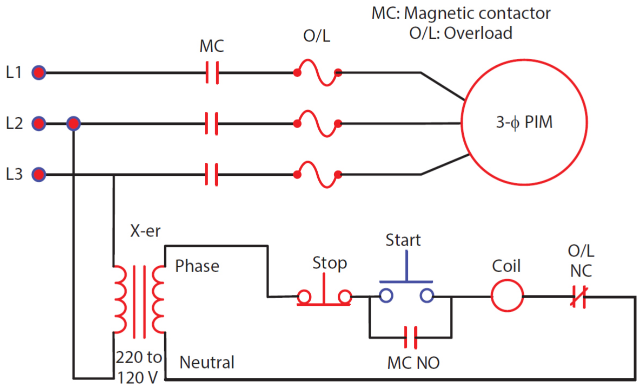


Fig. 8.6 Connection diagram of a simple magnetic starter.

The operations of this starter are as follows:

- When you press the start push button switch, the current will pass through the coil overload normally close contact (NC) and come back to the neutral of the transformer. As it closes the circuit, the magnetic contactor coil will be energized.
- The coil creates the force that closes normally open contacts to close contacts of the magnetic contactor. Because of the close contacts, the motor terminals will receive the voltage through an overload OL.
- At the same time, the auxiliary contact (AC) MC No will be closed. After starting the motor, the current flow will continue through the auxiliary contact, stop switch and an overload (OL) NC contact.
- The motor will run until it de-energizes the magnetic coil due to some reasons.
- If we press the stop push button switch, the current will not flow through the coil and then the magnetic coil will be de-energized. As a result, the main contacts will be opened again which will stop the motor.

- The normally closed contact of the overload will be opened if there is a higher current than the motor-rated current flows through the line. The O/L can not carry the higher value of the current which will cut the line and the motor will be stopped.

A simple magnetic starter to start the three-phase induction motor using Automation Studio software is shown in Fig. 8.7. When the start push button switch is pressed, the current passes through the coil which energizes it. Due to the coil energization, the normally open contacts of the magnetic contactor will be closed and a three-phase supply will be available at the motor terminals. As a result, the motor will be rotating in the clockwise direction as shown in Fig. 8.7.

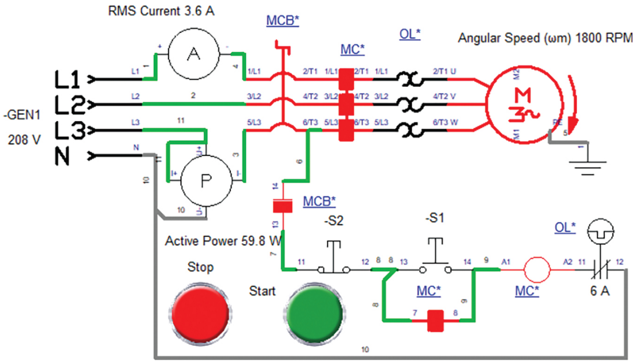

RMS Current 3.6 A
MCB*
MC*
OL*
Angular Speed (ωm) 1800 RPM
-GEN1
208 V
L1
L2
L3
N
A
P
M
3~
Active Power 59.8 W
Stop
Start
MCB*
-S2
-S1
MC*
MC*
OL*
6 A

Fig. 8.7 Coil energized when pressing the start push button switch.

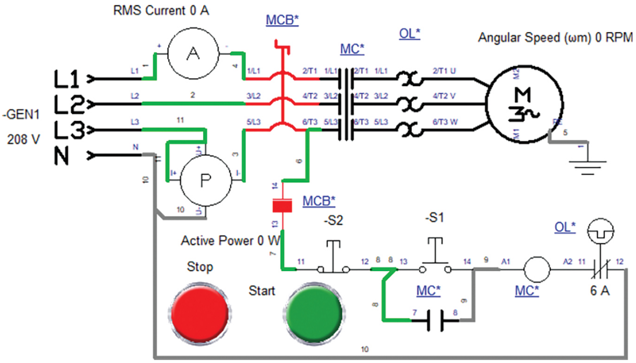

RMS Current 0 A
MCB*
MC*
OL*
Angular Speed (ωm) 0 RPM
-GEN1
208 V
L1
L2
L3
N
A
P
M
3~
Active Power 0 W
Stop
Start
MCB*
-S2
-S1
MC*
MC*
OL*
6 A

Fig. 8.8 Coil de-energized when pressing the stop push button switch.

If the stop push button is pressed, the control circuit will be disconnected. The current will not pass through the coil which de-energizes the coil. As a result, the motor will stop as shown in Fig. 8.8. A simple magnetic starter or direct online (DOL) magnetic stater is shown in Fig. 8.9. In Automation Studio software, click on a "Add new library" icon and add "Renewable Energy" library. Afterwards, drag and drop the necessary devices and wire them up. Select the customized motor by double-clicking on it. In this simulation, a 5 kW, 1800 rpm, 60 Hz three-phase induction motor is chosen. Set the circuit breaker to the ON position and press the start push button switch and the motor will rotate. Press the stop push push button switch to stop the motor from rotating.

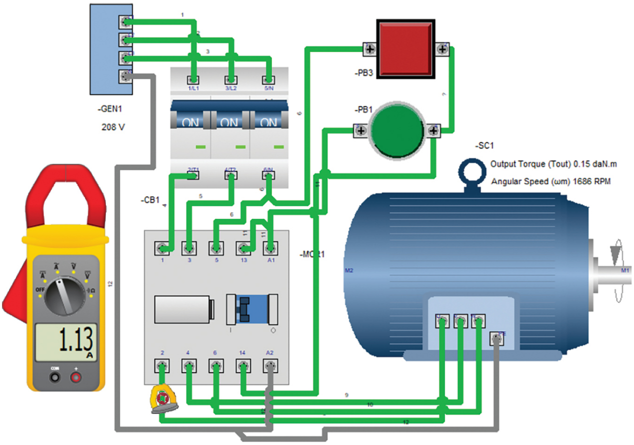

-GEN1
208 V
1
2
3
1/L1
3/L2
5/N
ON
ON
ON
2/T1
4/T2
6/N
-CB1
4
5
6
-PB3
-PB1
7
-SC1
Output Torque (Tout) 0.15 daN.m
Angular Speed (ωm) 1686 RPM
-MC1
1
3
5
13
A1
2
4
6
14
A2
11
12
9
10
M1
M2
OFF
1.13
A
COM

Fig. 8.9 A simple magnetic starter with the real devices.

8.7 Forward-Reverse Starter

This type of starter is normally used in the mechanical workshop to run the lathe machine, in the manufacturing industry, in a programmable logic controller laboratory, etc. The motor is required to run in a forward direction as well as a reverse direction for cutting or smoothing the object. In this starter, two sets of push button, magnetic contactors and stop push button switches are used. The connection diagrams of the forward-reverse starter are shown in Figs. 8.10 and 8.11. The operation steps of the forward-reverse starter are as follows:

- If we press the forward push button switch then the current will flow through the reverse contactor, normally close contact (MC-R NC), which energizes the forward contactor coil MC-F. After that, its three main contacts and normally open contacts will be closed.
- When the start push button is released, the closed MC-F NO will maintain the current flow to the coil MC-F Coil, thus holding or latching the "Forward" circuit in the ON state.
- The motor terminals will get the supply voltage through the overload. Finally, the motor will run in the forward (clockwise) direction.
- If we press the stop push button switch then current will not flow through the forward contactor coil. As a result, the forward coil will be de-energized and it will release the main contacts. Ultimately, the motor will be stopped.

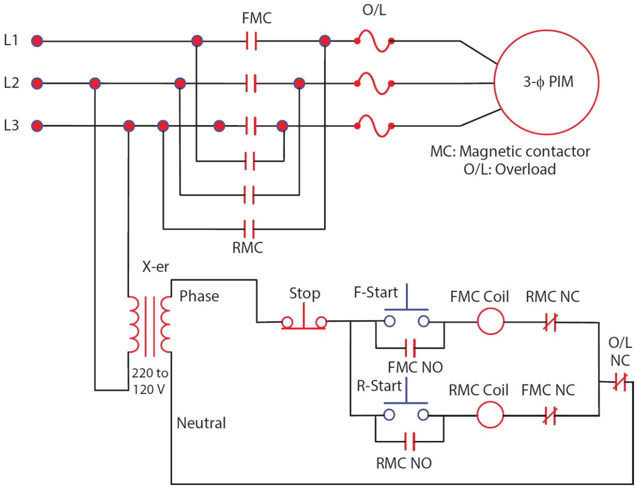

FMC
O/L
L1
L2
L3
3-ϕ PIM
MC: Magnetic contactor
O/L: Overload
RMC
X-er
Phase
Stop
F-Start
FMC Coil
RMC NC
O/L
NC
FMC NO
R-Start
RMC Coil
FMC NC
220 to
120 V
Neutral
RMC NO

Fig. 8.10 Connection diagram of a forward-reverse starter.

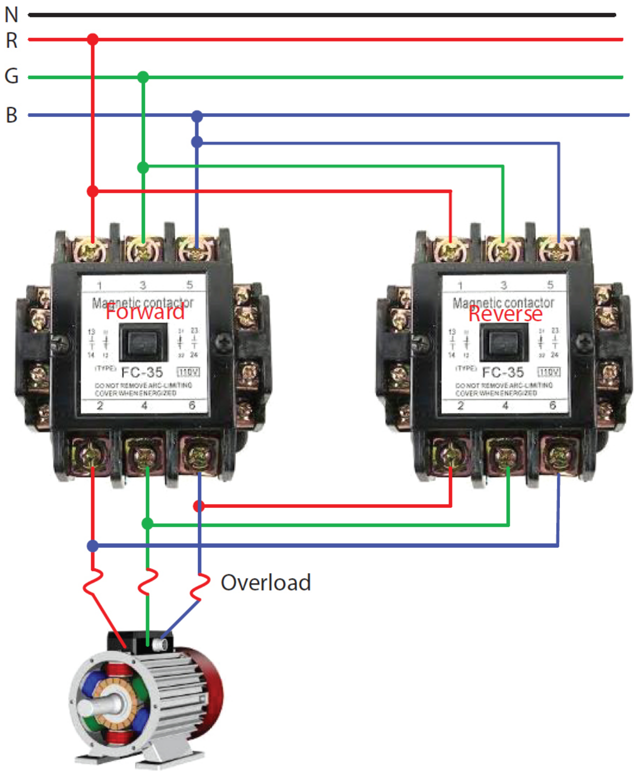

N
R
G
B
1
3
5
Magnetic contactor
Forward
FC-35
110V
DO NOT REMOVE ARC-LIMITING COVER WHEN ENERGIZED
2
4
6
Magnetic contactor
Reverse
FC-35
110V
Overload

Fig. 8.11 Connection diagram of a forward-reverse starter.

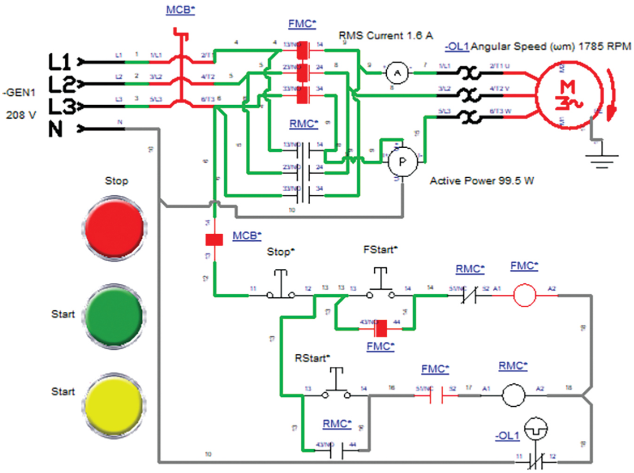

MCB*
FMC*
RMS Current 1.6 A
-OL1
Angular Speed (ωm) 1785 RPM
L1
L2
L3
N
-GEN1
208 V
RMC*
Active Power 99.5 W
Stop
Start
Start
MCB*
Stop*
FStart*
RMC*
FMC*
FMC*
RStart*
FMC*
RMC*
RMC*
-OL1

Fig. 8.12 Connection diagram of a forward starting.

- To start the motor in the reverse (counterclockwise) direction, we need to press the start push button switch. Then current will flow through the reverse contactor, normally close contact (MC-F NC), which energizes the reverse contactor coil MC-R. After that, its three main contacts and auxiliary contacts will be closed.

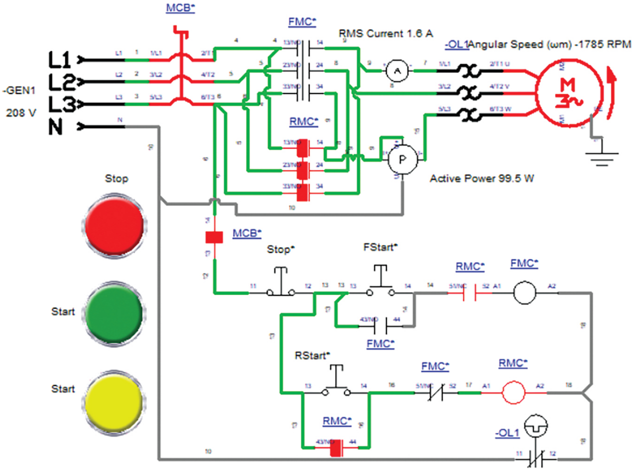

MCB
FMC
RMS Current 1.6 A
-OL1
Angular Speed (ωm) -1785 RPM
L1
L2
L3
N
-GEN1
208 V
M
3~
A
P
RMC
Active Power 99.5 W
Stop
Start
Start
MCB
Stop
FStart
RMC
FMC
FMC
RStart
FMC
RMC
RMC
-OL1

Fig. 8.13 Connection diagram of a reverse starting.

- The motor terminals will get the supply through the overload. Finally, the motor will run in the reverse (counterclockwise) direction. If we press the stop push button switch then the current will not flow through the reverse contactor coil. As a result, the reverse coil will be de-energized and it will release the main contacts. Hence, the motor will be stopped.

When the green push button switch is pressed, the forward coil is energized and the motor runs in the forward direction as shown in Fig. 8.12. Similarly, when the yellow push button switch is pressed, the reverse coil is energized and the motor runs in the reverse direction as shown in Fig. 8.13.

8.8 Cogging and Crawling

Cogging and crawling are the two important terms that need to be addressed. Cogging is often known as magnetic or teeth locking.

This magnetic locking happens when the number of stator teeth is equal to the root teeth and the motor refuses to start. At low supply voltage, the rotor of the squirrel cage induction motor refuses to start. In addition to that, the value of the reluctance of the magnetic path is much less when the stator teeth lock with the rotor teeth. Due to cogging, the motor may be overheated which might affect the performance of motor. The cogging of a squirrel cage induction motor can be overcome by making the rotor and the stator slots unequal.

Crawling

Sometimes, a squirrel cage induction motor shows a tendency to run stably at speeds as low as one-seventh of its synchronous speed. This phenomenon is known as the crawling of an induction motor. When the stator is energized by a three-phase supply, it does not create a pure sine wave flux. Instead, it creates a complex wave that consists of fundamental and odd harmonics such as 3rd, 5th, and 7th, etc. However, the fundamental flux or magnetic field can rotate at synchronous speed whereas the 3rd, 5th, and 7th harmonics flux may rotate in the forward and reverse directions at a speed of $N_S/3$, $N_S/5$ and $N_S/7$, respectively.

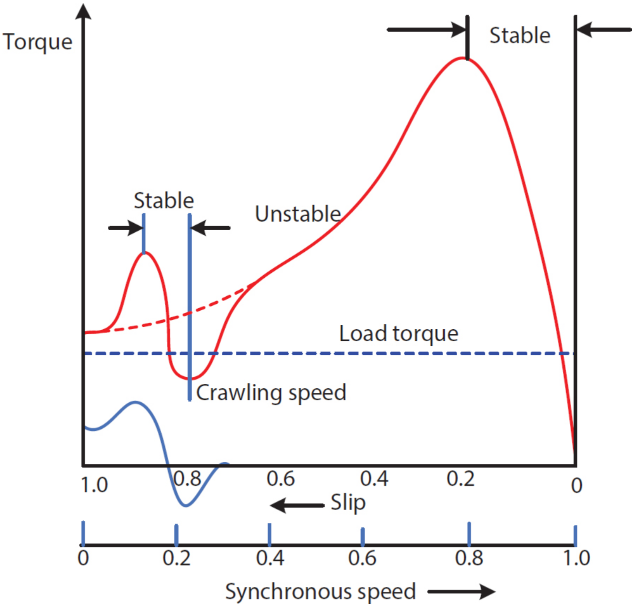


Fig. 8.14 Harmonic effect on torque.

In a balanced three-phase supply, the 3rd harmonics do not appear and it can be neglected. These harmonics generate rotor emf, current and torque same as the fundamental with the synchronous speeds $N_S/5$ (backward) and $N_S/7$ (Forward). The phase difference of the 5th harmonic current is calculated as,

$$5^{th}\,\text{harmonic} = 5\times 120^\circ = 600^\circ = 2\times 360^\circ - 120^\circ = -120^\circ \tag{8.38}$$

Due to a negative sign, the rotor will rotate in the reverse direction (Backward) at a speed of $N_S/5$.

Similarly, the phase difference of the 7th harmonic current is calculated as,

$$7^{th}\,\text{harmonic} = 7\times 120^\circ = 840^\circ = 2\times 360^\circ + 120^\circ = 120^\circ \tag{8.39}$$

Due to a positive sign, the rotor will rotate in the forward direction (Forward) at a speed of $N_S/7$.

The resultant torque-speed characteristics combined with fundamental and 7^{th} harmonics are shown in Fig. 8.14. The torque due to harmonic fluxes causes a dip or slope to the torque-speed characteristics curve. When a dip or slope occurs due to the 7th harmonic, the motor cannot accelerate to its full speed. Instead, starts crawling at the speed which refers to the dip point speed.

8.9 Importance of Deep Bar and Double Cage Rotor

The starting torque of a three-phase squirrel cage-type rotor is a concern when there is a demand to have a high starting torque. The starting torque of this type of motor during starting is usually low because of the lower rotor resistance. The rotor resistance can be increased by using a deep bar with a high-resistivity material. However, the higher value of the rotor resistance will increase ohmic loss, which decreases the performance of the motor. To maintain the same performance of the motor, the resistance of the rotor circuit is made higher at the time of starting and lower during running conditions. In the wound-type rotor starting, this condition is met by connecting an external resistance during starting and it is cut out in steps when the motor speed increases towards the full load speed. However, a in squirrel cage rotor, the rotor conductors are shorted by the two end-rings. The deep bar rotor and double cage rotor are used to achieve higher resistance during starting and lower resistance during running. Consider the rotor bar consisting of three narrow layers which are connected in parallel as shown in Fig. 8.15.

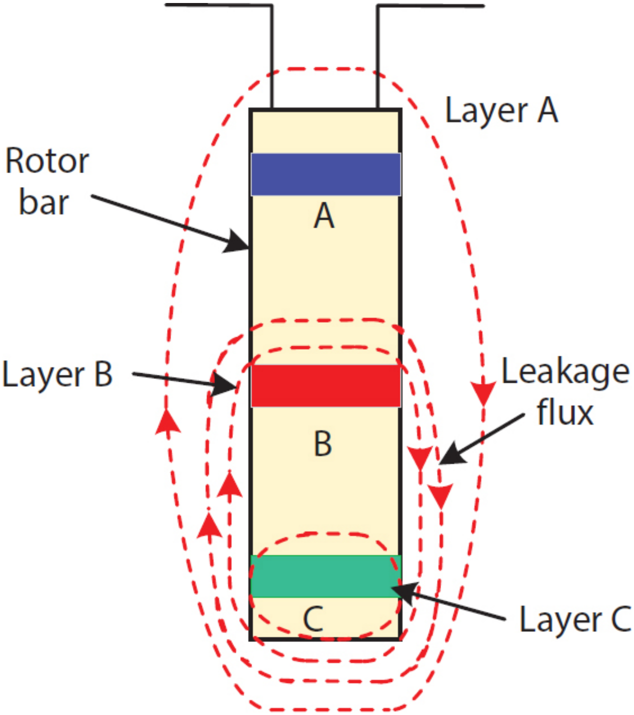


Fig. 8.15 Deep rotor bar with the parallel layers.

From Fig. 8.15, it is seen that layer A is placed on the top, B is placed at the middle and layer C is placed at the bottom. The rotor frequency is equal to the stator frequency during starting. The flux distributions are shown in the same Fig. 8.16.

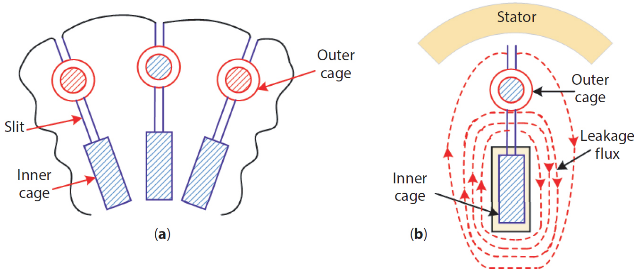


Fig. 8.16 Deep cage rotor bar with the distribution of leakage flux.

From Fig. 8.16(b), it is seen that the top layer A is linked with a minimum leakage flux so that it has a lower leakage inductance, whereas the layer C is placed at the bottom of the rotor bar which is linked with the highest leakage flux. As a result, the layer C has a large leakage inductance. Therefore, the maximum current will flow to the top layer A and the minimum current will flow to the bottom layer C. This effect is known as the skin effect. This unequal current flow will contribute to the rotor resistance to increase. During starting with a higher rotor resistance, the torque will be large and the current will be less. As the motor picks up to its normal speed, the frequency of the rotor current decreases to a steady state value. As a result, the skin effect diminishes and the current is uniformly distributed over the cross-section of the conductor. Finally, the rotor resistance will decrease gradually and become zero when the operates at the full load speed.

Double Cage Motor

A double cage induction motor’s rotor consists of two cages, the outer cage and the inner cage. In the same slot, the outer cage is placed on top of the inner cage as shown in Fig. 8.17. The outer cage bar has a lower cross-sectional area than the inner cage bar and it is usually made of brass, aluminum, bronze, etc., which has high resistance and lower reactance. The inner cage is deeply fixed inside the rotor and it is made of copper material. It has a higher cross-sectional area which provides lower resistance and higher reactance.

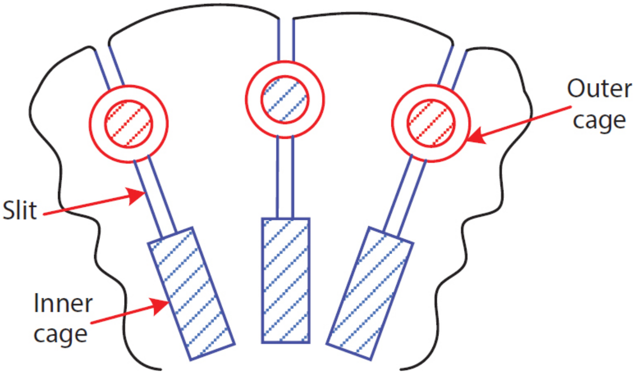


Fig. 8.17 Part of double squirrel cage rotor.

A slit is provided between the upper cage and lower cage conductors to provide high reluctance to the stator field. Therefore, the stator field, instead of linking the outer cage, links both the upper age and lower cage simultaneously.

When the motor is connected across the power supply, the frequency of the rotor circuit is high and is equal to the supply frequency. As a result, the inductance of the inner cage will be higher than the upper cage. The inner cage rotor bar will also provide higher impedance. Due to these circumstances, the upper cage rotor bar receives the majority of the current flow. This condition makes most of the current flow in the upper cage rotor bar. Therefore, the motor will develop a higher starting torque with a higher rotor resistance in the upper cage rotor bar.

8.10 Double Squirrel Cage Motor

A simple squirrel cage induction motor is more efficient than the wound type or slip ring induction motor. However, it has poor starting torque due to its small rotor resistance. To overcome this drawback, a double squirrel cage motor is designed to provide high starting torque with a low starting current. This motor is designed in such a way that the motor runs with a high rotor resistance during starting and a low rotor resistance during running time. In a double squirrel cage motor, the outer slot winding contains high resistance and low reactance. The inner slot of this motor contains low resistance and high reactance. A part of a double squirrel cage motor is shown in Fig. 8.18. The parameters of the two slots are normally connected in parallel. The following parameters are defined as,

R_{ocr} is the per-phase rotor resistance of the outer cage,

X_{ocr} is the per-phase rotor reactance of the outer cage,

R_{icr} is the per-phase rotor resistance of the inner cage,

X_{icr} is the per-phase rotor reactance of the inner cage,

I_{ocr} is the current that flows through the outer cage,

I_{icr} is the current that flows through the inner cage.

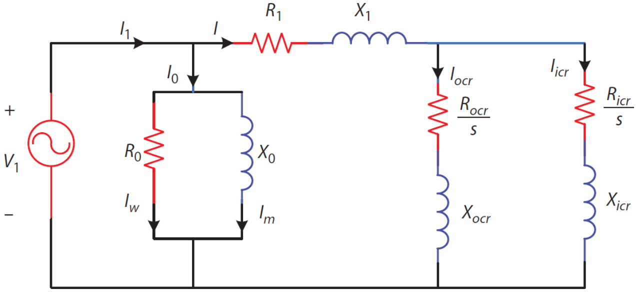


Fig. 8.18 Equivalent circuit with a no-load branch.

The equivalent circuit with a no-load branch of the double squirrel cage induction motor is shown in Fig. 8.19. Here, the no-load circuit is placed across the supply voltage and the turns ratio is considered as 1:1. At standstill condition (s =1), the combined impedance of the two cages is,

$$Z_{com} = \frac{Z_{ocr} Z_{icr}}{Z_{ocr} + Z_{icr}} \tag{8.40}$$

$$Z_{com} = \frac{(R_{ocr} + jX_{ocr})(R_{icr} + jX_{icr})}{R_{ocr} + jX_{ocr} + R_{icr} + jX_{icr}} \tag{8.41}$$

At any value of the slip, the combined impedance of the two cages is,

$$Z_{com-s}=\frac{\left(\frac{R_{ocr}}{s}+jX_{ocr}\right)\left(\frac{R_{icr}}{s}+jX_{icr}\right)}{\frac{R_{ocr}}{s}+jX_{ocr}+\frac{R_{icr}}{s}+jX_{icr}} \tag{8.42}$$

The equivalent circuit without the no-load circuit is drawn as shown in Fig. 8.18. The total impedance at standstill condition is written as,

$$Z_t=R_1+jX_1+Z_{com} \tag{8.43}$$

The total impedance at any slip is written as,

$$Z_{t-s}=R_1+jX_1+Z_{com-s} \tag{8.44}$$

At a standstill, the power input of the inner cage and outer cage are,

$$P_{icr}=I_{icr}{}^2R_{icr} \tag{8.45}$$

$$P_{ocr}=I_{ocr}{}^2R_{ocr} \tag{8.46}$$

At any slip, the power input of the inner cage and outer cage are,

$$P_{icr}=I_{icr}{}^2\frac{R_{icr}}{s} \tag{8.47}$$

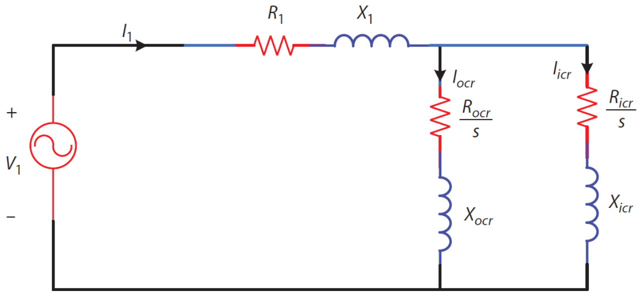


Fig. 8.19 Equivalent circuit without no-load branch.

$$P_{ocr} = I_{ocr}{}^{2} \frac{R_{ocr}}{s} \tag{8.48}$$

The ratio of the two torque of cages is expressed as,

$$\frac{T_{ocr}}{T_{icr}} = \frac{P_{ocr}}{P_{icr}} \tag{8.49}$$

Example 8.4

The inner cage and the outer cage impedance per phase at standstill are 0.04 + j0.5 Ω and 0.4 + j0.2 Ω, respectively. The per phase rotor induced voltage at standstill is 120 V. Determine the torques in synchronous watts at standstill and 6% slip.

Solution

At a standstill, the combined impedance is calculated as,

$$Z_{com} = \frac{(0.04 + j0.5)(0.4 + j0.2)}{0.04 + j0.5 + 0.4 + j0.2} = 0.16 + j0.22 = 0.27\angle 53.97^\circ\,\Omega \tag{8.50}$$

The value of the rotor current per phase is determined as,

$$I_2 = \frac{120}{0.27} = 444.44\text{ A} \tag{8.51}$$

The value of the combined rotor resistance is,

$$R_2 = 0.16\Omega \tag{8.52}$$

The torque at standstill condition is calculated as,

$$T = I_2{}^2 R_2 = 444.44^2 \times 0.16 = 31604.31\text{syn.watt} \tag{8.53}$$

At 6% slip, the combined impedance is calculated as,

$$Z_{com-6} = \frac{\left(\frac{0.04}{0.06} + j0.5\right)\left(\frac{0.4}{0.06} + j0.2\right)}{\frac{0.04}{0.06} + j0.5 + \frac{0.4}{0.06} + j0.2} \tag{8.54}$$

$$Z_{com-6} = \frac{(0.67 + j0.5)(6.67 + j0.2)}{0.67 + j0.5 + 6.67 + j0.2} = 0.63 + j0.41 = 0.75\angle 33.06^\circ\,\Omega \tag{8.55}$$

The value of the rotor current per phase is determined as,

$$I_2 = \frac{120}{0.75} = 160\,\text{A} \tag{8.56}$$

The value of the combined rotor resistance is,

$$R_2 = 0.63\,\Omega \tag{8.57}$$

The torque at 6% slip can be determined as,

$$T_6 = I_2{}^2 R_2 = 160^2 \times 0.63 = 16128\,\text{syn.watt}$$

Practice Problem 8.4

The inner cage and the outer cage impedance per phase at standstill are 0.05+ *j*0.6Ω and 0.5+ *j*0.3Ω respectively. The per phase rotor induced voltage at standstill is 100 V. Find the torques in synchronous watts at standstill and 5% slip.

8.11 Determination of Equivalent Circuit Parameters

The related equivalent parameters of a three-phase induction motor can be determined by the tests named as blocked rotor test and the no-load test.

8.11.1 Blocked Rotor Test

This test is very similar to the short circuit test of a single-phase transformer. In this test, the rotor is held fixed by applying external torque to its shaft. Under locked conditions, the rotor is not rotating and the stator current is almost six times the rated current. Since the rotor is not rotating, the slip is equal to 1 and the rotor resistance R_2/s is equal to R_2. Therefore, the no-load circuit resistance and reactance are considered as $R_0 >> (R_2+jX_2)$ and $X_0 >> (R_2+jX_2)$, respectively. Most of the current will flow through R_2 and X_2 instead of the no-load circuit. Therefore, the no-load circuit can be neglected and the final circuit looks like a series circuit as shown in Fig. 8.20.

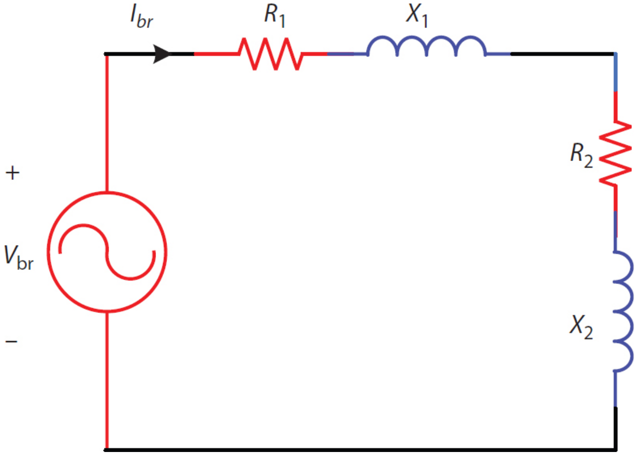


Fig. 8.20 Equivalent circuit under block rotor test.

The three-phase variable voltage is applied to the stator winding and it increases slowly until the motor picks up its rated current. By appropriate measuring instruments, we can measure the input voltage, line current and power. The typical connection diagram is shown in Fig. 8.21.

Under block rotor conditions, consider the per-phase rated voltage, rated current and power are V_{br}, I_{br} and P_{br}, respectively. Then, the equivalent resistance is calculated as,

$$R_{eq} = \frac{P_{br}}{I_{br}{}^2} \quad (8.58)$$

The per-phase rotor resistance is calculated as,

$$R_2 = R_{eq} - R_1 \tag{8.59}$$

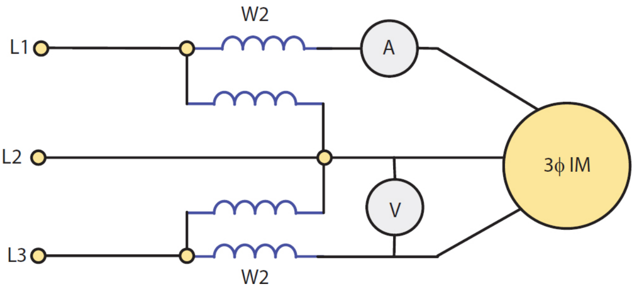


Fig. 8.21 Connection diagram of block rotor test.

To determine the stator resistance, consider the circuit as shown in Fig. 8.22. Let us consider R to be the DC value of the resistance between any two terminals of the motor.

For wye connection

The equivalent resistance between two terminals is calculated as,

$$R_1 + R_1 = R \tag{8.60}$$

$$R_1 = 0.5R \tag{8.61}$$

For delta connection

The equivalent resistance between two terminals is determined as,

$$\frac{(R_1 + R_1)R_1}{3R_1} = R \tag{8.62}$$

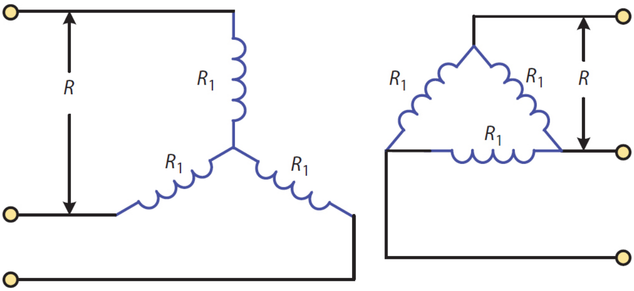


Fig. 8.22 Connection diagram of a star-delta circuit.

$$R_1 = 1.5R \tag{8.63}$$

The equivalent impedance is calculated as,

$$Z_{eq} = \frac{V_{br}}{I_{br}} \tag{8.64}$$

The equivalent reactance is determined as,

$$X_{eq} = \sqrt{{Z_{eq}}^2 - {R_{eq}}^2} \tag{8.65}$$

For practical cases, the reactances are usually calculated based on,

$$X_1 = X_2 = 0.5X_{eq} \tag{8.66}$$

Example 8.5

The blocked rotor test data of a three-phase 210 V, 50 Hz, 4-pole Y-connected induction motor is as follows: Input power 50 W, line current 2.5 A and line voltage 25 V. The DC resistance between any two terminals is 2.4 Ω. Determine the equivalent parameters.

Solution

The phase voltage is calculated as,

$$V_{br} = \frac{25}{\sqrt{3}} = 14.4\,\text{V} \tag{8.67}$$

The power per phase is determined as,

$$P_{br} = \frac{50}{3} = 16.7\,\text{W} \tag{8.68}$$

The value of the equivalent resistance is calculated as,

$$R_{eq} = \frac{P_{br}}{{I_{br}}^2} = \frac{16.7}{2.5^2} = 2.7\,\Omega \tag{8.69}$$

$$R_2 = R_{eq} - R_1 = 2.7 - \frac{2.4}{2} = 1.5\Omega \quad (8.70)$$

The equivalent impedance is calculated as,

$$Z_{eq} = \frac{V_{br}}{I_{br}} = \frac{14.4}{2.5} = 5.8\Omega \quad (8.71)$$

The equivalent reactance is determined as,

$$X_{eq} = \sqrt{5.8^2 - 1.5^2} = 5.6\Omega \quad (8.72)$$

$$X_1 = X_2 = 0.5X_{eq} = 0.5 \times 5.6 = 2.8\Omega \quad (8.73)$$

Practice Problem 8.5

The blocked rotor test data of a three-phase 215 V, 50 Hz, 4-pole delta-connected induction motor is as follows: Input power 90 W, line current 2.1 A and line voltage 30 V.

The DC resistance between any two terminals is 2.2 Ω. Find the equivalent parameters.

8.11.2 No-Load Test

This test is similar to an open circuit test of a single-phase transformer. The balanced voltage is applied to the stator terminals and the motor operates at no-load. The only load on the motor is the friction and windage loss, Pf_W. At no-load conditions, the slip of the motor is very small and the per-phase rotor impedance is almost infinite. Under the no-load condition, a per-phase equivalent circuit without a rotor circuit is shown in Fig. 8.23. The measured input power, voltage and rated current are P_0, V_0 and I_0 respectively. Due to friction and windage loss, the core power becomes,

$$P_{core} = P_0 - P_{fw} \quad (8.74)$$

Neglect the stator impedance and the no-load circuit resistance per phase is determined as,

$$R_0 = \frac{V_0^{\ 2}}{\frac{P_{core}}{3}} \tag{8.75}$$

where V_0 is the phase voltage?

The power factor at no load condition is determined as,

$$\cos\phi_0 = \frac{P_{core}}{\sqrt{3}\, V_0 I_0} \tag{8.76}$$

The power factor angle is calculated as,

$$\phi_0 = \cos^{-1} \frac{P_{core}}{\sqrt{3}\, V_0 I_0} \tag{8.77}$$

The magnetization reactance per phase is determined as,

$$X_0 = \frac{V_0}{I_0 \sin\phi_0} \tag{8.78}$$

The no-load circuit resistance per phase is calculated as,

$$R_0 = \frac{V_0}{I_0 \cos\phi_0} \tag{8.79}$$

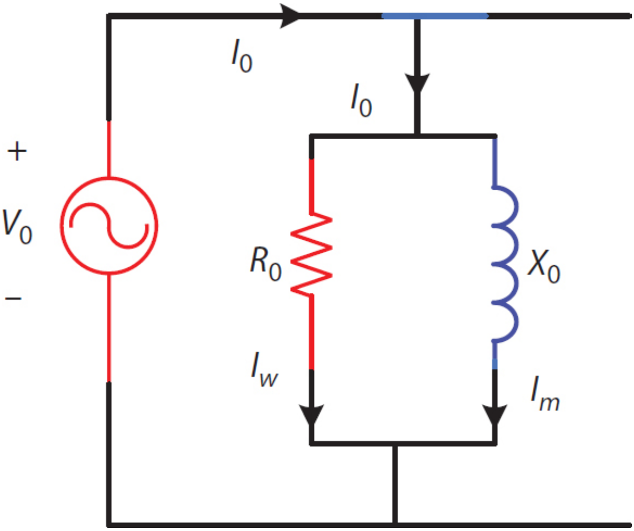


Fig. 8.23 No-load equivalent circuit.

The magnetization reactance or no-load circuit reactance is also calculated in an alternative way as,

$$Z_0 = \frac{V_0}{I_0} \tag{8.80}$$

$$X_0 = \sqrt{Z_0^{\ 2} - R_0^{\ 2}} \tag{8.81}$$

Example 8.6

The no-load test data of a three-phase, 4-pole, 210 V, 50 Hz, Y-connected induction motor are as follows:

No load test: power input 400 W, line current 1.2 A, line voltage 210 V.

The total windage and friction losses are 150 W. The stator resistance between any two terminals is 2.2 Ω. Find the equivalent circuit parameters.

Solution

The per-phase stator resistance is calculated as,

$$R_1 = \frac{2.2}{2} = 1.1\,\Omega \quad (8.82)$$

The stator copper loss is calculated as,

$$P_{scu} = 3I_0^{\ 2}R_1 = 3\times 1.2^2 \times 1.1 = 4.8\,\text{W} \quad (8.83)$$

The stator core loss is determined as,

$$P_{core} = 400 - 150 - 4.8 = 245.2\,\text{W} \quad (8.84)$$

The value of no-load circuit resistance is calculated as,

$$R_0 = \frac{V_0^{\ 2}}{\frac{P_{core}}{3}} = 3\frac{\left(\frac{210}{\sqrt{3}}\right)^2}{245.2} = 179.8\,\Omega \quad (8.85)$$

Alternate approach: The power factor angle is calculated as,

$$\phi_0 = \cos^{-1}\frac{P_{core}}{\sqrt{3}\,V_0 I_0} = \cos^{-1}\frac{245.2}{\sqrt{3}\times 210\times 1.2} = 55.8^\circ \quad (8.86)$$

The value of the no-load circuit resistance per phase is calculated as,

$$R_0 = \frac{V_0}{I_0\cos\phi_0} = \frac{\frac{210}{\sqrt{3}}}{1.2\times 0.56} = 180.4\,\Omega \quad (8.87)$$

The magnetization reactance per phase is calculated as,

$$X_0 = \frac{V_0}{I_0\sin\phi_0} = \frac{\frac{210}{\sqrt{3}}}{1.2\times \sin 55.8} = 122\,\Omega \quad (8.88)$$

Practice Problem 8.6

The no-load test data of a three-phase, 4-pole, 50 Hz, delta-connected induction motor are as follows:

No-load test: power input 425 W, line current 1.5 A, line voltage 208 V.

The total windage and friction losses are 120 W. The stator resistance between any two terminals is 2.4 Ω. Calculate the equivalent circuit parameters.

8.12 Speed Control of Induction Motor

The speed control option of a three-phase induction motor is adopted when the mechanical machine needs variable speed. In chapter 7, we have already derived the expression of rotor speed in terms of slip, frequency and number of poles. The expression of rotor speed is,

$$N_l = \frac{120f}{P}(1-s) \tag{8.89}$$

From equation (8.89), it is observed that the speed can be controlled with the variation of the frequency, slip and number of poles.

8.12.1 Frequency Control

The speed of the induction motor can be increased by increasing the frequency and can be decreased by decreasing the frequency of the supply voltage. The air gap field of the induction motor is constant and it rotates at a synchronous speed. The frequency of the power system is normally constant. Therefore, a special arrangement is necessary to get the variable frequency. Initially, an AC voltage is converted into DC output by using an SCR rectifier. Later on, this DC output is fed to the static inverter, incorporating the solid-state devices, which in turn delivers the variable frequency to the motor. This method provides a wide control of the speed of the induction motor. The simple arrangement to get the variable frequency is shown in Fig. 8.24.

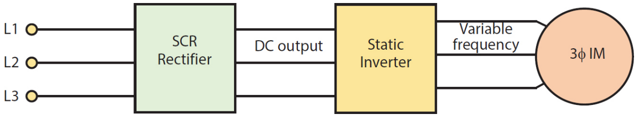


Fig. 8.24 Schematic for frequency control.

8.12.2 Pole Changing

The induction motor is normally wound with respect to its slots per pole per phase. Therefore, the poles of an induction motor are created by a suitable arrangement of the stator conductors. This method of speed control is suitable for squirrel cage induction motors rather than wound rotor motors because the squirrel cage motor automatically assumes the same number of poles as the stator. In this method, the stator conductors are connected in series and/or parallel to increase or decrease the speed.

8.12.3 Cascade Arrangements

In this method of speed control, two induction motors are used. One is a squirrel cage-type rotor and the other is a wound-type rotor. The stator windings of a wound-type rotor are connected to the three-phase supply and its rotor is connected to the stator windings of the squirrel cage motor as shown in Fig. 8.25. The revolving magnetic field produced by both motors is in such a way that they are in the same direction. In this case, the expression of combined synchronous speed is,

$$N_{cs} = \frac{120f}{P_1 + P_2} \tag{8.90}$$

where,

P_1 is the number of poles of the motor 1,

P_2 is the number of poles of the motor 2.

The synchronous speed of the motor 1 is,

$$N_{s1} = \frac{120f}{P_1} \tag{8.91}$$

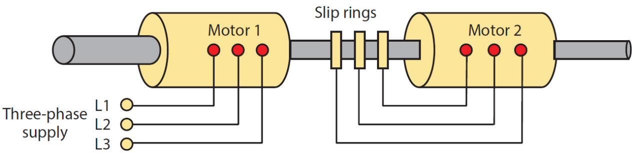


Fig. 8.25 Schematic of cascade connection.

The speed of the motor 1 at any load is,

$$N_{l1} = N_{s1}(1-s) \tag{8.92}$$

Substituting equation (8.91) into the equation (8.92) yields,

$$N_{l1} = (1-s)\frac{120f}{P_1} \tag{8.93}$$

The frequency of the second motor is,

$$f_2 = sf \tag{8.94}$$

The synchronous speed of the motor 2 is,

$$N_{s2} = \frac{120f_2}{P_2} \tag{8.95}$$

$$N_{s2} = \frac{120sf}{P_2} \tag{8.96}$$

The speed of motor 2 at any load is,

$$N_{l2} = N_{s2}(1-s_2) \tag{8.97}$$

Substituting equation (8.96) into equation (8.97) yields,

$$N_{l2} = (1-s_2)\frac{120sf}{P_2} \tag{8.98}$$

Here, in cascade arrangements, both motors are coupled. Therefore, the speed of both motors at any load must be the same. This relation becomes,

$$(1-s)\frac{120f}{P_1}=(1-s_2)\frac{120sf}{P_2} \tag{8.99}$$

$$(1-s)\frac{1}{P_1}=(1-s_2)\frac{s}{P_2} \tag{8.100}$$

$$P_2-sP_2=sP_1-sP_1s_2 \tag{8.101}$$

The product ss_2 is very small and it can be neglected. Equation (8.101) becomes,

$$P_2-sP_2=sP_1 \tag{8.102}$$

$$s=\frac{P_2}{P_1+P_2} \tag{8.103}$$

The speed of the motor at any load can be expressed as,

$$N_{l1}=N_{cs}=(1-\frac{P_2}{P_1+P_2})\frac{120f}{P_1} \tag{8.104}$$

$$N_{l1}=N_{cs}=\frac{P_1}{P_1+P_2}\times\frac{120f}{P_1} \tag{8.105}$$

$$N_{l1}=N_{cs}=\frac{120f}{P_1+P_2} \tag{8.106}$$

Example 8.7

A 6-pole 60 kW motor is connected in cascade with a 4-pole motor. The 6-pole motor is energized by a 50 Hz supple. Calculate the combined slip, combined synchronous speed, and output of the 4-pole motor. Neglect all kinds of losses.

Solution

The combined slip is determined as,

$$s = \frac{P_2}{P_1 + P_2} = \frac{4}{6+4} = 0.4 \quad (8.107)$$

The combined synchronous speed is calculated as,

$$N_{cs} = \frac{120f}{P_1 + P_2} = \frac{120 \times 50}{6+4} = 600\,\text{rpm} \quad (8.108)$$

The output of the 4-pole motor is calculated as,

$$P_0 = 60 \times \frac{4}{10} = 24\,\text{kW}$$

Practice Problem 8.7

A 4-pole motor is connected in cascade with an 8-pole motor. The 4-pole motor is energized by a 60 Hz supple. Find the combined slip, combined synchronous speed and output of the 8-pole motor if the input of the 4-pole motor is 50 kW. Neglect all kinds of losses.

8.13 Automatic Star-Delta Starter

A wiring diagram for an automatic star-delta starter is shown in Fig. 8.26. The control circuit of the star-delta starter is also shown in Fig. 8.27. When the start push button switch is ON, it energizes the main contactor coil, star contactor coil and timer coil.

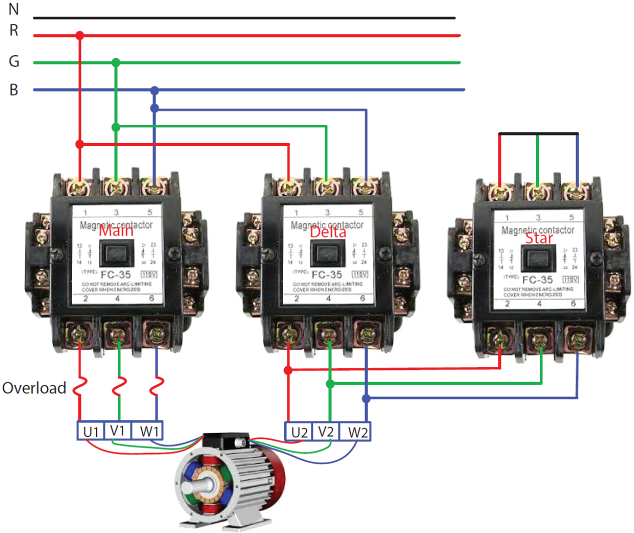

N
R
G
B
Magnetic contactor
Main
Magnetic contactor
Delta
Magnetic contactor
Star
FC-35
Overload
U1
V1
W1
U2
V2
W2

Fig. 8.26 Main wiring diagram of a star-delta starter.

As a result, the main and star magnetic contactor changes its position from NO to NC and the motor runs as a star. The auxiliary contact of the main contactor that is connected in parallel with the start push button switch will become NO to NC. The current will continue to flow through this path and activate or energize the main magnetic contactor coil and star contactor coil until the preset time of the timer is reached. After the preset timing of the timer, the auxiliary contact of the timer in the star coil circuit will change the position from NC to NO. At the same time, timer NO contact in the delta coil circuit will change the position to NC. The main contacts of the delta magnetic contactor will be closed and the motor will run as a delta or its full load speed. The stop push button switch is used to stop the motor from running when necessary. An automation studio software simulation of a star-delta starter is shown in Fig. 8.28.

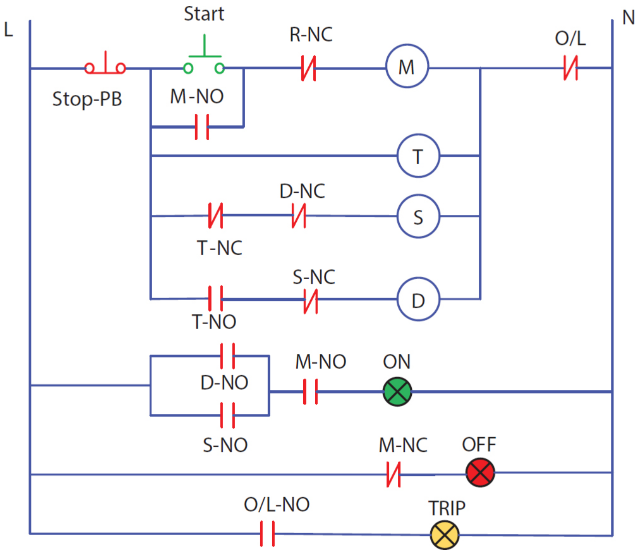

L
Start
N
R-NC
O/L
M
Stop-PB
M-NO
T
D-NC
S
T-NC
S-NC
D
T-NO
M-NO
ON
D-NO
S-NO
M-NC
OFF
O/L-NO
TRIP

Fig. 8.27 Main wiring diagram of a star-delta starter.

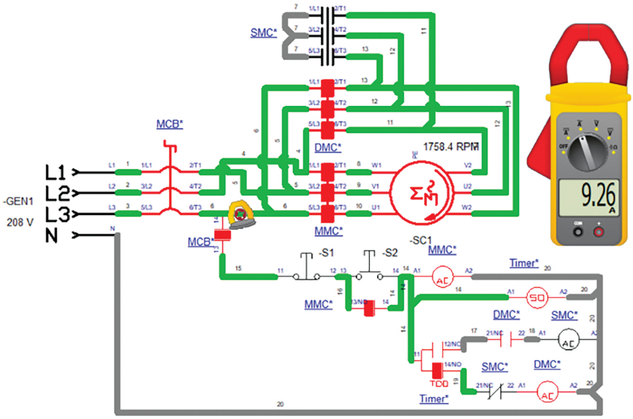

SMC*
MCB*
DMC*
1758.4 RPM
L1
L2
L3
N
-GEN1
208 V
MMC*
MCB*
-S1
-S2
-SC1
MMC*
Timer*
MMC*
DMC*
SMC*
SMC*
DMC*
Timer*
9.26
A

Fig. 8.28 Simulation circuit by an automation studio software.

Exercise Problems

8.1 The starting torque of a three-phase induction motor is 1.2 times the full load torque while running directly from the supply. Determine the value of the short circuit current in terms of the full load current if the full load slip is 5%.

8.2 The full load slip of a three-phase induction motor is 6% and it is started from the supply directly. The ratio of the starting or short circuit current to the full load current is 0.9. Determine the starting torque in terms of the full load torque.

8.3 The ratio of the starting torque to the full load torque is equal to the square root of the ratio of the short circuit current to the full load current. The motor is connected directly to the supply. Determine the full load slip if the short circuit current is 2 times the full load current.

8.4 The squirrel cage induction motor is connected directly from the supply and its full load slip is 6%. The starting torque of the motor is 60% of the full load torque. Calculate the suitable autotransformer tappings if the line current does not exceed 5.5 times the full load current.

8.5 The starting torque of an autotransformer is 80% of full load torque. The full load slip is5 % and the short circuit current in normal voltage is 6.5 times the full load current. Determine the suitable percentage tapings that are required on the autotransformer.

8.6 A squirrel cage induction motor is started by an autotransformer against 65% of full load torque. The full load slip is 3% and the percentage tappings on the autotransformer is 80%. Determine the ratio of the short circuit current to the full load current.

8.7 The full load current of a three-phase, 4-pole, 50 Hz induction motor is 50 A while running at 1400 rpm. The full load torque is 120 N-m. Determine the starting torque if the starting current at normal voltage is 160 A.

8.8 A star-delta starter is used to start a three-phase induction motor. During starting, the motor takes 120% of the full load line current and develops 25% of the full load torque. Determine the full load slip.

8.9 A three-phase induction motor is started by a star-delta starter. The short circuit current of the motor at normal voltage is 5.5 times the full load current and the full load slip is 5%. Determine the ratio of the starting torque to the full load torque.

8.10 The inner cage and the outer cage impedance per phase at a standstill are $0.05 + j0.6$ and $0.5 + j0.3$, respectively. The per phase rotor induced voltage at standstill is 140 V. Determine the torques in synchronous watts at standstill and 3% slip.

8.11 The blocked rotor test data of a three-phase 240 V, 50 Hz, 4-pole Y-connected induction motor is as follows: Input power 80 W, line current 3.4 A and line voltage 30 V. The DC resistance between any two terminals is 3.2 Ω. Determine the equivalent parameters.

8.12 The blocked rotor test data of a three-phase 415 V, 50 Hz, 4-pole delta-connected induction motor is as follows: Input power 90 W, line current 3.7 A and line voltage 45 V. The DC resistance between any two terminals is 1.1 Ω. Determine the equivalent parameters.

8.13 A 4-pole motor is connected in cascade with a 6-pole motor. The 4-pole motor is energized by a 60 Hz supple. Determine the combined synchronous speed and output of the 6-pole motor if the input of the 4-pole motor is 40 kW. Neglect all kinds of losses.

9
Synchronous Generator

9.1 Introduction

Synchronous machines are considered the largest energy converters in the world, and they are classified as synchronous generators and synchronous motors. The synchronous generator consists of the stator and the rotor, which are separated by an air gap. In the steady state, the synchronous generator rotates at a constant speed. In this state, the rotating air gap field and the rotor rotate at the same speed, called the synchronous speed. The synchronous generator with a power rating of several hundred mega volt-amperes is quite common in the power generating station. In the power stations, a three-phase AC generator is used to generate the three-phase voltage with a lower magnitude of around 11 kV. The lower magnitude of the three-phase voltage is then raised by using a power transformer with an MVA rating. This high magnitude of the voltage is then transmitted to the grid substation by transmission lines.

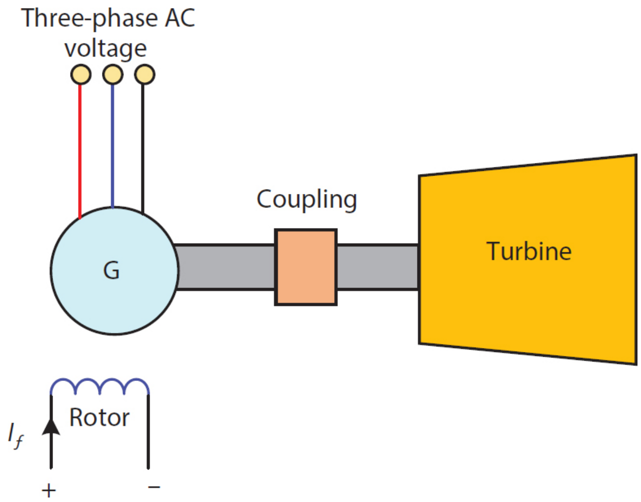


Fig. 9.1 Connection diagram of a generator and turbine.

The machine, which produces a three-phase electric power from the mechanical power, is called an alternator or synchronous generator. The shaft of the generator is coupled with the mechanical turbine. This turbine may be a gas, steam or water turbine. A synchronous machine with the no-load is called a synchronous condenser. A synchronous condenser is used in the factory to improve the power factor. The commercial synchronous generator is manufactured as the revolving-field synchronous generator and the stationary-field synchronous generator. In the revolving-field synchronous generator, the field is excited by a DC generator, which is mounted on the same shaft. The stationary-field synchronous generator is normally used when the power rating is less than 5 kVA. However, for larger output, the stationary-field synchronous generator is cheaper, more reliable and more efficient. The alternator with the mechanical turbine is shown in Fig. 9.1.

9.2 Construction

The basic elements of the synchronous machine are the stator and the rotor. The description of the two elements is as follows.

Stator: The stator is the stationary part of the alternator and is made of thin laminations of highly permeable steel to reduce the losses. The stator has a fixed number of slots in its inner periphery. The conductors are suitably placed in these slots. The ends of the conductors are connected either in a star or delta to form the balanced polyphase winding. The windings are distributed that are projected along the air gap. In addition to that, the number of poles is not equal to the number of slots and the schematic of the stator is shown in Fig. 9.2.

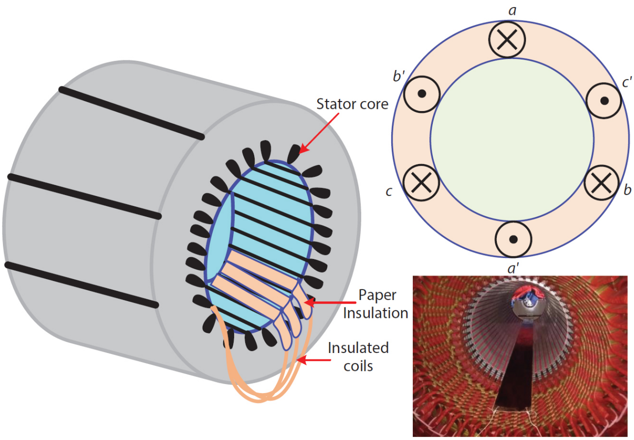


Fig. 9.2 Schematic of a stator.

Rotor: The rotor is the inner part of the alternator. The rotor contains the field windings that are connected in series, which are energized by a direct current from the separate DC source. There are two types of rotors used in the alternator:

- Salient or projecting pole-type,
- Non-salient or cylindrical pole-type.

Salient pole-type: Salient pole generators have many poles, as many as 50. For this reason, the salient pole rotor is used in the low and medium-speed alternators. The salient pole rotor is projected outward, which makes a nonuniform air gap as shown in Fig. 9.3. Due to the outward projection, the salient pole rotor cannot withstand heavy mechanical stress. In addition to that, if the salient pole rotor rotates at high speed, the windage and friction losses will be higher. The salient pole rotor has a short axial length and larger diameter to accommodate many poles. This type of alternator is driven by a diesel engine or by a water turbine. Here, the salient or projecting poles are fixed to the shaft of the alternator by bolts. In this type, the air gap between the stator and the rotor is not uniform. The air gap along the direct-axis (d-axis) is less, which makes the reluctance smaller, whereas the air gap along the quadrature axis (q-axis) is large, which makes the reluctance higher.

Non-salient or cylindrical pole-type: In this type, the rotor is made of smooth solid steel and the air gap is uniform. The reluctance is uniform and it does not depend on the orientation of the field poles with respect to the armature mmf. This type of rotor has a comparatively large axial length with a smaller diameter. The effect of armature reaction and leakage reactance is represented by the synchronous reactance. This type of rotor is used in a large generator with either two or four poles.

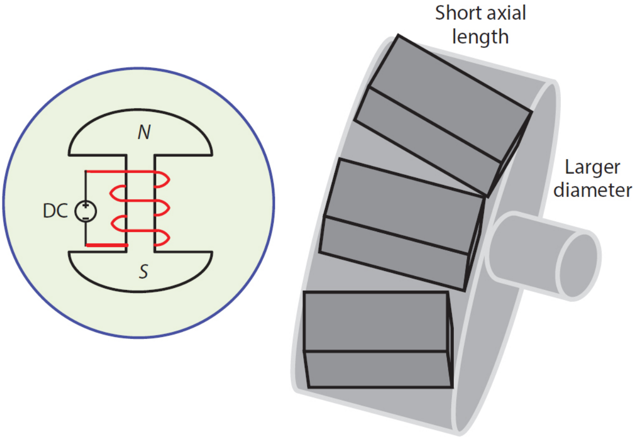


Fig. 9.3 Schematic of a salient pole-type rotor.

The steam turbine is used to drive this cylindrical pole rotor. This type of rotor normally runs at a high speed (3600 rpm or 1800 rpm). This type of rotor has a uniform air gap which provides noiseless operation at the higher speed. The cylindrical pole-type rotor is shown in Fig. 9.4.

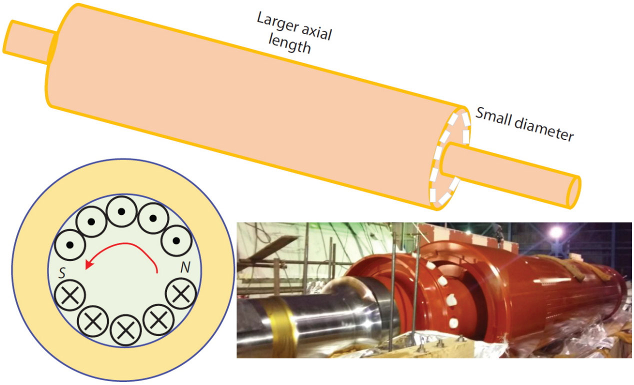


Fig. 9.4 Non-salient pole-type rotor.

The non-salient pole-type rotor gives balance, is more efficient, and has less windage loss.

9.3 Pole and Frequency

There is a relationship between the speed of the rotor, frequency of the generated voltage and the number of poles of an alternator. The number of poles of the alternator is normally assigned depending on the speed. The number of cycles per second, i.e., the frequency, is directly proportional to the speed of the rotor. One cycle of voltage is generated in each conductor if the rotor rotates one revolution. Therefore, if the rotor rotates at the speed of 60 rpm, then the frequency is 1 Hz. Similarly, if the rotor rotates at the speed of 3000 rpm, then the frequency is 50 Hz. The frequency in the armature conductors is directly proportional to the number of pairs of poles. Consider, a four-pole rotor, which rotates at a speed of 3000 rpm to generate the three-phase voltage and the corresponding frequency is,

$$f = \frac{4}{2} \times \frac{3000}{60} \tag{9.1}$$

In general, the expression of frequency can be expressed as,

$$f = \frac{P}{2} \times \frac{N}{60} \tag{9.2}$$

$$f = \frac{PN}{120} \tag{9.3}$$

Alternative approach: The number of cycles per revolution is expressed as,

$$C/R = \frac{P}{2} \tag{9.4}$$

The revolution per second is expressed as,

$$R/s = \frac{N}{20} \quad (9.5)$$

The cycles per second is known as frequency and it is expressed as,

$$f = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \quad (9.6)$$

where,

N is the synchronous speed of the alternator,

P is the number of poles,

f is the frequency of the generated voltage in Hz.

From equation (9.2), it is seen that the speed is fixed for a given number of poles and frequencies.

Example 9.1

The speed of a water turbine is 300 rpm and is connected to a synchronous generator. The frequency of the induced voltage is 50 Hz. Determine the number of poles of the generator.

Solution

The speed of the turbine is,

$$N = 300\,\text{rpm} \quad (9.7)$$

The number of poles is determined as,

$$P = \frac{120f}{N} = \frac{120 \times 50}{300} = 20\,\text{poles} \quad (9.8)$$

Alternatively, 10 pairs of north (N) and south (S) poles.

Practice Problem 9.1

The speed of a water turbine is 200 rpm and is connected to a synchronous generator. If the number of poles of the generator is 36 then find the frequency of the induced voltage.

9.4 Working Principle

Let us consider the stator conductors of the alternator are star-connected and the rotor is energized by the DC supply. The connection diagram and the generated voltage per phase are shown in Fig. 9.5. The DC exciter initially energizes the rotor winding of the alternator. Then a direct current flows through the rotor field windings that establish the magnetic field (flux) with the north pole and the south pole in the rotor circuit. This flux is static, which means it does not vary with time.

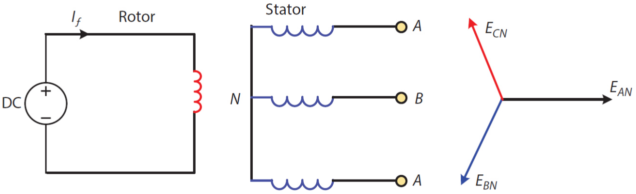


Fig. 9.5 Connection diagram of rotor and stator.

A rotating field of constant magnitude is produced in the air gap when the rotor rotates by a prime mover at synchronous speed. As a prime mover, different types of turbines (water and steam) and diesel engines are used. Then the armature conductors of the synchronous generator are cut by this rotating magnetic flux.

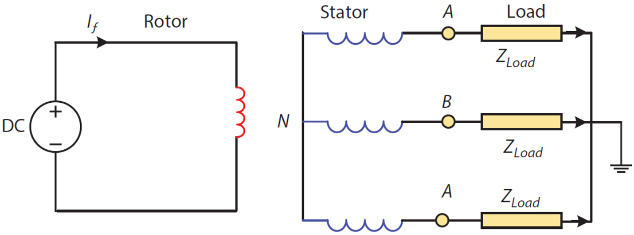


Fig. 9.6 Synchronous generator with load.

Consequently, a voltage is induced in the armature conductors due to an electromagnetic induction, which is known as Faraday's laws of electromagnetic induction. If the rotor rotates continuously, then the three-phase alternating voltage will be induced in the armature conductors. Since the three-stator windings are equally spaced from each other, therefore, the induced voltages have the same magnitudes but are phase-shifted by 120 electrical degrees. The frequency of the induced voltage depends on the relation, which is mentioned in the equation (9.3). The expressions of induced voltages are as follows,

$$e_a = v_a = V_{an} \sin \omega t \tag{9.9}$$

$$e_b = v_b = V_{bn} \sin(\omega t - 120^\circ) \tag{9.10}$$

$$e_c = v_c = V_{cn} \sin(\omega t - 240^\circ) \tag{9.11}$$

If the three-phase load is connected across the stator windings as shown in Fig. 9.6, then the three-phase currents will flow through the load.

9.5 Full-Pitch and Short-Pitch Windings

The coils of an alternator are distributed according to the number of slots per pole per phase. The coil span or coil pitch is the distance between the two sides of the coil. The coil also has two endpoints as shown in Fig. 9.7.

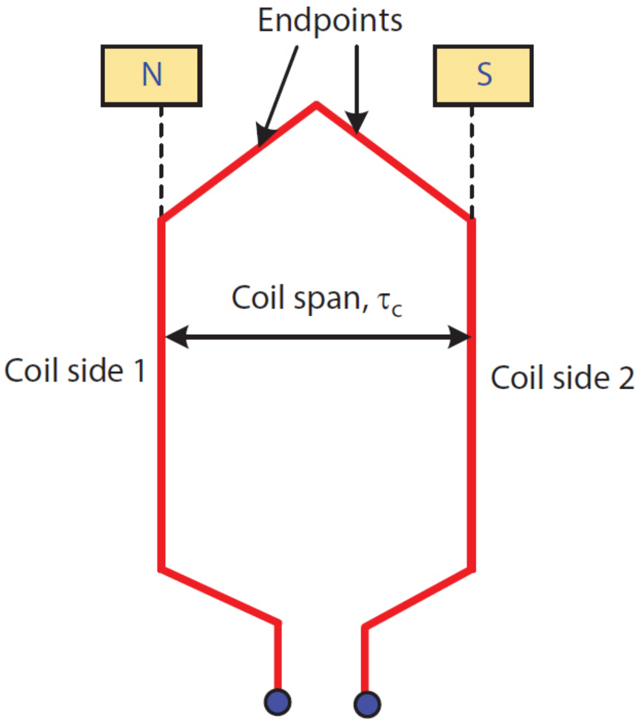

Endpoints
N
S
Coil span, τ_c
Coil side 1
Coil side 2

Fig. 9.7 Coil sides with endpoints.

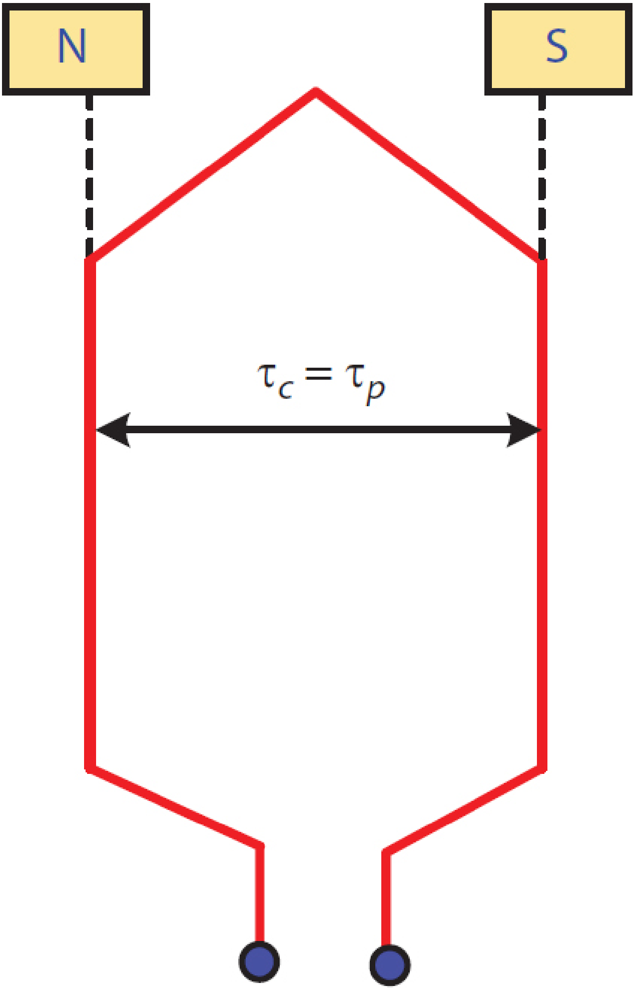


Fig. 9.8 Schematic of full-pitch coil.

The distance between the two sides of the coil is known as the coil span. The two sides of the coil are placed in the slots in such a way that the span is obtained 180 electrical degrees. This span is known as pole pitch and it is denoted by τp as shown in Fig. 9.8.

In other words, the coils having span, which is equal to one pole pitch, i.e., the spanning over 180 electrical degrees is known as full-pitch winding. The coils having a span which is less than the full pitch winding is called short-pitch winding. The full-pitch and short-pitch windings together are shown in Fig. 9.9.

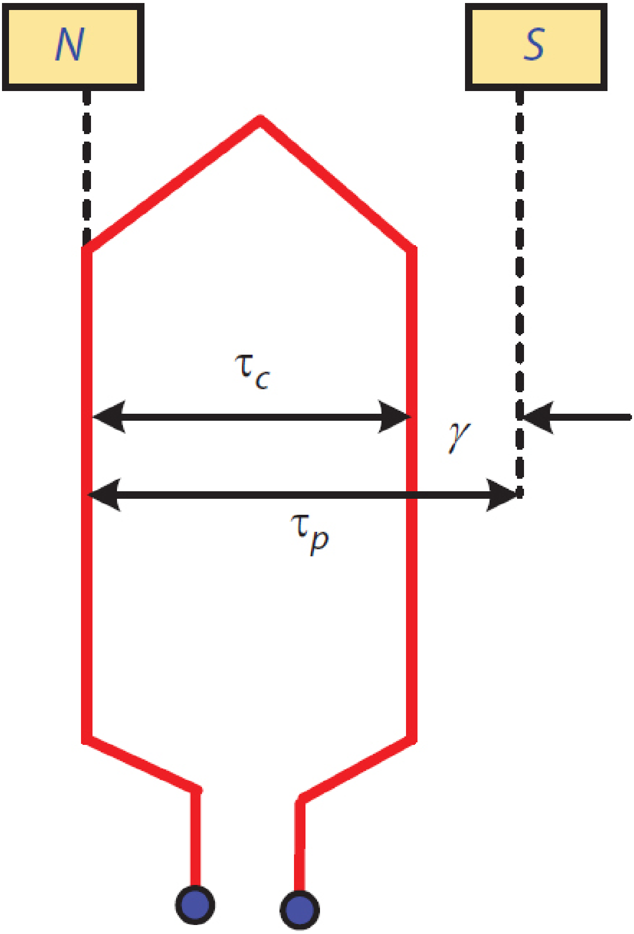


Fig. 9.9 Schematic of short-pitch and full-pitch coils.

Let,

S is the total number of slots,

P is the total number of poles,

m is the number of phases,

α is the angle between two successive slots,

τ_p is the pole pitch,

τ_c is the coil pitch,

γ is the short pitch angle,

n is the number of slots per pole per phase.

The expression of the number of slots (S) per pole (P) per phase (m) is,

$$n = \frac{S}{P \times m} \tag{9.12}$$

The angle between two successive slots can be determined as,

$$\alpha = \frac{360}{S} \times \frac{P}{2} \tag{9.13}$$

$$\alpha = \left(\frac{180P}{S} \right)^{\circ} \text{elec.} \tag{9.14}$$

The expression of pole pitch is written as,

$$\tau_p = \frac{S}{P} = 180^{\circ}\,\text{elec.} \tag{9.15}$$

For a full-pitch coil, the relation between the pole pitch and the coil pitch is,

$$\tau_c = \tau_p \tag{9.16}$$

For a short-pitch coil, the relation between the pole pitch and the coil pitch is expressed as,

$$\tau_c < \tau_p \tag{9.17}$$

The short pitch angle is determined as,

$$\gamma = \tau_p - \tau_c = (180 - \tau_c)^{0}\,\text{elec.} \tag{9.18}$$

The short-pitch coil has a few advantages which are mentioned below:

- Saves copper for end connection.
- Generated voltage can be made pure sine wave more easily.

- High-frequency harmonics are reduced.
- Eddy current and hysteresis losses depend on high-frequency harmonics. Hence, these losses are also reduced and thereby increase efficiency.

9.6 Pitch or Chording Factor

In the synchronous generator, the voltage generated in the short-pitch winding is always less than that of the full-pitch winding. The pitch factor is defined as the ratio of voltages generated by the short-pitch winding to the voltage, which is generated in the full-pitch winding. The coil factor is always less than unity. It is also known as the coil span factor or chording factor. Mathematically, it can be expressed as,

$$k_p = \frac{\text{Voltages induced in the short pitch coil}}{\text{Voltages induced in the full pitch coil}} \tag{9.19}$$

$$k_p = \frac{E_{cr-SP}}{E_{cr-PP}} \tag{9.20}$$

The phasor diagrams of the full-pitch coil and the short-pitch coil are shown in Figs. 9.10 and 9.11.

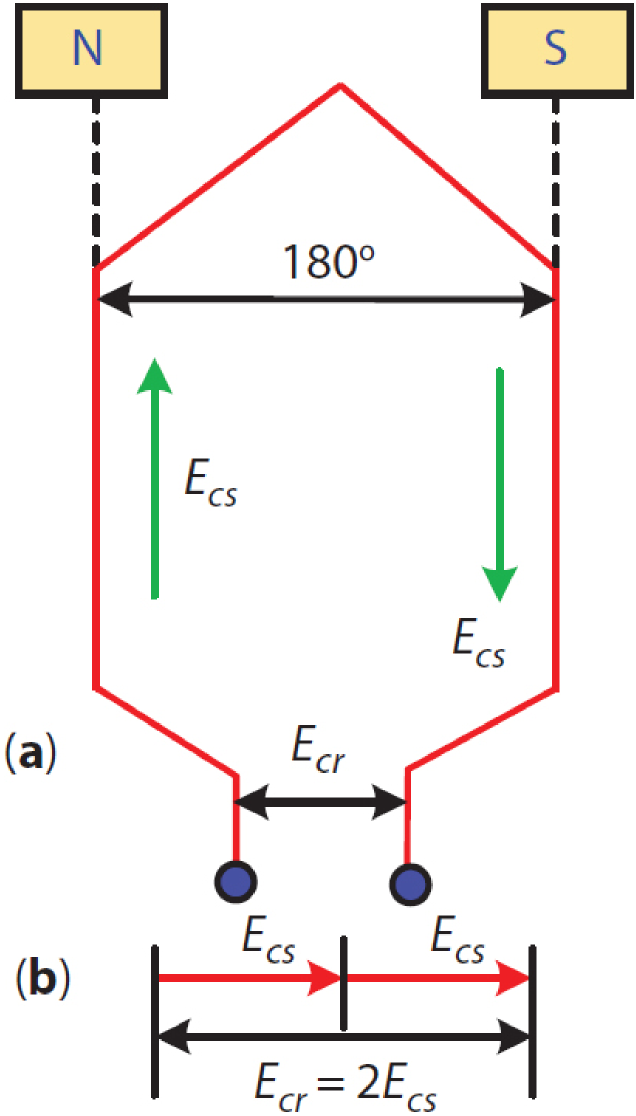

N
S
180°
E_{cs}
E_{cs}
(a)
E_{cr}
(b)
E_{cs}
E_{cs}
$E_{cr} = 2E_{cs}$

Fig. 9.10 Phasor diagram for the full-pitch coil.

The following notation can be defined as,

E_{cs} is the rms value of the coil side voltage,

E_{cr} is the rms value of the resultant coil side voltage.

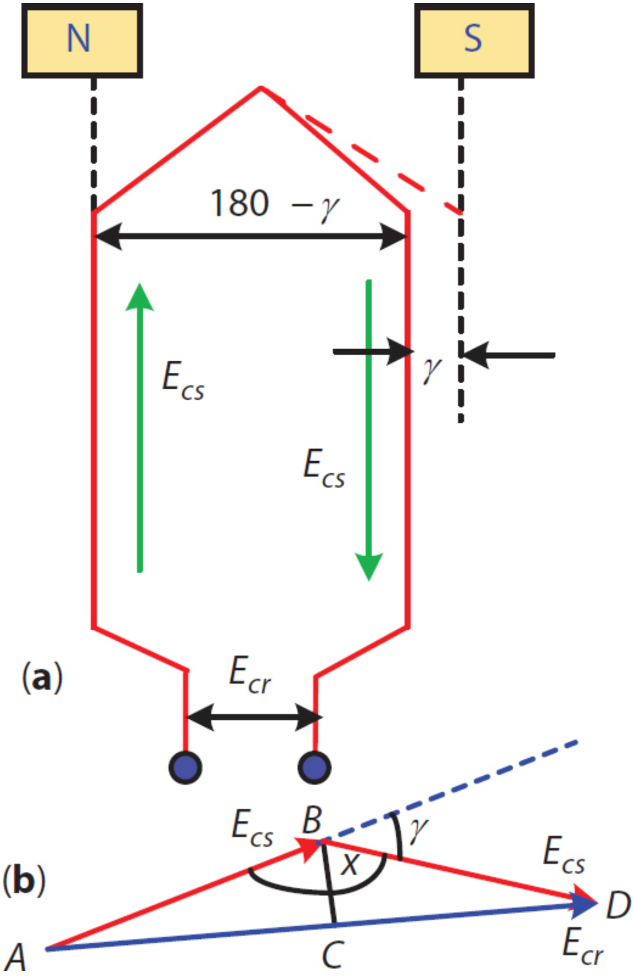

N
S
180 −γ
E_{cs}
E_{cs}
γ
(a)
E_{cr}
E_{cs}
B
γ
x
E_{cs}
(b)
D
A
C
E_{cr}

Fig. 9.11 Phasor diagram for the short-pitch coil.

From Fig. 9.10(b), the expression of resultant voltage for the full-pitch coil is written as,

$$E_{cr-FP} = 2E_{cs} \tag{9.21}$$

Again, from Fig. 9.11(b), the expression of the angle is determined as,

$$x = 180^\circ - \gamma \tag{9.22}$$

$$\angle ABC = \frac{x}{2} \tag{9.23}$$

For the triangle, *ABC*, the sum of the three angles is expressed as,

$$y + \frac{x}{2} + 90^\circ = 180^\circ \tag{9.24}$$

$$y = 90^\circ - \frac{x}{2} \tag{9.25}$$

Substituting equation (9.22) into equation (9.25) yields,

$$y = 90^\circ - \frac{180^\circ - \gamma}{2} = \frac{\gamma}{2} \tag{9.26}$$

From Fig. 9.11(b), the cosine angle is written as,

$$\cos\frac{\gamma}{2} = \frac{AC}{AB} = \frac{AC}{E_{cs}} \tag{9.27}$$

$$AC = E_{cs}\cos\frac{\gamma}{2} \tag{9.28}$$

For the short-pitch coil, the resultant voltage is,

$$AD = 2AC \tag{9.29}$$

$$E_{cr-SP} = 2AC \tag{9.30}$$

Substituting equation (9.28) into equation (9.30) yields,

$$E_{cr-SP} = 2E_{cs}\cos\frac{\gamma}{2} \tag{9.31}$$

Again, substituting equations (9.21) and (9.31) into equation (9.20) and the expression of pitch factor becomes, (9.32) $$k_p = \frac{2E_{cs}\cos\frac{\gamma}{2}}{2E_{cs}}$$

(9.33) $$k_p = \cos\frac{\gamma}{2}$$

Example 9.2

The number of slots of a three-phase, 6-pole alternator is 144. Determine the pitch factor if the winding is shorted by two slot pitches.

Solution

The number of poles is given by,

(9.34) $$P = 8$$

The number of phases is given by,

(9.35) $$m = 3$$

The number of slots is given by,

(9.36) $$S = 144$$

The short pitch angle is given by,

(9.37) $$\gamma = 2\text{slots-pitch}$$

The value of the pole pitch is calculated as,

(9.38) $$\tau_p = \frac{S}{P} = \frac{144}{8} = 18\,\text{slots}$$

Now, the following relation can be written as:

(9.39) $$18\text{ slots}=180^\circ$$

$$1\,\text{slots}=\frac{180^\circ}{18}=10^\circ \tag{9.40}$$

The value of the short pitch angle is determined as,

$$\gamma = 2\,\text{slots-pitch} = 2\times 10^\circ = 20^\circ \tag{9.41}$$

The value of the pitch factor is determined as,

$$k_p = \cos\frac{\gamma}{2} = \cos\frac{20^0}{2} = 0.98 \tag{9.42}$$

Example 9.3

The number of slots of a three-phase, 4-pole alternator is 40. Determine the pitch factor if the coil spans from 1 to 9.

Solution

The number of poles is given as,

$$P = 4 \tag{9.43}$$

The number of phases is given as,

$$m = 3 \tag{9.44}$$

The number of slots is given,

$$S = 40 \tag{9.45}$$

The value of the pole pitch is calculated as,

$$\tau_p = \frac{S}{P} = \frac{40}{4} = 10\,\text{slots} \tag{9.46}$$

The coil spans 1 to 9, i.e., coil pitch is calculated as,

$$\tau_c = 9 - 1 = 8\,\text{slots} \tag{9.47}$$

Now,

$$10\ \text{slots}=180^\circ \tag{9.48}$$

$$1\text{slots}=\frac{180^\circ}{10}=18^\circ \tag{9.49}$$

The short pitch angle is calculated as,

$$\gamma=\tau_p-\tau_c=10-8=2\ \text{slots} \tag{9.50}$$

It is represented in terms of angle is,

$$\gamma=2\times 18^\circ=36^\circ \tag{9.51}$$

The value of the pitch factor is determined as,

$$k_p=\cos\frac{\gamma}{2}=\cos\frac{36^\circ}{2}=0.95 \tag{9.52}$$

Practice Problem 9.2

The number of slots of a three-phase, 4-pole alternator is 120. Find the pitch factor if the winding is shorted by two slot pitches.

Practice Problem 9.3

A three-phase, 4-pole alternator has 36 slots. Find the pitch factor if the winding is shorted by 1 slot-pitch and the coil span is 1 to 9.

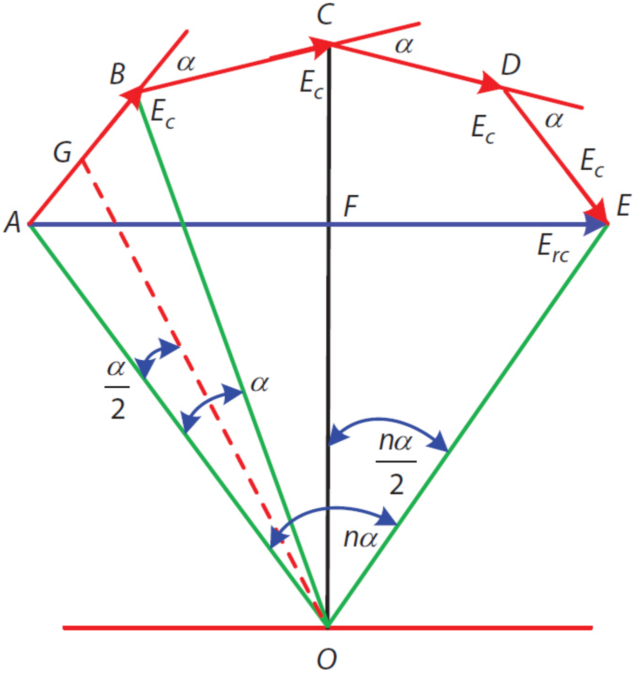
9.7 Distribution Factor

In an alternator, the coils are distributed properly for the double-layer windings. In these windings, the total induced voltage will be equal to the sum of the induced voltage in each coil. The voltages in each coil are not in phase but are displaced with some angles due to the different positions of the slots. Therefore, the total phasor sum of the voltages is always less than the arithmetic sum of the voltages. The distribution factor is defined as the ratio of the phasor sum of the voltages in each coil to the arithmetic sum of the coil voltages. Mathematically, it can be expressed as,

$$k_d = \frac{\text{Phasor sum of the coil voltages}}{\text{Arithematic sum of the coil voltages}} \quad (9.53)$$

$$k_d = \frac{E_{phasor-sum}}{E_{Arithm-sum}} \quad (9.54)$$

Let us consider that n numbers of coils are connected in series to form a phase group. The phase angle between the slots is α and the induced voltage per coil is Ec. The side voltages and the resultant voltages are shown in Fig. 9.12.



C
B
α
α
E_c
D
E_c
E_c
α
G
E_c
A
F
E
E_{rc}
$\frac{\alpha}{2}$
α
$\frac{n\alpha}{2}$
$n\alpha$
O

Fig. 9.12 Side voltages and resultant voltage.

From the triangle AOG, the following relation is written as,

$$\sin\frac{\alpha}{2} = \frac{\frac{E_c}{2}}{OA} \quad (9.55)$$

$$E_c = 2(OA)\sin\frac{\alpha}{2} \tag{9.56}$$

Again using the triangle *EOF*, the following relation is written as,

$$\sin\frac{n\alpha}{2} = \frac{\frac{E_{rc}}{2}}{OE} \tag{9.57}$$

$$E_{rc} = 2(OA)\sin\frac{n\alpha}{2} \tag{9.58}$$

The distribution factor is written as,

$$k_d = \frac{AE}{nAB} = \frac{E_{rc}}{nE_c} \tag{9.59}$$

Substituting equations (9.56) and (9.58) into equation (9.59) yields,

$$k_d = \frac{2(OA)\sin\frac{n\alpha}{2}}{n2(OA)\sin\frac{\alpha}{2}} \tag{9.60}$$

$$k_d = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} \tag{9.61}$$

Example 9.4

A 4-pole alternator has 48 slots where double-layer windings are placed. Find the distribution factor.

Solution

The value of the pole pitch is determined as,

$$\tau_p = \frac{S}{P} = \frac{48}{4} = 12\text{slots} \quad (9.62)$$

Now, the following relation can be written as:

$$12 \text{ slots}=180^\circ \quad (9.63)$$

$$1\text{slots} = \frac{180^\circ}{12} = 15^\circ \quad (9.64)$$

The number of slots or coils per pole per phase is calculated as,

$$n = \frac{S}{P \times 3} = \frac{48}{4 \times 3} = 4 \quad (9.65)$$

The value of the distribution factor is,

$$k_d = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \frac{\sin\frac{4\times 15}{2}}{4\sin\frac{15}{2}} = 0.96 \quad (9.66)$$

Practice Problem 9.4

An 8-pole, three-phase alternator has 144 slots for double-layer windings. Determine the distribution factor.

9.8 Effect of Harmonics on Pitch and Distribution Factors

The induced voltage in the armature windings will contain harmonics due to non-sinusoidal space flux density distribution. These harmonics also affect the pitch factor as well as the distribution factor. Let us consider γ the short pitch angle for the fundamental flux wave. Then the pitch and distribution factors for *hth* harmonic are,

$$k_{ph} = \cos\frac{h\gamma}{2} \tag{9.67}$$

$$k_{dh} = \frac{\sin\frac{nh\alpha}{2}}{n\sin\frac{h\alpha}{2}} \tag{9.68}$$

For fundamental, the pitch factor is expressed as,

$$k_{p1} = \cos\frac{\gamma}{2} \tag{9.69}$$

For the third harmonic, the pitch factor is written as,

$$k_{p3} = \cos\frac{3\gamma}{2} \tag{9.70}$$

For the fifth harmonic, the pitch factor is written as,

$$k_{p5} = \cos\frac{5\gamma}{2} \tag{9.71}$$

and so on.

For fundamental, the distribution factor is expressed as,

$$k_{d1} = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} \tag{9.72}$$

For the third harmonic, the distribution factor is written as,

$$k_{d3} = \frac{\sin\frac{n3\alpha}{2}}{n\sin\frac{3\alpha}{2}} \tag{9.73}$$

For the fifth harmonic, the distribution factor is expressed as,

(9.74) $$k_{d5} = \frac{\sin\frac{n5\alpha}{2}}{n\sin\frac{5\alpha}{2}}$$

9.9 EMF Equation of an Alternator

The flux created by the rotor revolves at a synchronous speed within the air gap when the rotor is rotating by the turbine. This flux varies sinusoidally and its expression can be written as,

(9.75) $$\phi = \phi_m \sin \omega t$$

According to Faraday's laws of electromagnetic induction, the voltage induced in a single turn of the coil is,

(9.76) $$e = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_m \sin \omega t)$$

(9.77) $$e = \phi_m \omega \cos \omega t$$

(9.78) $$e = \phi_m \omega \sin(90^\circ - \omega t)$$

The rms value of the induced voltage in a single turn of the coil is,

(9.79) $$E = \frac{\phi_m \omega}{\sqrt{2}} = 4.44 f \phi_m$$

The rms value of induced voltage for T turns of the coil is,

(9.80) $$E = 4.44 f \phi_m T$$

The general expression of emf per phase can be obtained by incorporating the pitch factor and distribution factor. Then equation (9.80) can be modified as,

$$E = 4.44 k_p k_d f \phi_m T \tag{9.81}$$

The winding factor is the product of the pitch factor and the distribution factor. Therefore, equation (9.78) can be modified as,

$$E = 4.44 k_w f \phi_m T \tag{9.82}$$

where,

k_p is the pitch factor,

k_d is the distribution factor,

$k_w = kpkd$ is the winding factor,

ϕ_m is the maximum value of the flux in Wb,

f is the frequency in Hz,

T is the number of turns in any phase.

The number of turns can be determined as,

$$T = \frac{1}{2} \times \text{No. of conductors per phase} \tag{9.83}$$

$$T = \frac{1}{2} \times (\text{slot per phase}) \times (\text{conductors per slot}) \tag{9.84}$$

Example 9.5

The number of slots of a three-phase, 12-pole, Y-connected alternator is 180. The number of conductors per slot is 12 and the conductors are connected in series. The coil is in full-pitch winding and the flux per pole is 0.05 Wb. If the machine runs at a speed of 600 rpm, determine the line voltage.

Solution

The slots per pole are calculated as,

$$\tau_p = \frac{S}{P} = \frac{180}{12} = 15 \text{ slots} \tag{9.85}$$

Now, the following relation can be written as:

$$15 \text{ slots} = 180^\circ \tag{9.86}$$

$$1\text{slots}=\frac{180^\circ}{15}=12^\circ \tag{9.87}$$

The number of slots or coils per pole per phase is determined as,

$$n=\frac{S}{P\times 3}=\frac{180}{12\times 3}=5 \tag{9.88}$$

The distribution factor is calculated as,

$$k_d=\frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}}=\frac{\sin\frac{5\times 12}{2}}{5\sin\frac{12}{2}}=0.95 \tag{9.89}$$

The pitch factor is given as,

$$k_p=1 \tag{9.90}$$

The total number of conductors per phase is calculated as,

$$Z=\frac{180}{3}\times 12=720 \tag{9.91}$$

The number of turns per phase is determined as,

$$T=\frac{720}{2}=360 \tag{9.92}$$

The frequency is calculated as,

$$f=\frac{PN}{120}=\frac{12\times 600}{120}=60\,\text{Hz} \tag{9.93}$$

The induced voltage per phase is,

$$E = 4.44 k_p k_d f \phi_m T = 4.44 \times 0.95 \times 1 \times 60 \times 0.05 \times 360 = 4555.4\,\text{V} \quad (9.94)$$

The line voltage is calculated as,

$$E_L = \sqrt{3} \times 4555.4 = 7890\,\text{V} \quad (9.95)$$

Example 9.6

A three-phase, 50 Hz, 4-pole, Y-connected alternator has 6 slots per pole per phase. The armature has double layers of windings. The windings are arranged as 5 conductors per layer. The coil pitch is 5/6th of the full pitch and the value of the flux is 0.05 Wb. Find the voltage per phase.

Solution

The number of poles is,

$$P = 4 \quad (9.96)$$

The number of phases is,

$$m = 3 \quad (9.97)$$

The slots per pole are calculated as,

$$\tau_p = \frac{S}{P} = 6 \times 3 = 18\,\text{slots} \quad (9.98)$$

Now, the following relation can be written as:

$$18\text{ slots} = 180^\circ \quad (9.99)$$

$$1\,\text{slots} = \frac{180^\circ}{18} = 10^\circ \quad (9.100)$$

The value of the coil pitch is calculated as,

$$\tau_c = \frac{5}{6} \times 18 = 15\,\text{slots} \quad (9.101)$$

The short pitch angle is determined as,

$$\gamma = \tau_p - \tau_c = 18 - 15 = 3 \text{ slots} \tag{9.102}$$

In terms of angle, it is represented as,

$$\gamma = 3 \times 10^\circ = 30^\circ \tag{9.103}$$

The pitch factor is calculated as,

$$k_p = \cos\frac{\gamma}{2} = \cos\frac{30^\circ}{2} = 0.96 \tag{9.104}$$

The number of slots or coils per pole per phase is given as,

$$n = 6 \tag{9.105}$$

The distribution factor is determined as,

$$k_d = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \frac{\sin\frac{6\times 10}{2}}{6\sin\frac{10}{2}} = 0.956 \tag{9.106}$$

The number of turns in any phase is,

$$T = \frac{1}{2} \times \text{No. of conductors per phase} \tag{9.107}$$

$$= \frac{1}{2} \times (\text{slots/pole/phase})(\text{No. of poles})(\text{condctors/slots}) \tag{9.108}$$

$$= \frac{1}{2} \times 6 \times 4 \times (\text{No. of layers})(\text{condctors/layer}) \tag{9.109}$$

$$T = \frac{1}{2} \times 6 \times 4 \times 2 \times 5 = 120 \tag{9.110}$$

The induced voltage per phase is calculated as,

$$E = 4.44 k_p k_d f \phi_m T = 4.44 \times 0.956 \times 0.95 \times 50 \times 0.05 \times 120 = 1209.72\,\text{V} \tag{9.111}$$

Example 9.7

A three-phase, 10-pole, 50 Hz synchronous generator has 3 slots per pole per phase. The armature has double layers of windings having 5 conductors per layer. The coil span is 150°. The values of the fundamental and third harmonic components of the flux are 0.05 Wb and 0.006 Wb respectively. Calculate the rms value of the induced voltage per phase voltage.

Solution

The number of poles is given as

$$P = 10 \tag{9.112}$$

The number of phases is given as,

$$m = 3 \tag{9.113}$$

The slots per pole are calculated as,

$$\tau_p = \frac{S}{P} = 3 \times 3 = 9\,\text{slots} \tag{9.114}$$

Now, the following relation can be written as:

$$9 \text{ slots} = 180^\circ \tag{9.115}$$

$$1\text{slots} = \frac{180^\circ}{9} = 20^\circ \tag{9.116}$$

The short pitch angle is calculated as,

$$\gamma = \tau_p - \tau_c = 180^\circ - 150^\circ = 30^\circ \tag{9.117}$$

The number of turns is determined as,

$$T = \frac{1}{2} \times 10 \times 3 \times 5 \times 2 = 150 \tag{9.118}$$

The number of slots or coils per pole per phase is,

$$n = 3$$

For fundamental, the pitch and distribution factors are calculated as,

$$k_{p1} = \cos\frac{\gamma}{2} = \cos\frac{30^\circ}{2} = 0.96 \tag{9.119}$$

$$k_{d1} = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \frac{\sin\frac{3\times 20}{2}}{3\sin\frac{20}{2}} = 0.96 \tag{9.120}$$

The fundamental emf per phase is determined as,

$$E_1 = 4.44 k_p k_d f \phi_m T = 4.44\times 0.96\times 0.96\times 50\times 0.05\times 150 = 1534.5\,\mathrm{V} \tag{9.121}$$

For the third harmonic, the pitch factor and distribution factor are calculated as,

$$k_{p3} = \cos\frac{3\gamma}{2} = \cos\frac{90^\circ}{2} = 0.71 \tag{9.122}$$

$$k_{d3} = \frac{\sin\frac{3n\alpha}{2}}{n\sin\frac{3\alpha}{2}} = \frac{\sin\frac{3\times 3\times 20}{2}}{3\sin\frac{3\times 20}{2}} = 0.66 \tag{9.123}$$

The value of the voltage is calculated as,

$$E_3 = 4.44 k_p k_d f \phi_m T = 4.44\times 0.71\times 0.66\times 50\times 0.006\times 150 = 93.6\,\mathrm{V} \tag{9.124}$$

The induced voltage per phase is determined as,

$$E = \sqrt{E_1^2 + E_3^2} = \sqrt{(1534.5)^2 + (93.6)^2} = 1537.4\,\mathrm{V} \tag{9.125}$$

Practice Problem 9.5

The number of slots of a three-phase, 4-pole, star-connected alternator is 36. The number of conductors per slot is 8 and the conductors are connected in series.

The coil is full pitch winding and the flux per pole is 0.06 Wb. If the machine runs at 600 rpm, determine the line voltage.

Practice Problem 9.6

The number of slots per pole per phase of a three-phase, 50 Hz, 4-pole, Y-connected alternator is 4. The armature has double layers of windings having 4 conductors per layer. The coil pitch is 3/4th of the full pitch and the value of the flux is 0.03 Wb. Calculate the phase voltage and the line voltage.

Practice Problem 9.7

A three-phase, 6-pole, 60 Hz synchronous generator has 2 slots per pole per phase. The armature has double layers of windings having 4 conductors per layer. The coil span is 150°. The value of the fundamental flux is 0.05 Wb and the third harmonic component of the flux is 30% of the fundamental flux. Find the rms value of the induced voltage per phase voltage.

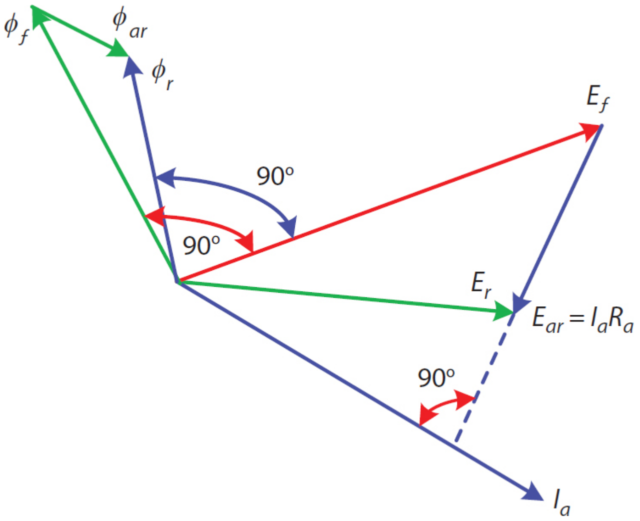
9.10 Equivalent Circuit of Synchronous Generator

Consider the per-phase armature winding to draw the equivalent circuit of a synchronous generator. The armature winding has a per phase small resistance, R_a. This resistance is known as effective resistance. The per-phase leakage reactance is the combined effects of the armature leakage reactance as well as the armature reaction reactance. The current will flow through the armature winding when it is connected to the load. This current will establish a constant magnitude rotating flux in the air gap. This resultant flux (ϕ_r) is the sum of the field flux (ϕ_f) and the armature reaction flux (ϕ_r). These fluxes induce the field voltage (E_f) and armature reaction voltage (E_{ar}). These induced voltages lag their respective fluxes by 90 electrical degrees (90°). The phasor diagram of the loaded alternator is shown in Fig. 9.13. The sum of the armature leakage reactance and armature reaction reactance is known as the synchronous reactance. It can be expressed as,

$$X_s = X_l + X_{ar} \tag{9.126}$$

The synchronous impedance of the alternator is defined as,

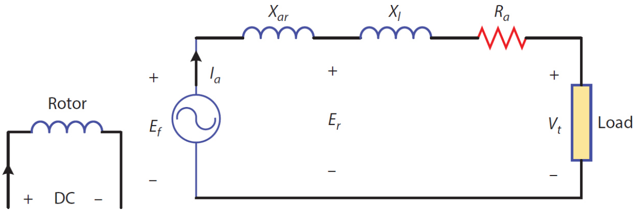
$$Z_s = R_a + jX_s \tag{9.127}$$



ϕ_f
ϕ_{ar}
ϕ_r
E_f
90°
90°
E_r
$E_{ar} = I_a R_a$
90°
I_a

Fig. 9.13 Phasor diagram of a loaded alternator.

The per-phase equivalent circuit with armature leakage reactance and armature reaction reactance of an alternator (three-phase synchronous generator) is shown in Fig. 9.14.



X_{ar}
X_l
R_a
Rotor
+
I_a
+
+
E_f
E_r
V_t
Load
–
–
–
+ DC –

Fig. 9.14 Equivalent circuit with armature reactance and leakage reactance.

In the equivalent circuit, the following parameters are defined as,

E_f is the induced emf per phase,

V_t is the terminal voltage per phase,

I_a is the armature current per phase,

X_{ar} is the armature reaction reactance per phase,

X_l is the leakage reactance per phase,

X_s is the synchronous reactance per phase.

From Fig. 9.14, the expression of induced emf per-phase can be calculated as,

$$E_f = V_t + I_a R_a + jI_a (X_{ar} + X_l) \tag{9.128}$$

Similarly, from Fig. 9.15, the expression of induced emf per phase can be calculated as,

$$E_f = V_t + I_a R_a + jI_a X_s \tag{9.129}$$

The magnitude of induced emf per-phase can be determined as,

$$E_f = \sqrt{(V_t + I_a R_a)^2 + (I_a X_s)^2} \tag{9.130}$$

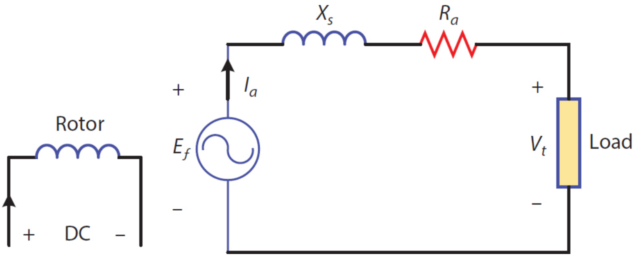


Fig. 9.15 Equivalent circuit with synchronous reactance.

9.11 Phasor Diagrams

Three types of phasor diagrams are considered in an alternator. These phasor diagrams are the unity power factor, lagging power factor and leading power factor. In the entire phasor diagrams, the terminal voltage is considered as the reference phasor.

9.11.1 Unity Power Factor Phasor Diagram

When an alternator driving a unit power factor load, the armature current is in phase with the terminal voltage, V_t. The resistive drop I_aR_a is in phase with the armature current I_a and the reactive drop I_aX_s leads the resistive drop by an angle of 90°. The phasor diagram with a unity power factor is shown in Fig. 9.16.

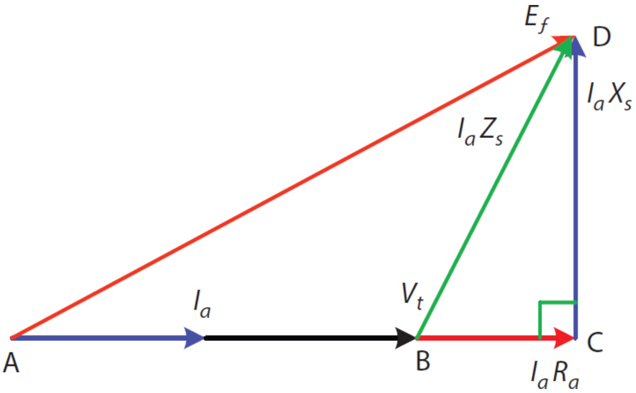


Fig. 9.16 Phasor diagram with unity power factor.

From the right-angle triangle ACD, the expression for induced emf per phase can be written as,

$$AD^2 = AC^2 + CD^2 \tag{9.131}$$

$$AD^2 = (AB + BC)^2 + CD^2 \tag{9.132}$$

$$E_f = \sqrt{(V_t + I_aR_a)^2 + (I_aX_s)^2} \tag{9.133}$$

9.11.2 Lagging Power Factor Phasor Diagram

Here, the armature current I_a lags behind the terminal voltage V_t by an angle of ϕ. The resistive drop I_aR_a is always in phase with the armature current and the reactive drop I_aX_s leads the resistive drop I_aR_a by an angle of 90°. The phasor diagram with a lagging power factor is shown in Fig. 9.17.

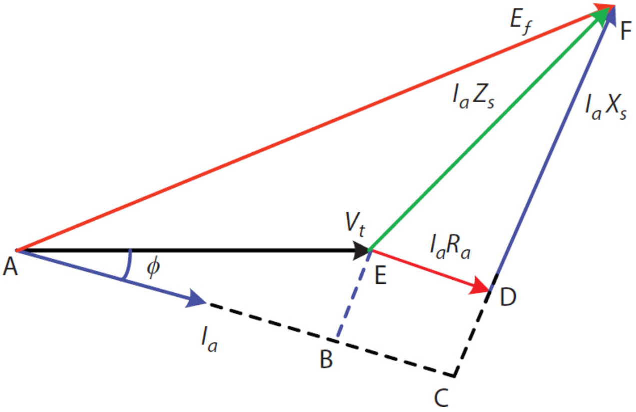


Fig. 9.17 Phasor diagram with lagging power factor.

From the triangle ACF, the induced emf per phase is written as,

$$AF^2 = AC^2 + CF^2 \tag{9.134}$$

$$AF^2 = (AB + BC)^2 + (CD + DF)^2 \tag{9.135}$$

From the ABE triangle, the following relations are written as,

$$AB = V_t \cos\phi \tag{9.136}$$

$$BE = V_t \sin\phi = CD \tag{9.137}$$

$$ED = BC = I_a R_a \tag{9.138}$$

Substituting equations (9.133) to (9.135) into equation (9.132) yields,

$$E_f = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi + I_a X_s)^2} \tag{9.139}$$

9.11.3 Leading Power Factor Phasor Diagram

Here, the armature current I_a leads the terminal voltage V_t by an angle of ϕ. The resistive drop I_aR_a is in phase with the armature current and the reactive drop I_aX_s leads the resistive drop I_aR_a by an angle of 90º. The phasor diagram with a leading power factor is shown in Fig. 9.18.

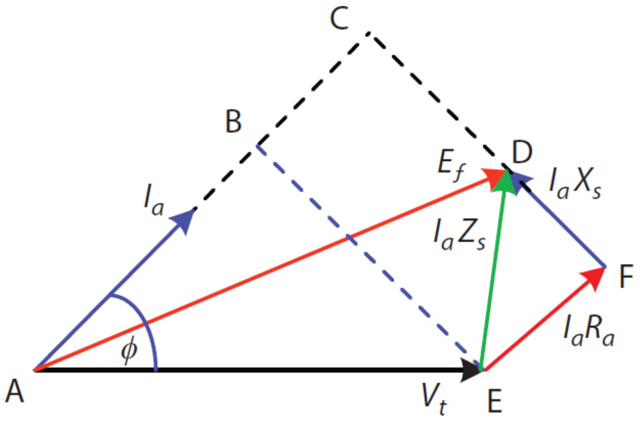


Fig. 9.18 Phasor diagram with leading power factor.

From the triangle ACD, the induced emf per phase is written as,

$$AD^2 = AC^2 + CD^2 \tag{9.140}$$

$$AD^2 = (AB + BC)^2 + (CF - CD)^2 \tag{9.141}$$

$$AD^2 = (AB + BC)^2 + (BE - CD)^2 \tag{9.142}$$

From the ABE triangle, the following relations are written as,

$$AB = V_t \cos\phi \tag{9.143}$$

$$BE = V_t \sin\phi = CF \tag{9.144}$$

$$EF = BC = I_a R_a \tag{9.145}$$

Substituting equations (9.143) to (9.145) into equation (9.142) yields,

$$E_f = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi - I_a X_s)^2} \tag{9.146}$$

9.12 Voltage Regulation

The terminal voltage of an alternator always changes with a change in load. The voltage regulation of an alternator is defined as the rise in terminal voltage when the load is decreased from full-load voltage to zero voltage by keeping the speed and the field excitation constant. In other words, the voltage regulation (*VR*) of a synchronous generator is defined as the change in magnitude of the terminal voltage between no-load to full-load voltage divided by the full-load voltage. Therefore, the percentage voltage regulation may be expressed as follows,

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100 = \frac{E_f - V_t}{V_t} \times 100 \quad (9.147)$$

$$VR = \frac{E_f - V_t}{V_t} \times 100 \quad (9.148)$$

where,

V_{nl} is the no-load terminal voltage,

V_{fl} is the full-load terminal voltage.

For unity and lagging power factor, the rise in terminal voltage when the load is removed. Hence, the voltage regulation is positive. However, in a leading power factor (capacitive load), the terminal voltage may drop if the load is removed.

Example 9.8

A 50 kVA, 220 V, 50 Hz, single-phase alternator is being used in the power station to generate the voltage. This alternator has an effective resistance of 0.011 Ω and a synchronous reactance of 0.09 Ω. Calculate the no-load induced voltage per-phase and voltage regulation at rated current and 0.85 power factor lagging.

Solution

The value of the armature current is calculated as,

$$I_a = \frac{50 \times 1000}{220} = 227.3\,\text{A} \quad (9.149)$$

The induced voltage per phase can be calculated using the following formula,

$$E_f = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi + I_a X_s)^2} \tag{9.150}$$

$$E_f = \sqrt{(220\times 0.85 + 227.3\times 0.011)^2 + (220\times 0.52 + 227.3\times 0.09)^2} \tag{9.151}$$

$$E_f = 232.6\,\mathrm{V} \tag{9.152}$$

The voltage regulation can be determined as,

$$VR = \frac{232.6 - 220}{220}\times 100 = 5.7\% \tag{9.153}$$

Practice Problem 9.8

A three-phase, 60 kVA, 440 V, star-connected, 50 Hz alternator has an armature resistance of 0.2 Ω and a synchronous reactance of 2.3 Ω. Find the no-load induced voltage per phase and the voltage regulation at the unity power factor.

9.13 Tests of a Synchronous Generator

In the synchronous generator, the following tests are conducted to determine its necessary parameters.

9.13.1 Determination of Resistance

The armature resistance of the alternator is determined by the voltmeter and ammeter method. Let *R* be the equivalent resistance between any two terminals of a star connection alternator as shown in Fig. 9.19. For the star-connection, the value of the armature resistance is,

$$R_a + R_a = R \tag{9.154}$$

$$R_a = \frac{1}{2}R = 0.5R \tag{9.155}$$

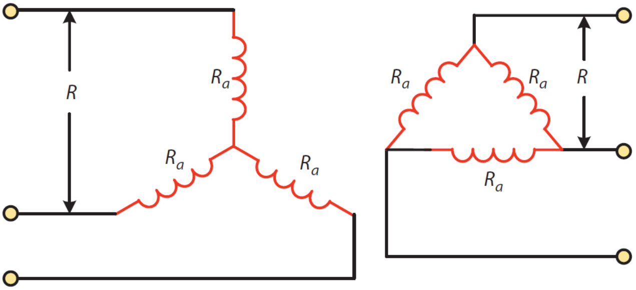

R
R_a
R_a
R_a
R_a
R_a
R
R_a

Fig. 9.19 Wye and delta connection.

For delta connection, the value of the armature resistance is,

$$R = R_a \parallel (R_a + R_a) \tag{9.156}$$

$$R = \frac{2}{3R_a} \tag{9.157}$$

$$R_a = \frac{3}{2}R = 1.5R \tag{9.158}$$

9.13.2 Open Circuit Test

The rotor field winding is connected to the DC power supply through a variable resistor (Rheostat). A voltmeter is connected between the neutral point and the line R to measure the phase voltage. Other three voltmeters are connected between the two lines to measure the line voltage as shown in Fig. 9.20.

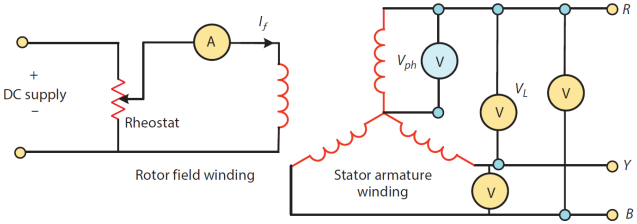

+
DC supply
−
Rheostat
A
I_f
Rotor field winding
V_{ph}
V
V_L
V
V
Stator armature winding
V
R
Y
B

Fig. 9.20 Connection for open circuit test.

To obtain the open circuit characteristics (OCC) curve, the generator is driven at no-load and its field current is set to zero.

Then the field current is increased in steps by varying the field rheostat. The line voltages across the generator terminals are recorded for every change of field current. These line voltages are plotted as ordinate (y-axis) and the field current as abscissa (x-axis). This curve is known as the open circuit characteristics curve as shown in Fig. 9.21.

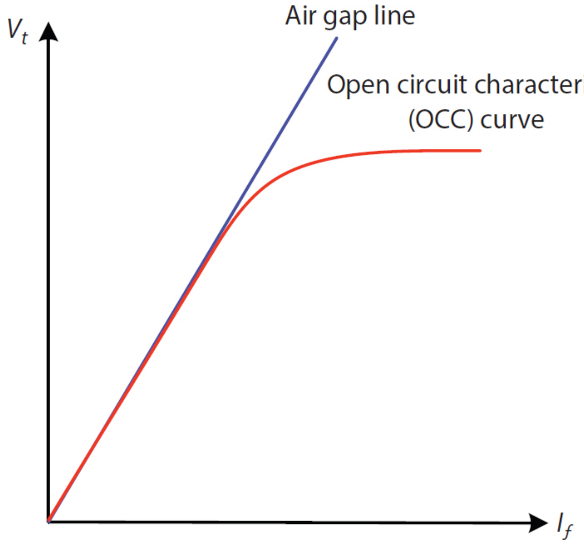


Fig. 9.21 OCC curve of a synchronous generator.

9.13.3 Short Circuit Test

In this test, the alternator terminals are shorted either by the wire or the ammeter to measure the short circuit current as shown in Fig. 9.22. Initially, the field current is set to zero and to rotate the generator to its rated speed. Then the field current is increased in steps by varying the rheostat and the corresponding armature current/short circuit currents are recorded.

These armature/short circuit currents are plotted as ordinate with the field current as abscissa. This curve is known as the short circuit characteristic curve.

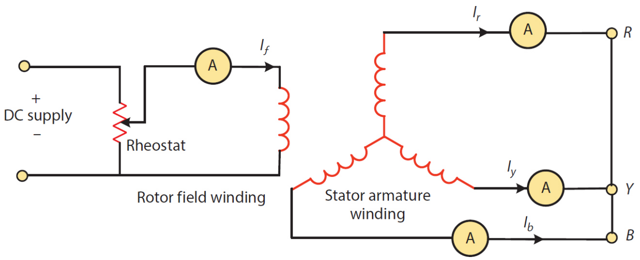


Fig. 9.22 Connection for open circuit test.

The open circuit characteristics curve and short circuit characteristics curve are plotted together as shown in Fig. 9.23 to find the synchronous impedance. At a fixed value of field current I_{f1}, the value of no-load voltage per phase is E_{f1} and the value of the short circuit current is I_{sc1}. During the short circuit, the terminal voltage is zero. Therefore the induced voltage in the generator is utilized to circulate the short-circuit current. Hence, the value of the synchronous impedance can be determined as,

$$Z_s = \left.\frac{\text{Open circuit voltage}}{\text{Short circuit current}}\right|_{\text{same } I_f} = \left.\frac{E_{f1}}{I_{sc1}}\right|_{\text{same } I_f} \tag{9.159}$$

The value of the synchronous impedance is not constant and is varied with the field current as shown in a dotted line in Fig. 9.23.

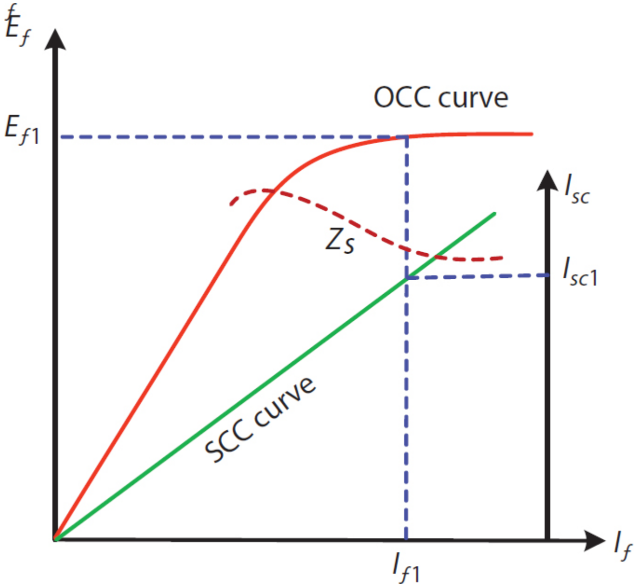


Fig. 9.23 OCC and SCC curves.

Practically, the DC resistance of a synchronous generator is measured by supplying a small value of DC voltage to one of the phases and the corresponding current is measured by the ammeter as shown in Fig. 9.24.

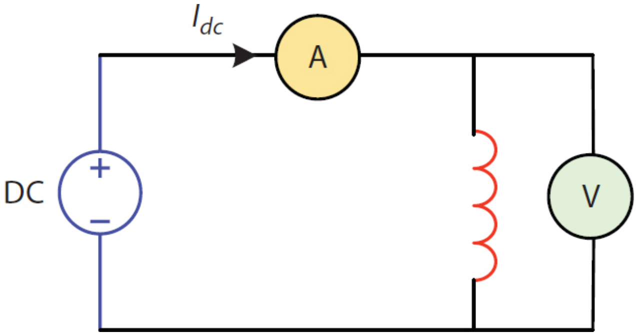


Fig. 9.24 Circuit for resistance measurement.

Then the value of the armature resistance is determined as,

$$R_a = \frac{V_{dc}}{I_{dc}} \tag{9.160}$$

The value of the synchronous reactance can be calculated as,

$$X_s = \sqrt{Z_s^{\ 2} - R_a^{\ 2}} \tag{9.161}$$

Example 9.9

A three-phase, 200 kVA, 50 Hz, 33 kV, star-connected alternator is installed in the power station to generate electricity. The per-phase armature resistance is 0.54 Ω. The test data are given below.

SC test: I_f = 15 A, a voltage between lines is 415 V

OC test: I_f = 15 A, short circuit current is 25 A

Find the synchronous impedance, synchronous reactance, and full load voltage regulation at a 0.9 power factor lagging.

Solution

The value of the phase voltage during the short circuit test is calculated as,

$$V_p = \frac{415}{\sqrt{3}} = 239.6\,\text{V} \tag{9.162}$$

The value of the synchronous impedance is calculated as,

$$Z_s = \frac{239.6}{25} = 9.6\,\Omega \tag{9.163}$$

The value of the synchronous reactance is determined as,

$$X_s = \sqrt{9.6^2 - 0.54^2} = 9.58\,\Omega \tag{9.164}$$

The value of the full load current is calculated as,

$$I_a = \frac{200}{\sqrt{3} \times 33} = 3.5\,\text{A} \tag{9.165}$$

The voltage per phase of the alternator is determined as,

$$V_t = \frac{33}{\sqrt{3}} = 19052\,\text{V} \tag{9.166}$$

The no-load voltage per phase is determined as,

$$E_f = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_t \sin\phi + I_a X_s)^2} \tag{9.167}$$

$$E_f = \sqrt{(19052 \times 0.9 + 3.5 \times 0.54)^2 + (19052 \times 0.44 + 3.5 \times 0.54)^2} \tag{9.168}$$

$$E_f = 19088\,\text{V} \tag{9.169}$$

The voltage regulation is calculated as,

$$VR = \frac{19088 - 19052}{19052} \times 100 = 0.18\% \tag{9.170}$$

Practice Problem 9.9

A 100 kVA, 50 Hz, 11 kV, three-phase, a delta-connected alternator is installed in the power station to generate alternating voltage. The armature resistance between the two terminals is 0.24 Ω. The test data are given below.

SC test: $I_f = 25$, short circuit current is 40 A, OC test: $I_f = 25$, the voltage between lines is 400 V

Calculate the synchronous impedance, synchronous reactance, and full load voltage regulation at a 0.85 power factor leading.

9.14 Power and Torque Expressions

A synchronous generator is a synchronous machine that converts its mechanical input power into three-phase electrical power. Consider the per-phase equivalent circuit of a synchronous generator to derive the expressions of power and torque. Fig. 9.25 shows the per-phase equivalent circuit, where bar *Vt* is the terminal voltage across the load. This terminal voltage is considered the reference vector. Let,

$$\overline{V_t} = V_t\angle 0 \tag{9.171}$$

$$\overline{E}_f = E_f\angle\delta \tag{9.172}$$

$$\overline{Z}_s = Z_s\angle\phi \tag{9.173}$$

The expression of the three-phase complex power at the load terminals is written as,

$$S = 3V_t I_a^{\,*} \tag{9.174}$$

Apply KVL to the circuit and it becomes,

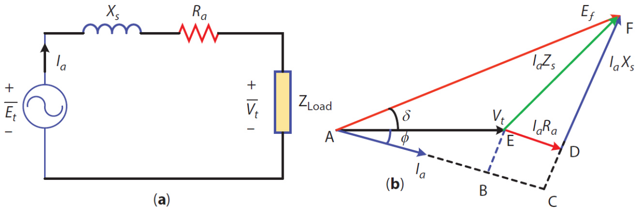
$$-\overline{E}_f + I_a\overline{Z}_s + \overline{V}_t = 0 \tag{9.175}$$

$$I_a = \frac{\overline{E}_f - \overline{V_t}}{\overline{Z}_s} \tag{9.176}$$

$$I_a = \frac{E_f\angle\delta - V_t\angle 0}{Z_s\angle\phi} \tag{9.177}$$

The conjugate of the armature current is expressed as,

$$I_a^{\,*} = \frac{E_f\angle{-\delta} - V_t\angle 0}{Z_s\angle{-\phi}} \tag{9.178}$$



X_s
R_a
I_a
+
$\overline{E_t}$
−
+
$\overline{V_t}$
−
Z_{Load}
(a)
E_f
F
I_aZ_s
I_aX_s
δ
ϕ
A
V_t
E
I_aR_a
D
I_a
(b)
B
C

Fig. 9.25 Per phase equivalent circuit and phasor diagram.

Substituting equation (9.178) into equation (9.174) yields,

$$S=\frac{3V_tE_f}{Z_s}\underline{|\phi-\delta}-\frac{3V_t^2}{Z_s}\underline{|\phi} \tag{9.179}$$

$$P+jQ=\frac{3V_tE_f}{Z_s}[\cos(\phi-\delta)+j\sin(\phi-\delta)]-\frac{3V_t^2}{Z_s}[\cos\phi+j\sin\phi] \tag{9.180}$$

Now equate the real and imaginary parts from both sides of the equation (9.180). The expression of the real power is written as,

$$P=\frac{3V_tE_f}{Z_s}\cos(\phi-\delta)-\frac{3V_t^2}{Z_s}\cos\phi \tag{9.181}$$

The expression of the reactive power is written as,

$$Q=\frac{3V_tE_f}{Z_s}\sin(\phi-\delta)-\frac{3V_t^2}{Z_s}\sin\phi \tag{9.182}$$

In an alternator, the armature resistance is very small compared to its synchronous reactance and it can be neglected for simplification. In this case, the impedance becomes,

$$Z_s=0+jX_s=X_s\underline{|90^\circ} \tag{9.183}$$

Substituting equation (9.180) into equations (9.178) and (9.179) yields,

$$P=\frac{3V_tE_f}{X_s}\cos(90^\circ-\delta)-0 \tag{9.184}$$

$$P=\frac{3V_tE_f}{X_s}\sin\delta \tag{9.185}$$

$$Q=\frac{3V_tE_f}{X_s}\cos\delta-\frac{3V_t^2}{X_s} \tag{9.186}$$

The per-phase induced voltage, E_f and the terminal voltage, V_t are constant. The value of the power angle can be changed by varying the mechanical driving torque. Theoretically, the power will be maximum, if δ = 90º. Then the expression of the maximum power becomes,

$$P_{\max} = \frac{3V_t E_f}{X_s} \tag{9.187}$$

Alternative approach: Assume the armature resistance Ra is neglected to exclude the copper loss. Under this condition, the lagging phasor diagram is modified as shown in Fig. 9.26.

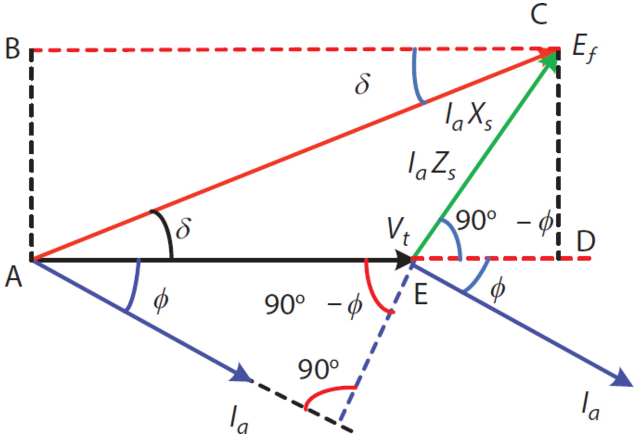


Fig. 9.26 Phasor diagram without armature resistance.

From the *ABC* triangle of Fig. 9.26, the following relation is written as,

$$AB = \sin\delta \tag{9.188}$$

Similarly, from the *CDE* triangle, the following relation is written as,

$$CD = I_a X_s \sin(90^\circ - \phi) \tag{9.189}$$

$$CD = I_a X_s \cos\phi \tag{9.190}$$

However, the length of *AB* is equal to the length of *CD* and it is written as,

$$(9.191)\,AB = CD$$

Substituting equations (9.188) and (9.190) into equation (9.191) yields,

$$(9.192)\,I_a X_s \cos\phi = E_f \sin\delta$$

$$I_a \cos\phi = \frac{E_f \sin\delta}{X_s} \qquad (9.193)$$

The three-phase output real power is expressed as,

$$(9.194)\,P = 3V_t I_a \cos\phi$$

Substituting equation (9.190) into equation (9.191) yields,

$$P = 3V_t \frac{E_f \sin\delta}{X_s} \qquad (9.195)$$

From equation (9.195), it is concluded that the real power of the synchronous is directly proportional to the $E_f \sin\delta$ and $I_a \cos\phi$.

Substituting equation (9.184) into equation (9.192) yields,

$$(9.196)\,P = P_{\max} \sin\delta$$

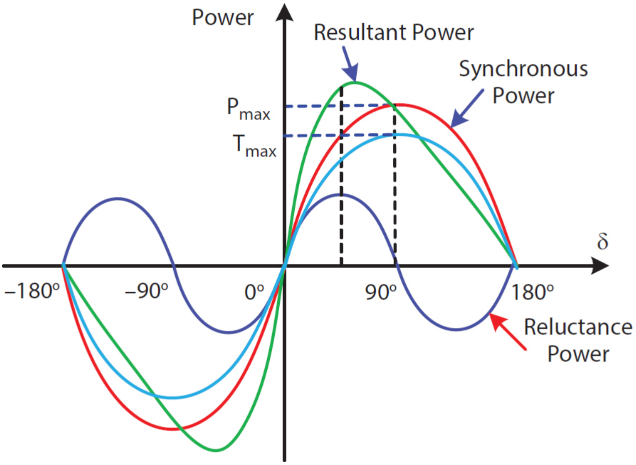


Fig. 9.27 Power and torque characteristics curves.

The torque is defined by the following expression,

$$T = \frac{P}{\omega_s} \tag{9.197}$$

Substituting equation (9.196) into equation (9.197) yields,

$$T = \frac{P_{\max}}{\omega_s}\sin\delta = T_{\max}\sin\delta \tag{9.198}$$

$$T = T_{\max}\sin\delta \tag{9.199}$$

where the expression of maximum torque is written as,

$$T_{\max} = \frac{3V_t E_f}{X_s \omega_s} \tag{9.200}$$

From equations (9.196) and (9.199), it is observed that the power and torque vary sinusoidally with a power or torque angle of δ. If we further increase the prime-mover input to the generator then the power output will decrease. As a result, the excess power goes to accelerate the generator, thereby increasing its speed and causing it to pull out of synchronism. Hence, the steady-state stability limit is reached when δ = 90°. For normal steady-state operating conditions, the value of the power angle or torque angle is less than 90°. However, it is between 15° to 30°. The power and torque characteristics are shown in Fig. 9.27.

Example 9.10

The per-phase synchronous reactance of a 30 MVA, 20 kV, 1800 rpm, three-phase alternator is 6 Ω. The induced voltage per phase is 15 kV and per phase terminal voltage is 10 kV. Determine the three-phase real power delivered to the load at a power angle of 40° and three-phase maximum power.

Solution

Three-phase power delivered to the load can be determined as,

$$P = \frac{3V_t E_f}{X_s}\sin\delta = 3\frac{15\times10}{6}\sin 40^\circ = 48.21\,\text{MW} \tag{9.201}$$

Three-phase maximum power is calculated as,

$$P_{\max} = 3\frac{15\times10}{6} = 75\,\text{MW} \tag{9.202}$$

Practice Problem 9.10

A 30 MVA, 20 kV, 1800 rpm, three-phase alternator is delivering a power of 80 MW to the load. The induced voltage per phase is 20 kV and per phase terminal voltage is 15 kV. Determine the value of synchronous reactance if the torque angle of 40º.

9.15 Salient Pole Synchronous Generator

In 1899, Professor André Blondel, French electrical engineer and physicist, developed a two-rection theory to understand and analyze the salient pole synchronous machine. The two-reaction was proposed to resolve armature magnetomotive force F_a into two mutually perpendicular components. The one component that is aligned or placed along the rotor axis is known as the polar axis or direct axis or *d*-axis. The other component is placed 90º apart from the *d*-axis. This component is known as the interpolar axis or quadrature axis or q-axis. The magnetomotive force in the *d*-axis is denoted by F_d and in the *q*-axis is denoted by F_q. The *d*-axis component F_d is either a magnetizing or demagnetizing effect depending on the direction of the field flux, whereas the *q*-axis component F_q is always a cross-magnetizing effect. A synchronous generator with the two-pole salient rotor is shown in Fig. 9.27. According to Fig. 9.28, the air gap is less along the *d*-axis, while it is more along the *q*-axis. As the reluctance is directly proportional to the length ($R = l/\mu A$), the reluctance will be less on the *d*-axis and more on the *q*-axis. In this alignment of the rotor, the inductance ($L = N^2/R$) will be more along the *d*-axis, while it will be less along the *q*-axis. Therefore, the reactance will be higher in the *d*-axis and will be lower in the *q*-axis as shown in Fig. 9.29.

The voltage induced in the stator due to the d-axis component is known as the direct-axis voltage. Similarly, the voltage induced in the stator due to the q-axis is called the q-axis voltage. These induced voltages are written as,

$$E_{ad} = jI_d X_{ad} \tag{9.203}$$

$$E_{aq} = jI_q X_{aq} \tag{9.204}$$

Consider the generator is delivering power with lagging and leading loads. When a load is connected with the generator armature current will flow. This armature current is resolved into two components. These are direct axis component current, I_d and quadrature axis component current, I_q.

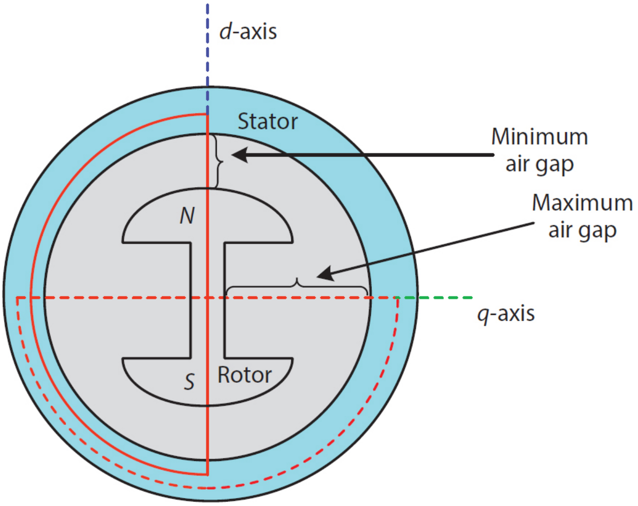


Fig. 9.28 Direct axis and quadrature axis components.

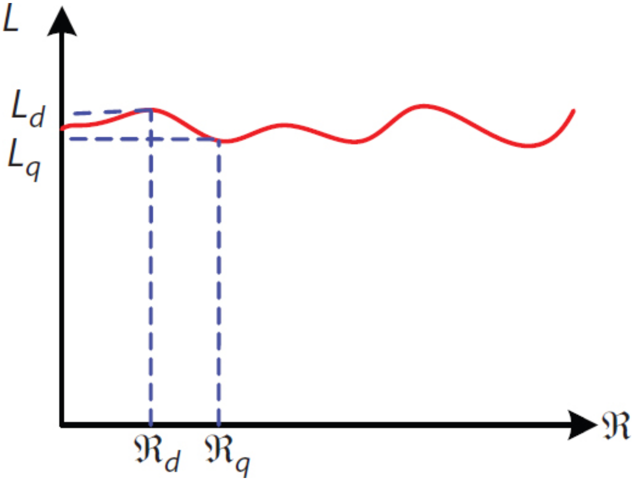


Fig. 9.29 Direct axis and quadrature axis inductances.

The current I_d produces a direct-axis component of armature reaction flux ϕ_{ard} while I_q produces a quadrature-axis component of armature reaction flux ϕ_{arq} as shown in Fig. 9.30. The inductive effects of the direct-axis and quadrature-axis armature reaction flux waves can be accounted for by the direct-axis and quadrature-axis magnetizing reactances X_{ard} and X_{arq}, respectively.

The direct axis and the quadrature axis synchronous reactances are then given by the following expressions,

$$X_d = X_l + X_{ard} \tag{9.205}$$

$$X_q = X_l + X_{arq} \tag{9.206}$$

But $X_d > X_a$ and the relation between the direct axis and quadrature axis is,

$$X_q = 0.6X_d \tag{9.207}$$

From Fig. 9.30, it is seen that the vector sum of ϕ_{ard} and X_{ard} gives the armature reaction flux X_{ar}. The total flux makes an angle of β with the field flux as shown in Fig. 9.30.

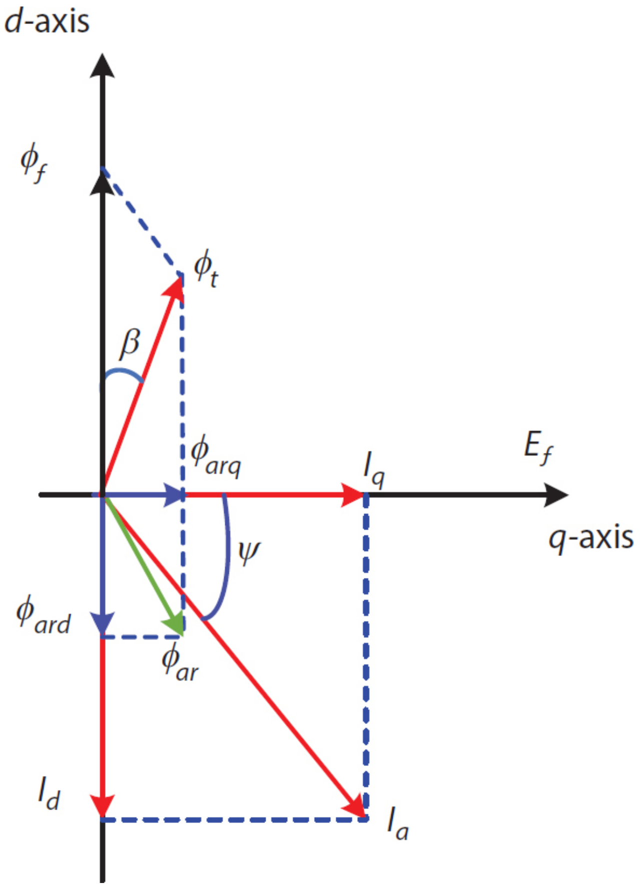

d-axis
ϕ_f
ϕ_t
β
ϕ_{arq}
I_q
E_f
q-axis
ψ
ϕ_{ard}
ϕ_{ar}
I_d
I_a

Fig. 9.30 Different fluxes in a salient pole generator.

Again, draw the lagging and leading power factor phasor diagram to identify the demagnetization and magnetization effects as shown in Figs. 9.31 and 9.32, respectively. In Fig. 9.31, the armature current lags the induced voltage E_f by the angle ψ. This angle is known as the internal power factor angle.

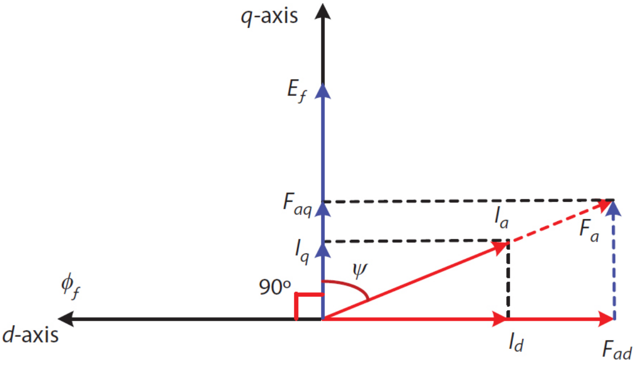

q-axis
E_f
F_{aq}
I_a
F_a
I_q
ψ
ϕ_f
90°
d-axis
I_d
F_{ad}

Fig. 9.31 Armature current lags the induced voltage.

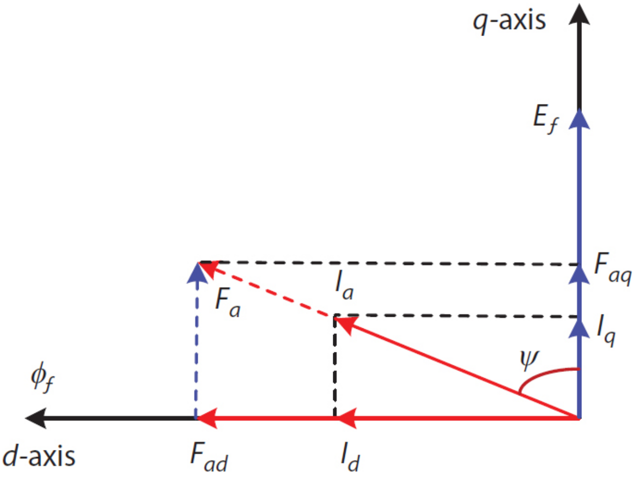

q-axis
E_f
F_{aq}
I_a
F_a
I_q
ψ
ϕ_f
d-axis
F_{ad}
I_d

Fig. 9.32 Armature current leads the induced voltage.

It is also seen that the magnetomotive force F_{ad} is the opposite of the main field flux ϕ_f. In other words, the flux ϕ_{ad} opposes the main field flux, which is known as the demagnetizing effect.

From Fig. 9.32, it is seen that the magnetomotive force due to the direct-axis component is in the same direction as the main field flux ϕ_f. Therefore, this flux aids the main field flux, which is known as the magnetizing effect.

9.16 Power of a Salient Pole Generator

The phasor diagram is required to derive the power expression of a salient pole synchronous generator. Initially, the per-phase armature resistance is considered to draw a lagging power factor phasor diagram as shown in Fig. 9.33. The resistive drop is drawn from the endpoint of terminal voltage V_t and is in phase with I_a. The resistive drop is resolved into two components $I_q R_a \cos\psi$ and $I_d R_a \sin\psi$. Then $I_d X_d$ is drawn perpendicular to $I_d R_a \sin\psi$. Afterward, $I_q X_q$ is drawn perpendicular to $I_q R_a \cos\psi$ which is the same to draw with $I_d X_d$.

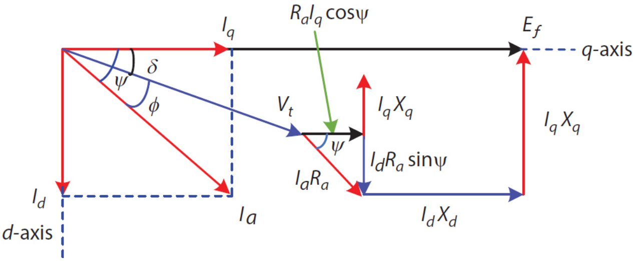


Fig. 9.33 Phasor diagram of a salient pole generator.

The effect of per-phase armature resistance is considered negligible to developing the expression of the power. Here, ϕ is the lagging power factor angle which means the current lags the terminal voltage by this angle, whereas the ψ is the internal power factor angle between the induced emf Ef and armature current Ia. The induced voltage per phase can be calculated as,

$$E_f = V_t + I_a R_a + jI_d X_d + jI_q X_q \tag{9.208}$$

The approximate phasor diagram of the salient pole generator with a lagging power factor is shown in Fig. 9.34. From Fig. 9.34, the expression of the direct axis component of the current is written as,

$$I_d = I_a \sin(\delta + \phi) \tag{9.209}$$

The expression of the quadrature axis component of the current is written as,

$$I_q = I_a \cos(\delta + \phi) \tag{9.210}$$

Again, from Fig. 9.34, the following relations can also be written as,

$$ad = V_t \sin\delta \tag{9.211}$$

$$oa = V_t \cos\delta \tag{9.212}$$

$$bc = I_q X_q \tag{9.213}$$

The lengths *ad* and *bc* are the same and the relation becomes,

$$V_t \sin\delta = I_q X_q \tag{9.214}$$

Substituting equation (9.210) into equation (9.214) yields,

$$V_t \sin\delta = I_a X_q \cos(\delta + \phi) \tag{9.215}$$

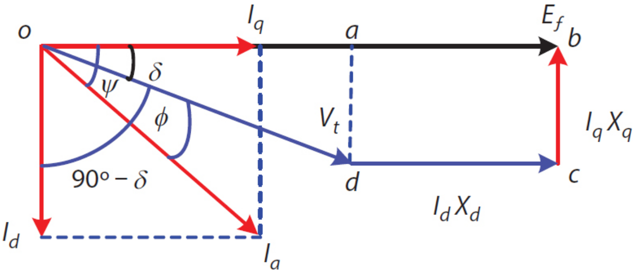


Fig. 9.34 Approximate phasor diagram of a salient pole generator.

$$V_t \sin\delta = I_a X_q (\cos\delta \cos\phi - \sin\delta \sin\phi) \tag{9.216}$$

$$(V_t + I_a X_q \sin\phi)\sin\delta = I_a X_q \cos\delta \cos\phi \tag{9.217}$$

$$\frac{\sin\delta}{\cos\delta} = \frac{I_a X_q \cos\phi}{(V_t + I_a X_q \sin\phi)} \tag{9.218}$$

$$\tan\delta = \frac{I_a X_q \cos\phi}{V_t + I_a X_q \sin\phi} \tag{9.219}$$

Again, from equation (9.214), the expression of the quadrature component of current can be rearranged as,

$$I_q = \frac{V_t \sin\delta}{X_q} \tag{9.220}$$

From Fig. 9.33, the following relation can be written as,

$$ob = oa + ab \tag{9.221}$$

The length of *ab* is expressed as,

$$ab = I_d X_d \tag{9.222}$$

Substituting equations (9.212), (9.222) and *ob* = E_f into equation (9.221) yields,

$$E_f = V_t \cos\delta + I_d X_d \tag{9.223}$$

$$I_d = \frac{E_f - V_t \cos\delta}{X_d} \tag{9.224}$$

Multiplying equation (9.209) by sinδ and equation (9.210) by cosδ and adding them yields,

$$I_d \sin\delta + I_q \cos\delta = I_a \sin(\delta + \phi)\sin\delta + I_a \cos(\delta + \phi)\cos\delta \tag{9.225}$$

$$I_d \sin\delta + I_q \cos\delta = \frac{I_a}{2}[\cos\phi - \cos(2\delta + \phi)] + \frac{I_a}{2}[\cos(2\delta + \phi) + \cos\phi] \tag{9.226}$$

$$I_d \sin\delta + I_q \cos\delta = \frac{I_a}{2}\cos\phi + \frac{I_a}{2}\cos\phi \tag{9.227}$$

Therefore, the relation among the currents is expressed as,

$$I_a \cos\phi = I_d \sin\delta + I_q \cos\delta \tag{9.228}$$

The expression of three-phase output power is written as,

$$P = 3V_t(I_a \cos\phi) \tag{9.229}$$

Substituting equation (9.228) into equation (9.229) yields,

$$P = 3V_t(I_d \sin\delta + I_q \cos\delta) \quad (9.230)$$

$$P = 3V_t I_d \sin\delta + 3V_t I_q \cos\delta \quad (9.231)$$

Substituting equations (9.220) and (9.224) into equation (9.231) yields,

$$P = 3V_t\left[\frac{V_t \sin\delta}{X_q}\cos\delta + \frac{E_f - V_t\cos\delta}{X_d}\sin\delta\right] \quad (9.232)$$

$$P = \frac{3V_t^2 \sin\delta\cos\delta}{X_q} + \frac{3V_t E_f \sin\delta}{X_d} - \frac{3V_t^2 \sin\delta\cos\delta}{X_d} \quad (9.233)$$

$$P = \frac{3V_t E_f \sin\delta}{X_d} + \frac{3V_t^2 (X_d - X_q)\sin 2\delta}{2X_d X_q} \quad (9.234)$$

Alternative Method:

The power output of a salient pole synchronous generator is,

$$P = 3P_d + 3P_q \quad (9.235)$$

The single-phase d-axis component power (P_d) is expressed as,

$$P_d = V_t I_d \cos(90^\circ - \delta) \quad (9.236)$$

The single-phase q-axis component power (P_q) is expressed as,

$$P_q = V_t I_q \cos\delta \quad (9.237)$$

Substituting equations (9.236) and (9.237) into equation (9.235) yields,

$$P = 3V_t I_d \cos(90^\circ - \delta) + 3V_t I_q \cos\delta \quad (9.238)$$

$$P = 3V_t I_d \sin\delta + 3V_t I_q \cos\delta \quad (9.239)$$

Substituting equation (9.220) and (9.224) into equation (9.239) yields,

$$P = 3V_t \sin\delta \left(\frac{E_f - V_t \cos\delta}{X_d} \right) + 3V_t \cos\delta \left(\frac{V_t \sin\delta}{X_q} \right) \tag{9.240}$$

$$P = \frac{3V_t E_f \sin\delta}{X_d} - \frac{3V_t^2}{X_d} \sin\delta \cos\delta + \frac{3V_t^2}{X_q} \sin\delta \cos\delta \tag{9.241}$$

$$P = \frac{3V_t E_f \sin\delta}{X_d} + \frac{3V_t^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) 2\sin\delta \cos\delta \tag{9.242}$$

$$P = \frac{3V_t E_f \sin\delta}{X_d} + \frac{3V_t^2 (X_d - X_q) \sin 2\delta}{2X_d X_q} \tag{9.243}$$

Differentiating equation (9.243) with respect to torque angle it becomes,

$$\frac{dP}{d\delta} = \frac{3V_t E_f \cos\delta}{X_d} + \frac{3V_t^2 (X_d - X_q) \sin 2\delta}{X_d X_q} \tag{9.244}$$

where $dP/d\delta$ is known as the synchronizing coefficient.

Again, consider that the generator delivers power with a leading power factor. The armature current I_a leads the terminal voltage V_t by a power factor angle ϕ. However, the armature current lags the induced voltage E_f by an internal angle ψ as shown in Fig. 9.35.

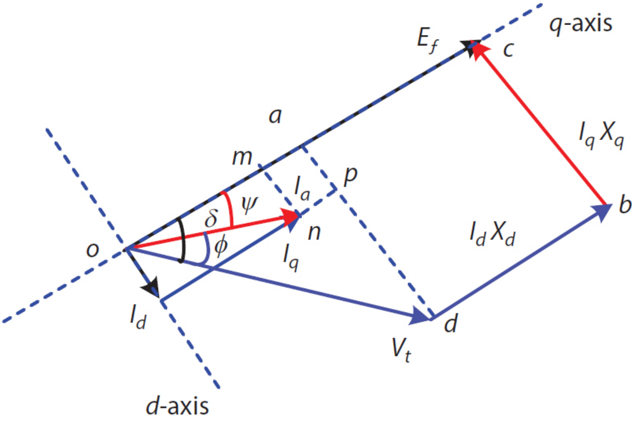


Fig. 9.35 Phasor diagram of a salient pole generator with a leading power factor.

From Fig. 9.35, the relations are written as,

$$\delta = \psi + \phi \tag{9.245}$$

$$\psi = \delta - \phi \tag{9.246}$$

$$om = I_q = I_a \cos\psi \tag{9.247}$$

$$mn = I_d = I_a \sin\psi \tag{9.248}$$

Substituting equation (9.246) into equations (9.247) and (9.248) yields,

$$I_q = I_a \cos(\delta - \phi) \tag{9.249}$$

$$I_d = I_a \sin(\delta - \phi) \tag{9.250}$$

Again, from Fig. 9.34, the following relation is written as follows,

$$V_t \sin\delta = I_q X_q \tag{9.251}$$

Substituting equation (9.249) into equation (9.251) yields,

$$V_t \sin\delta = X_q I_a \cos(\delta - \phi) \tag{9.252}$$

$$V_t \sin\delta = X_q I_a \cos\delta\cos\phi + X_q I_a \sin\delta\sin\phi \tag{9.253}$$

$$\left(V_t - X_q I_a \sin\phi\right)\sin\delta = X_q I_a \cos\delta\cos\phi \tag{9.254}$$

$$\frac{\sin\delta}{\cos\delta} = \frac{X_q I_a \cos\phi}{V_t - X_q I_a \sin\phi} \tag{9.255}$$

$$\tan\delta = \frac{X_q I_a \cos\phi}{V_t - X_q I_a \sin\phi} \tag{9.256}$$

In equation (9.219), it is seen that the sign of the denominator is positive for a lagging power factor, while in equation (9.256), it is negative for a leading power factor. However, the power expression will be the same for leading and lagging power factors.

Example 9.11

A 25 kVA, 440 V, 50 Hz, star-connected, three-phase salient pole alternator is delivering its rated load at a lagging power factor of 0.8. The machine parameters per phase are $Ra = 0.3\Omega$, $Xd = 5\Omega$, $Xq = 3\Omega$. Calculate the torque angle, induced voltage per phase, and voltage regulation (9.268).

Solution

The terminal voltage per phase is determined as,

$$V_t = \frac{440}{\sqrt{3}} = 254\,\text{V} \tag{9.257}$$

The armature current is calculated as,

$$I_a = \frac{25000}{\sqrt{3} \times 440} = 32.8\,\text{A} \tag{9.258}$$

The torque angle is determined as,

$$\tan\delta = \frac{I_a X_q \cos\phi}{V_t + I_a X_q \sin\phi} = \frac{32.8 \times 3 \times 0.8}{254 + 32.8 \times 3 \times 0.6} \tag{9.259}$$

$$\delta = 14.12^\circ \tag{9.260}$$

The direct axis component of the current is calculated as,

$$I_d = I_a \sin(\delta + \phi) = 32.8 \sin(14.12 + 36.7) = 25.43\,\text{A} \tag{9.261}$$

The induced voltage per phase is determined as,

$$E_f = V_t \cos\delta + I_d X_d = 254 \times 0.97 + 25.43 \times 5 = 373.53\,\text{V} \tag{9.262}$$

The voltage regulation is calculated as,

$$VR = \frac{E_f - V_t}{V_t} \times 100 = \frac{373.53 - 254}{254} \times 100 = 47.05\% \tag{9.263}$$

Practice Problem 9.11

A 30 kVA, 415 V, 50 Hz, delta-connected, three-phase salient pole alternator is delivering its rated load at a lagging power factor of 0.95. The machine parameters per phase are $Ra = 0.13\Omega$, $Xd = 6\Omega$, $Xq = 4\Omega$. Find the torque angle, induced voltage per phase and voltage regulation.

9.17 Parallel Operation of an Alternator

In the power station, several alternators are connected in parallel to generate the required magnitude of electrical power to fulfill the consumers' load demand. It is possible to reduce the generation and operation costs when the synchronous generators are connected in parallel. When any alternator is shut down due to a fault and for scheduled maintenance, then the remaining alternator connected in parallel can supply the load smoothly. Therefore, there is an important need for parallel operation among the alternators. The connection of the parallel alternators with the common infinite bus is known as synchronization or paralleling the alternators. The following conditions should be satisfied before making the parallel connection:

- The terminal voltage of the incoming alternator and the voltage of the infinite busbar must be the same.
- The speed of the incoming alternator must be such that its output voltage frequency must be equal to the busbar voltage frequency.
- The phase angle of the incoming alternator must be the same as the running alternator.
- The phase sequence of the incoming alternator voltage must be the same as the busbar voltage. That means the phase sequence R, Y, and B of the incoming alternator must be the same as the busbar sequence as shown in Fig. 9.36.

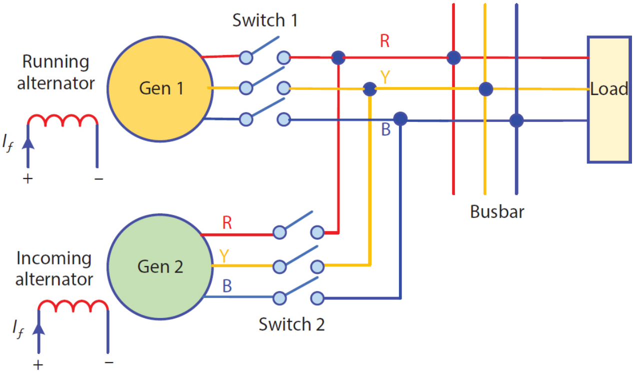

Switch 1
Running alternator
Gen 1
I_f
+
−
R
Y
B
Load
Busbar
R
Y
B
Incoming alternator
Gen 2
I_f
+
−
Switch 2

Fig. 9.36 Two generators are in parallel.

Consider two alternators with the same load-torque characteristics are connected in parallel across the load as shown in Fig. 9.37. The following parameters can be defined as,

V_t is the terminal voltage,

E_1 is the induced voltage per phase of the alternator 1,

E_2 is the induced voltage per phase of the alternator 2.

The impedance and current of generator 1 are Z_1 and I_1. The same parameters of generator 2 are Z_2 and I_2, respectively. From Fig. 9.37, the following equations can be written as,

$$E_1 - I_{a1}Z_1 = V_t \tag{9.264}$$

$$E_2 - I_{a2}Z_2 = V_t \tag{9.265}$$

$$V_t = IZ \tag{9.266}$$

$$I = I_{a1} + I_{a2} \tag{9.267}$$

Equating equations (9.264) and (9.266) yields,

$$E_1 - I_{a1}Z_1 = IZ \tag{9.268}$$

Substituting equation (9.267) into equation (9.268) and it becomes,

$$E_1 - I_{a1}Z_1 = (I_{a1} + I_{a2})Z \tag{9.269}$$

$$E_1 = I_{a1}(Z + Z_1) + I_{a2}Z \tag{9.270}$$

Similarly, using equations (9.265), (9.266) and (9.267) yields,

$$E_2 = I_{a1}Z + (Z + Z_2)I_{a2} \tag{9.271}$$

Multiplying equation (9.271) by Z and equation (9.270) by $(Z + Z_2)$ and then subtracting yields,

$$E_1(Z + Z_2) - E_2Z = I_{a1}[(Z + Z_1)(Z + Z_2) - Z^2] \tag{9.272}$$

$$I_{a1} = \frac{(E_1 - E_2)Z + E_1Z_2}{Z_1Z_2 + Z(Z_1 + Z_2)} \tag{9.273}$$

Substituting equation (9.273) into equation (9.270) and the expression of armature current for the second alternator is written as,

$$I_{a2} = \frac{(E_2 - E_1)Z + E_2Z_1}{Z_1Z_2 + Z(Z_1 + Z_2)} \tag{9.274}$$

Substituting equations (9.273) and (9.274) into equation (9.267) and the expression of load current is written as,

$$I = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2 + Z(Z_1 + Z_2)} \tag{9.275}$$

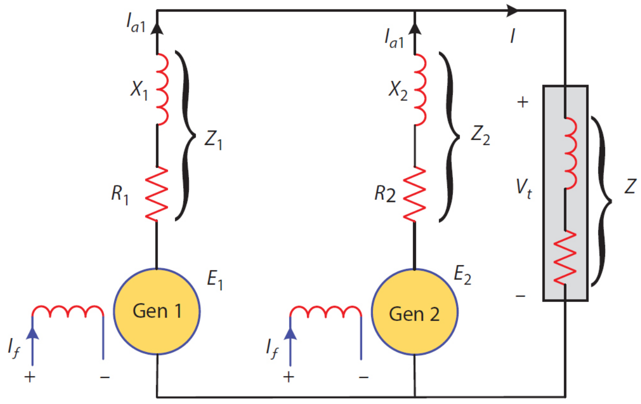


Fig. 9.37 Equivalent circuit when two generators are in parallel.

Example 9.12

Two alternators are connected in parallel to deliver electrical power of the load 5Ω. The necessary parameters of the alternators are,

$$E_1 = 220\,\text{V},\ Z_1 = j3\,\Omega,\ E_2 = 220\angle 5^\circ\,\text{V},\ Z_2 = j4\,\Omega$$

Determine the load current, terminal voltage and power per phase delivered by the first alternator.

Solution

The value of the load current is determined as,

$$I = \frac{E_1Z_2 + E_2Z_1}{Z_1Z_2 + Z(Z_1 + Z_2)} = \frac{j220\times 4 + 220\angle 5^\circ \times j3}{37\angle 109^\circ} \tag{9.276}$$

$$I = \frac{1538.5\angle 92^\circ}{37\angle 109^\circ} = 41.6\angle 17^\circ\,\text{A} \tag{9.277}$$

The terminal voltage is calculated as,

$$V_t = 41.6\angle 17^\circ \times 5 = 208\angle 17^\circ\,\text{V} \tag{9.278}$$

The value of the armature current is determined as,

$$I_{a1} = \frac{(E_1 - E_2)Z + E_1Z_2}{Z_1Z_2 + Z(Z_1 + Z_2)} = \frac{(220 - 220\angle 5^\circ)5 + 220\times j4}{j3\times j4 + 6(j3 + j4)} \tag{9.279}$$

$$I_{a1} = \frac{(220 - 219 - j19)5 + j880}{-12 + j35} = 23.3\angle 19^\circ\,\text{A} \tag{9.280}$$

The value of the power is calculated as,

$$P_1 = V_tI_{a1}\cos\phi_1 = 208\times 41.6\cos(17 - 19) = 8.6\,\text{kW} \tag{9.281}$$

Practice Problem 9.12

Two alternators are connected in parallel to deliver a load to 8Ω. The necessary parameters of the alternators are,

$$E_1 = 210\,\text{V}, Z_1 = j5\,\Omega, E_2 = 210\angle 15^\circ\,\text{V}, Z_2 = j6\,\Omega$$

Find the load current, terminal voltage and three-phase power delivered by the second alternator.

9.18 Load Sharing of Alternators

Initially, consider the two alternators sharing an equal load. In this case, both the armature currents I_1 and I_2 will be the same as shown in Fig. 9.38. From Fig. 9.38(a), it is seen that both the alternators share equally the real power (kW) and the reactive power (kVAR), i.e., kW1 = kW2 and kVAR1 = kVAR2. That means each alternator will supply half of the real power in kW and the reactive power in kVAR. Similarly, the power factor angle will be the same, i.e., $\phi_1 = \phi_2$.

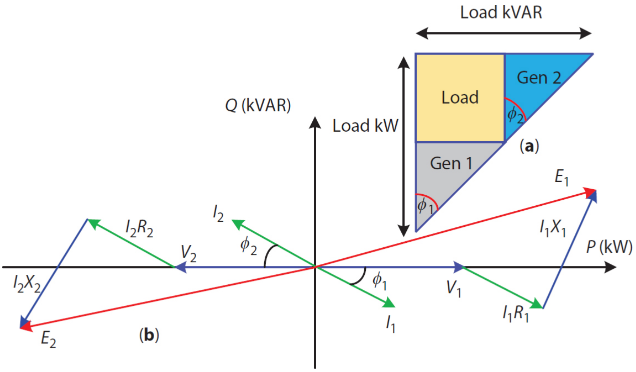


Fig. 9.38 Alternators share an equal load.

Now let the excitation of the alternator 1 increase by increasing the speed. As a result, induced emf E_1 will be higher than the induced emf E_2. This difference of induced emf $Ec = E_1 - E_2$ will make a circulating current I_c flow through both of the alternators as shown in Fig. 9.39(a). This circulating current I_c will added vectorially with the armature current of alternator 1 and subtracted vectorially with the armature current of alternator 2 as shown in Fig. 9.39(b).

These can be written as,

$$(9.282)\quad I_1' = I_1 + I_c$$

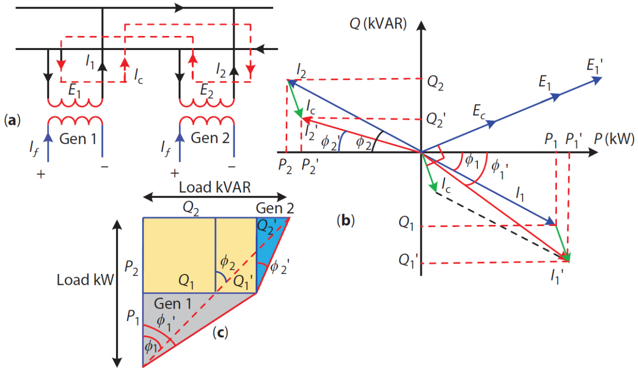

(a)
I_1
I_c
I_2
E_1
E_2
Gen 1
Gen 2
I_f
+
−
I_f
+
−
Load kVAR
Q_2
Gen 2
Q_2'
Load kW
P_2
Q_1
ϕ_2
Q_1'
ϕ_2'
Gen 1
P_1
ϕ_1'
ϕ_1
(c)
Q (kVAR)
I_2
Q_2
E_1'
E_1
I_c
Q_2'
E_c
I_2'
ϕ_2'
ϕ_2
P_1
P_1'
P (kW)
P_2
P_2'
ϕ_1
ϕ_1'
I_c
I_1
(b)
Q_1
Q_1'
I_1'

Fig. 9.39 Alternators share an unequal load.

$$(9.283)\quad I_2' = I_2 - I_c$$

From Fig. 9.39(b), it is seen that the changes in real power ($P_1 \approx P_1'$) by alternator 1 is very less compared to reactive powers ($Q_1 < Q_1'$). In alternator 1, the power factor angle increases and the power factor decreases based on the following equation.

$$(9.284)\quad pf_1 = \frac{P}{\sqrt{P^2 + Q^2}}$$

The real power is almost constant and the reactive power increases which reduces the power factor. Similarly, from Fig. 9.39(c), it is seen that the power factor angle and the reactive power of the alternator 2 reduces which increases the power factor based on equation (9.284).

9.19 Synchronization of Alternators

An incoming alternator connects with a busbar where one alternator is already connected. Three lamps are connected in each phase of the incoming alternator as shown in Fig. 9.40. The incoming alternator rotates clockwise at its rated speed by the prime mover. Then adjust the field current to induce voltages equal to the busbar voltage. During this time, the flickering of the lamps will occur at the rate of the difference of the frequencies between the busbar voltage and incoming alternator voltage. When the generated voltages are equal to the busbar voltage, then turn ON the switch and the voltage across the lamps will be zero. When the phase R_2 of the incoming alternator coincides with busbar voltage R_1, the voltage across the lamp L_1 and the phase displacement is zero. Similarly, other phases will also be zero. During rotation, when phase displacement and voltage across the phases increase, the lamp brightness will also increase. This is known as the dark and brightness lamp method as shown in Fig. 9.40.

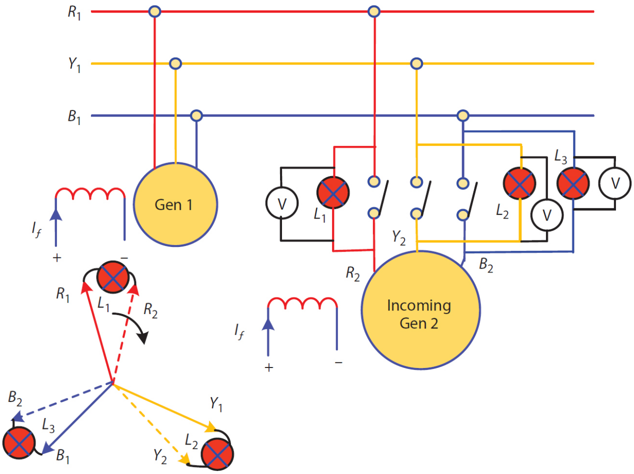


Fig. 9.40 Three lamps dark and bright method.

The dark and bright method is not able to identify the correct phase sequence of the incoming alternator. In this case, two lamps need to be connected with other phases. This method is known as the two bright and one dark method. The lamp L_1 is connected between phase R_1 and incoming generator R_2. Whereas the lamp L_2 is connected between Y_1 and B_2, the lamp L_3 is connected between B_1 and Y_2 as shown in Fig. 9.41.

The field excitation of the incoming generator is adjusted and rotates nearer to the rated speed so that the generated voltages of the incoming generator are the same as the busbar voltages. At this moment, the light L1 connected straight to the phase R_1 and R_2 will be dark and other lights L_2 and L_3 connected cross will be bright equally. Then immediately close the switch, which means the incoming alternator is synchronized with the correct phase sequence. The voltmeter readings across the lamp L_1 will be zero and other lamps L_2 and L_3 will be larger. Therefore, the incoming generator is ready to deliver the load.

In the lamp method, the identification of synchronization depends on the operator's judgement. Therefore, this method is not accurate and there is a need to identify synchronization automatically. This can be done by equipment known as a synchroscope. The synchroscope can identify the frequency, phase and phase angle of the incoming alternator and the busbar voltage.

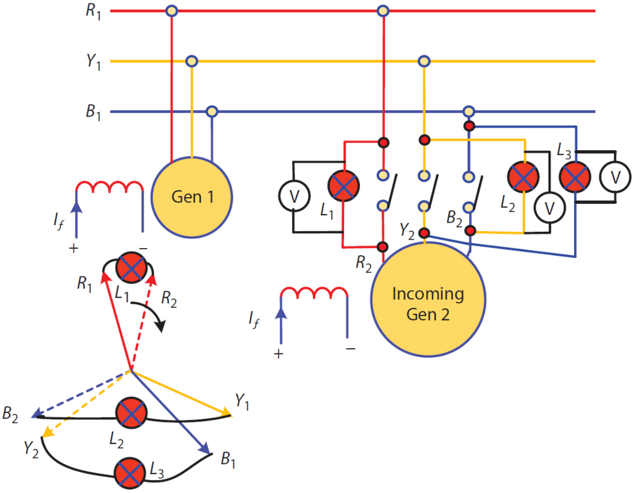


Fig. 9.41 One dark and two bright lamps method.

The synchroscope consists of the stator and rotor as shown in Fig. 9.42. The two phases of the incoming alternator supply power to the two poles of the stator terminals and the existing busbar gives power to the rotor.

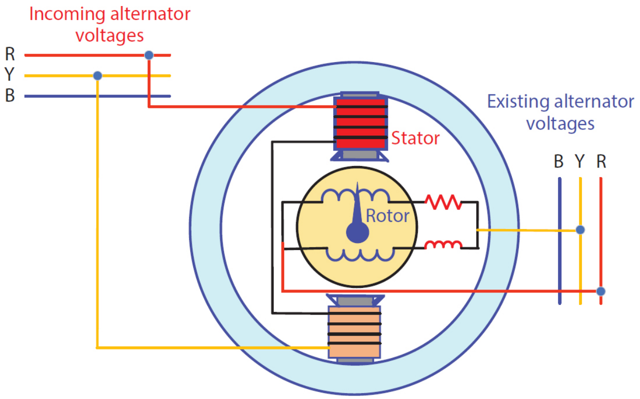


Fig. 9.42 Synchroscope connection diagram.

The armature winding (rotor) has two windings connected in parallel. One winding is connected in series with the resistor and the other winding is connected with the inductor to make the phase difference to 90º. These currents enter the rotor circuit which creates the rotating magnetic field with the interaction of stator poles. This rotating magnetic field rotates in clockwise and anticlockwise directions, which rotates the pointer of the synchroscope. The direction of the pointer depends on the speed and frequency of the incoming alternator. The meter has a pointer, that normally moves either at zero or 180 electrical degrees. The zero position is mentioned at the top of the meter and 180 electrical degrees at the bottom side. The alternators will be in the same phase if the pointer is at zero position. If the oncoming alternator is faster than the running one, then the needle will move in the clockwise direction.

Conversely, if the oncoming alternator is slower than the running one, then the needle will move in the counterclockwise direction. The switch can shut to connect the system if the voltages are in the same phase. It means that the needle of the meter is in the vertical position. In the modern power station, synchronization is normally done automatically. The physical appearance of the synchronoscope meter is shown in Fig. 9.43.

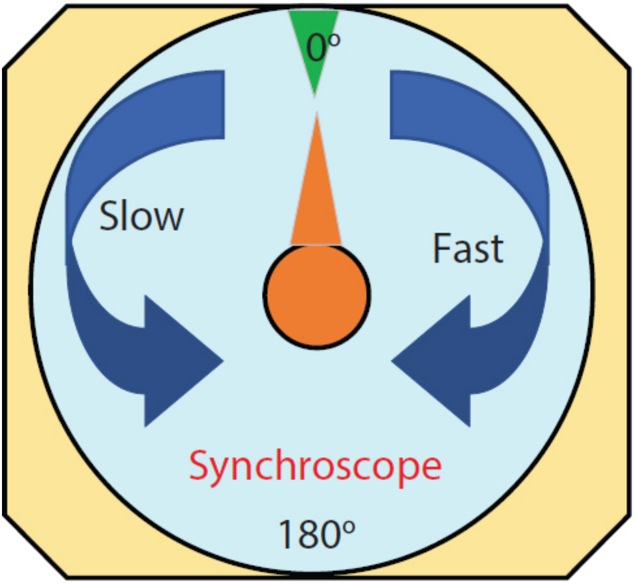


Fig. 9.43 Schematic of synchronoscope.

9.20 Capability Curve of Alternators

The boundaries within which the generator can supply reactive power continuously without overheating are known as the generator capability curve. In other words, the graphical representation of the heating of the stator winding, rotor winding and end part of the generator is known as the capability curve. It is often known as operating charts or capability charts of the synchronous generator. The information about maximum generation limit, steady-state stability, protection system set-up, minimum generation, etc., can be gathered from the capability curve. Generators can deliver reactive power without overheating the three important parameters limit such as stator current limit, rotor field current limit and the stator end part heating limit. The armature current in the stator windings will create heat due to the ohmic losses $I_a^2R_a$. Similarly, in the rotor windings, heat will be generated due to flowing the rotor field current. Due to this rotor heat, the generator can supply or absorb the maximum value of the reactive power. The stator end part puts limits when the generator operates with a leading power factor. Under the leading power factor, the generator is under excited, therefore, more field current is required to maintain the working flux for running the generator.

The capability curve of a generator is a plot of apparent power, real power and reactive power. From the knowledge of AC circuits, the expressions of those powers are written as,

$$P = 3V_t I_a \cos\theta \tag{9.285}$$

$$Q = 3V_t I_a \sin\theta \tag{9.286}$$

$$S = 3V_t I_a \tag{9.287}$$

The generated voltage is expressed as,

$$E_f \angle\delta = V_t + I_a \angle -\theta (R_a + jX_s) \tag{9.288}$$

If the armature resistance is negligible, then equation (9.288) becomes,

$$E_f \angle\delta = V_t + jX_s I_a \angle -\theta \tag{9.289}$$

Based on equation (9.289), the phasor diagram is drawn by considering the terminal voltage as a reference phasor as shown in Fig. 9.44.

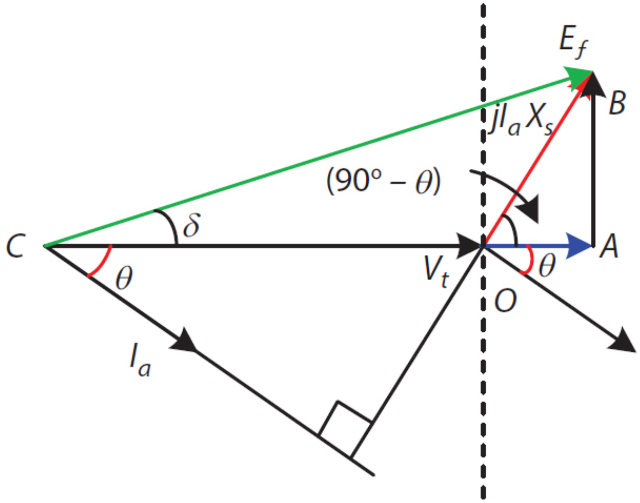


Fig. 9.44 Phasor diagram.

Let the point *O* be the origin and its coordinate is (0, 0). Then the coordinate of the point *C* is (-*Vt*, 0). From the *OBC* triangle in Fig. 9.43, the following relations are written as,

$$OA = I_a X_s \cos(90^\circ - \theta) = I_a X_s \sin\theta \tag{9.290}$$

$$AB = I_a X_s \sin(90^\circ - \theta) = I_a X_s \cos\theta \tag{9.291}$$

Multiplying by 3*Vt*/*Xs* to convert lines AB and OA in power format. Equations (9.290) and (9.291) are modified as,

$$Q_{OA} = I_a X_s \sin\theta \times \frac{3V_t}{X_s} \tag{9.292}$$

$$Q_{OA} = 3I_a V_t \sin\theta \tag{9.293}$$

$$P_{AB} = I_a X_s \cos\theta \times \frac{3V_t}{X_s} \tag{9.294}$$

$$P_{AB} = 3VI_a \cos\theta \tag{9.295}$$

The coordinate *C* in the voltage phasor diagram can be modified by multiplying by 3*Vt*/*Xs* to convert it to a reactive power line *Q* yields,

$$Q_C = (-V_t, 0) \times \frac{3V_t}{X_s} = \left(-\frac{3V_t^2}{X_s}, 0\right) \tag{9.296}$$

$$CB = E_f \times \frac{3V_t}{X_s} = \frac{3E_f V_t}{X_s} \tag{9.297}$$

$$S_{OB} = I_a X_s \times \frac{3V_t}{X_s} = 3V_t I_a \tag{9.298}$$

A circle is drawn with a radius of $S_{OB} = 3V_{tIa}$ with a blue colour as shown in Fig. 9.45(a). This circle represents the stator armature current heating limit. Similarly, another circle is drawn with a red colour with a radius of CB with a center Q_C. The generated voltage E_f is directly proportional to the flux and angular speed, i.e., $E_f = k\phi\omega$. The field flux depends on the rotor field current. Therefore, this red circle represents the rotor field current heating limit. The final generator capability curve is drawn as shown in Fig. 9.45(b).

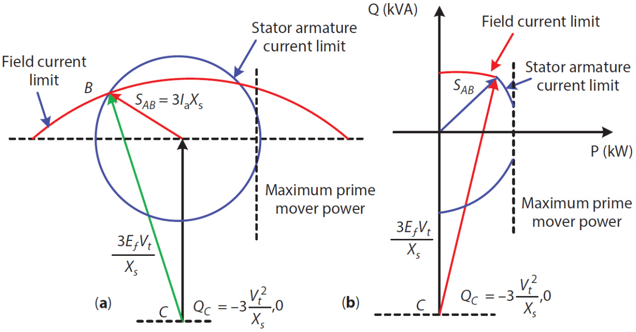

Field current limit
B
$S_{AB} = 3I_aX_s$
Stator armature current limit
Maximum prime mover power
$\frac{3E_fV_t}{X_s}$
(a)
C
$Q_C = -3\frac{V_t^2}{X_s},0$
Q (kVA)
Field current limit
Stator armature current limit
S_{AB}
P (kW)
Maximum prime mover power
$\frac{3E_fV_t}{X_s}$
(b)
C
$Q_C = -3\frac{V_t^2}{X_s},0$

Fig. 9.45 Generator capability curve.

Exercise Problems

9.1 Calculate the pitch factor for an alternator having 72 slots, 4-pole and coil spanning 14 slots (coil span 1 to 15).

9.2 Determine the pitch factor for an alternator having 48 slots, 4-pole and a coil span of 1 to 7.

9.3 A three-phase, 12-pole alternator has 108 slots for double-layer windings. Determine the distribution factor.

9.4 A three-phase, 4-pole alternator has 72 slots for double-layer windings. Determine the distribution factor.

9.5 The number of slots of a three-phase, 4-pole, 50 Hz synchronous generator is 24. The coil pitch is 5 slots. Determine the pitch factor and the distribution factor.

9.6 The number of slots of a 6-pole, three-phase, 50 Hz alternator is 54. If the winding is short-chorded by one slot-pitch, determine the pitch factor.

9.7 The number of slots of a three-phase, Y-connected, 8-pole synchronous generator is 72. The conductors per slot are 6 and the flux per pole is 30 mWb. If the speed of the rotor is 750 rpm, find the frequency and the line voltage. Consider full-pitch winding.

9.8 The number of slots of a three-phase, Y-connected, 4-pole 50 Hz synchronous generator is 60. The conductors per slot are 4 and the flux per pole is 836 mWb. The coils are short-pitched by 3 slots and the phase spread is 60º. Determine the phase voltage and line voltage.

9.9 The number of slots of a three-phase, 8-pole, Y-connected synchronous generator is 96. The conductors per slot are 6 and the flux per pole is 0.5 Wb, which is distributed sinusoidally. Determine the phase voltage at 60 Hz. Assume full-pitch winding.

9.10 A three-phase, 4-pole synchronous generator has 36 slots and each slot contains 6 conductors. The speed of the rotor is 1500 rpm and the flux per pole is 0.2 Wb sinusoidally distributed. Calculated the line voltage, when the generator phases are wye- and delta-connected. Assume the winding factor is 0.96.

9.11 A three-phase, 8-pole, star-connected alternator has 72 slots with 10 conductors per slot. The winding factor is 0.97 and the rotor revolves at 900 rpm. Determine the flux per pole if the voltage across the line is 2400 V.

9.12 The phase voltage of a three-phase, 4-pole, star-connected alternator is 1034.4 V. The speed of the rotor is 1500 rpm and the flux per pole is 0.05 Wb. Determine the number of conductors per phase if the winding factor is 0.94.

9.13 A 70 kVA, 230 V, 50 Hz, the single-phase alternator has an effective resistance of 0.04 Ω and a synchronous reactance of 0.06 Ω. Determine the no-load induced voltage per phase and voltage regulation at rated current and 0.9 power factor leading.

9.14 A three-phase, 1500 kVA, 11 kV, star-connected, 50 Hz alternator has an armature resistance of 0.08 Ω and a synchronous reactance of 2.2 Ω. Determine the no-load induced voltage per phase and the voltage regulation at 0.85 power factor lagging.

9.15 The armature resistance of an alternator is 0.15 Ω. A certain magnitude of field current produces a short circuit current of 150 A and generates an open circuit per phase voltage of 60 V. Determine the synchronous impedance, synchronous reactance, and no-load voltage per phase at 90 A rated current, 0.8 power factor lagging and the terminal voltage of 200 V per phase.

9.16 An alternator of 18 A field current produces a short circuit current of 180 A and generates an open circuit voltage of 500 V. Find the no-load voltage per phase with a rated current of 60 A, 0.85 power factor leading and terminal voltage of 220 V per phase. Consider the armature resistance 0.35Ω.

9.17 A certain magnitude of field current produces a short circuit current of 100 A and generates a per-phase open circuit voltage of 90 V. The armature resistance of an alternator is 0.11 Ω. Determine the synchronous impedance, synchronous reactance and voltage regulation at a unity power factor and the terminal voltage of 240 V per phase.

9.18 The synchronous reactance of a 35 MVA, 20 kV, 1800 rpm, three-phase, wye-connected alternator is 4.5 Ω. The induced voltage per phase is 20 kV and the line-to-line terminal voltage is 15 kV. Find the per-phase real power delivered to the load at a power angle of 30° and three-phase maximum power.

9.19 A 30 MVA, 20 kV, 1800 rpm, star-connected, three-phase alternator is delivering a power of 75 MW to the load. The induced voltage per phase is 22 kV and the line-to-line terminal voltage is 12 kV. Determine the value of synchronous reactance if the torque angle is 45°.

9.20 The synchronous reactance of a 20 MVA, 15 kV, 1800 rpm, delta-connected, three-phase alternator is 3 Ω and delivers a power of 80 MW to the load. The line-to-line terminal voltage is 10 kV. Determined the induced voltage per phase if the torque angle of 25°.

9.21 A 20 kVA, 415 V, 50 Hz, star-connected, three-phase salient pole alternator is delivering its rated load at a lagging power factor of 0.93. The machine parameters per phase are,

$$R_a = 0.35\Omega,\ X_d = 5.5\Omega,\ X_q = 3.4\Omega$$

Calculate the torque angle, induced voltage per phase and voltage regulation.

9.22 A 25 kVA, 440 V, 50 Hz, delta-connected, three-phase salient pole alternator is delivering its rated load at a leading power factor of 0.95. The machine parameters per phase are,

$$R_a = 0.25\Omega,\ X_d = 8\Omega,\ X_q = 4.8\Omega$$

Find the torque angle, induced voltage per phase and voltage regulation.

9.23 Two alternators are connected in parallel to deliver a load to the 10 Ω resistance. The necessary parameters of the alternators are:

$$E_1 = 210\text{V},\ Z_1 = 2 + j3.5\Omega,\ E_2 = 210\angle 6^\circ\ \text{V},\ Z_2 = 3 + j5\Omega$$

Calculate the load current, terminal voltage and power delivered by the second alternator.

10

Synchronous Motor

10.1 Introduction

The synchronous machine is normally considered a synchronous generator and synchronous motor. The synchronous generator has already been described in Chapter 9. The synchronous generator can run as a synchronous motor if the generator armature windings are connected to a three-phase supply. The synchronous motor runs at a constant speed known as synchronous speed. The synchronous speed of the motor depends on the supply frequency and the number of poles. The speed of the synchronous motor is also independent of the load. The torque of the motor increases with the increase of the load. Therefore, the motor will stop when the load torque exceeds the maximum torque. The synchronous motors are normally rated from 200HP to above. These high-rated synchronous motors are mainly used in the industry to perform heavy duty. Sometimes, synchronous motors are operated at no-load to improve the power factor, which is called the synchronous condenser. Some remarkable characteristics of a synchronous motor are: (i) it is not self-starting, (ii) it runs either at synchronous speed or less than the synchronous speed, and (iii) it is operated under a wide range of power factors for both lagging and leading. The details of the synchronous motor are discussed here.

10.2 Construction

The construction of the synchronous motor is identical to the construction of the synchronous generator or a three-phase induction motor. It has two components; namely, stator and rotor.

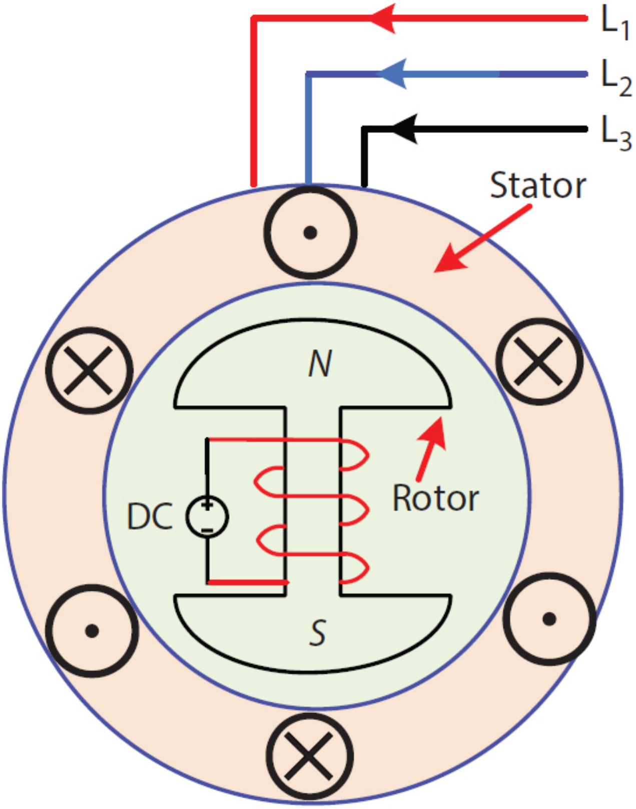


Fig. 10.1 Stator and rotor of a synchronous motor.

Stator: The stator is the stationary part of the synchronous motor and provides a suitable number of slots. This slot carries a three-phase lap winding and these windings are connected with a three-phase supply as shown in Fig. 10.1.

Rotor: The rotor is the inner part of the synchronous motor. The rotor contains field windings, which are connected in series with the slip rings. These windings are energized by direct current from a separate DC source as shown in Fig. 10.1.

10.3 Working Principle

Consider a two-pole synchronous motor having the stator poles N_s and S_s, respectively. By applying the DC supply, the magnetic poles created in the rotor are N_r and S_r, respectively. A three-phase voltage is applied to the stator windings, the current flows through the stator windings. A magnetic field of constant magnitude is produced due to the currents in the stator windings. This magnetic field moves in a clockwise direction at a synchronous speed.

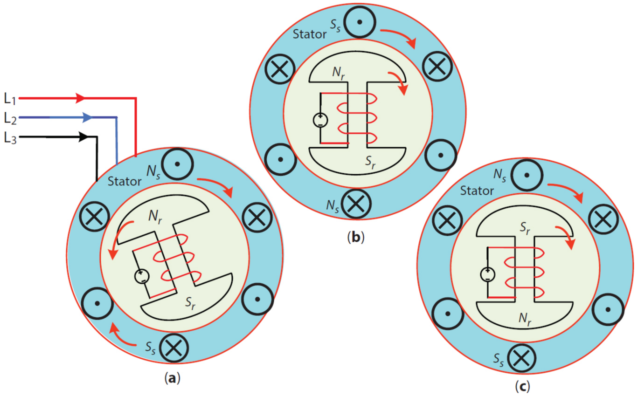


Fig. 10.2 Different positions of stator and rotor.

In Fig. 10.2(a), the similar poles of the stator and the rotor will repel each other. As a result, the rotor will move in the anticlockwise direction, whereas the stator will move in the clockwise direction and the toque will be produced in an anticlockwise direction. The stator poles rotate faster at a synchronous speed and the rotor poles rotate slower due to inertia. The pole of the stator and the rotor change their positions and the new position of the poles is shown in Fig. 10.2(b). In this position, the opposite poles of the stator and rotor will attract each other. Under this condition, the torque will be produced which rotates the motor in a clockwise direction. Therefore, the torque produced in a three-phase synchronous motor is changing rapidly and is not unidirectional. That is why the synchronous motor is not self-starting.

To start the synchronous motor, the rotor poles must be brought closer to synchronous speed by an external device which also changes the polarity of the rotor poles after half a cycle. Rotor magnetic poles lock into the step with the stator magnetic field once the rotor magnetic field is brought closer to the synchronous speed. As a result, the rotor rotates at the same speed as the stator's rotating magnetic field, which is known as synchronous speed.

Again, the poles of the stator and the rotor are shifted to the new position as shown in Fig. 10.2(c). Here, the poles of the stator and the rotor will repel each other and the rotor will move in the clockwise direction. If the process is continued, the synchronous motor will run continuously.

10.4 Starting of Synchronous Motor

The stator windings are normally energized by a three-phase AC supply and the rotor windings are excited by the separate DC source. If we energize the stator and rotor at the same time, the created poles will rotate at synchronous speed and they will lock each other. In this case, the average torque is zero. That is why the synchronous motor is not self-starting. In order to make the motor self-starting, the following methods are considered.

Amortisseur windings: In this method, additional windings are placed in the pole faces of the motor.

These windings are also known as damper windings or squirrel cage windings and are made of copper bars. The damper windings are placed in the pole faces and are shorted by the end rings as shown in Fig. 10.3.

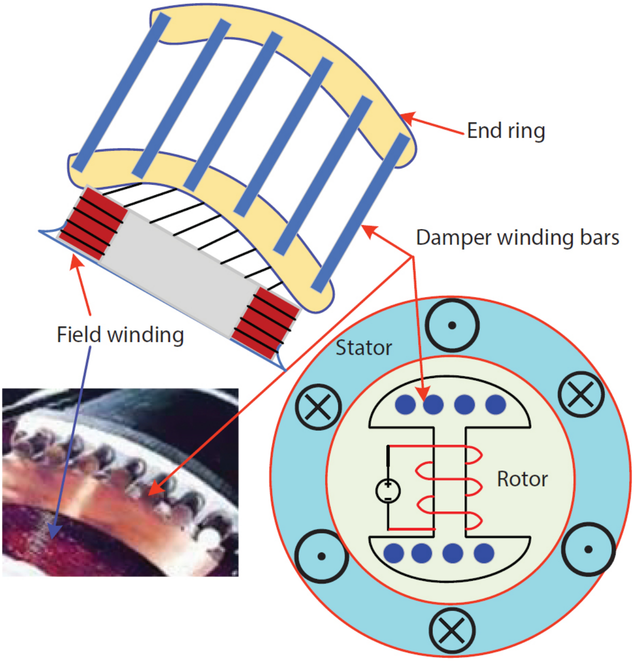


Fig. 10.3 Damper windings with end rings.

A synchronous motor with the damper windings will work as a squirrel cage induction motor. Initially, a three-phase supply is applied to the stator windings and the rotor is not energized. In this case, the rotating field is created by the stator. This field produces current in the damper windings. As a result, the motor starts as an induction motor. As soon as the motor approaches near synchronous speed, the field excitation is applied to the rotor. This field excitation produces rotor poles, which are of opposite polarity to the stator poles. The stator poles attract the rotor poles with a strong force. Therefore, the rotor poles move at the speed of the stator poles, i.e., at synchronous speed.

External prime mover: In this method, an external motor is attached to the shaft of the synchronous motor as shown in Fig. 10.4. The synchronous motor runs up to its full load speed by the extra motor. After starting the synchronous motor, the extra motor is switched off. Once the synchronous machine is synchronized, then the machine works as an asynchronous motor.

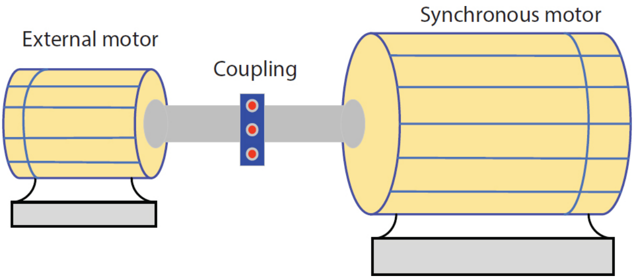


Fig. 10.4 External motor to start the synchronous motor.

Reducing frequency: The number of poles of the synchronous motor is fixed. In this method, the speed of the stator rotating magnetic field is reduced enough so that the rotor can accelerate and lock in with it quickly. The speed of the stator field is varied gradually by increasing the frequency up to its normal value of 50 Hz. Some electronic configurations, like inverter and cycloconverter, are used to perform this task. This method is used only for variable frequency speed control.

10.5 Equivalent Circuit

Consider the non-salient or cylindrical pole rotor for drawing the per-phase equivalent circuit. Here, the following parameters are defined as,

R_a is the effective armature resistance,

X_s is the synchronous reactance,

Z_s is the synchronous impedance,

V_t is the terminal voltage per-phase,

E_f is the excitation or induced voltage per phase.

The equivalent circuit of a synchronous motor is the same as the synchronous generator except that the direction of current flow is reversed. Consider per phase equivalent circuit, which is shown in Fig. 10.5. Apply KVL to the circuit in Fig. 10.4 and it becomes,

$$-V_t + (R_a + jX_s)I_a + E_f = 0 \tag{10.1}$$

$$V_t = I_a(R_a + jX_s) + E_f \tag{10.2}$$

$$E_f = V_t - I_aR_a - jI_aX_s \tag{10.3}$$

$$V_t = I_aZ_s + E_f \tag{10.4}$$

where the expression of the synchronous impedance is written as,

$$Z_s = R_a + jX_s \tag{10.5}$$

From equation (10.3), the expression of armature current is written as,

$$I_a = \frac{V_t - E_f}{Z_s} \tag{10.6}$$

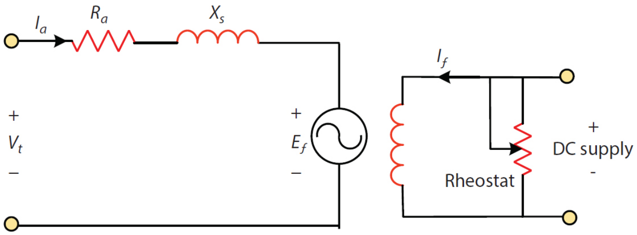


Fig. 10.5 Per phase equivalent circuit.

The terminal voltage will be equal to the open circuit voltage when the armature current is zero. From equation (10.3), it can be derived as,

$$V_t = 0 \times Z_s + E_f \tag{10.7}$$

$$V_t = E_f = E_{oc} \tag{10.8}$$

10.6 Phasor Diagrams

The terminal voltage V_t is considered the reference vector for drawing the phasor diagrams of the synchronous motor. The phasor diagrams for different power factors are discussed below.

10.6.1 Unity Power Factor

For the unity phasor diagram, the armature current is in phase with the terminal voltage. Consider the terminal voltage as a reference phasor. The resistive drop $I_a R_a$ is drawn at the same line as the terminal voltage V_t, which is parallel and opposite to the armature current I_a. The reactive drop $I_a X_s$ lags the resistive drop $I_a R_a$ by an angle of 90°. The excitation voltage E_f is the phasor sum of the terminal voltage, resistive drop and reactive drop as shown in Fig. 10.6. The phase difference between the terminal voltage and the excitation voltage is known as power angle or torque angle δ.

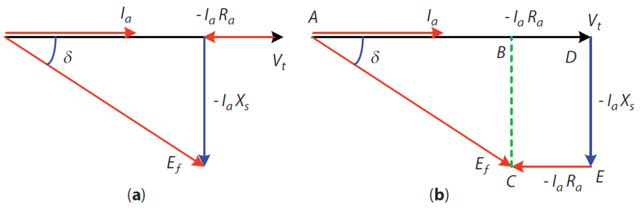


Fig. 10.6 Phasor diagram with a unity power factor.

Draw reactance drop first and resistive drop as shown in Fig. 10.6(b). From Fig. 10.6(b), the following relation is written as,

$$(10.9)\ AC^2 = AB^2 + BC^2$$

$$(10.10)\ AC^2 = (AD - BD)^2 + BC^2$$

$$(10.11)\ E_f^{\ 2} = (V_t - I_a R_a)^2 + (I_a X_s)^2$$

$$(10.12)\ E_f = \sqrt{(V_t - I_a R_a)^2 + (I_a X_s)^2}$$

10.6.2 Lagging Power Factor

In this phasor diagram, the armature current I_a lags the terminal voltage V_t by an angle of ϕ. The resistive drop $I_a R_a$ is drawn from the end of the terminal voltage V_t, which is parallel and opposite to the armature current I_a. The reactive drop $I_a X_s$ lags the resistive drop $I_a R_a$ by an angle of 90°. The excitation voltage is the phasor sum of the terminal voltage, resistive and reactive drop. The phasor diagram with a lagging power factor is shown in Fig. 10.7.

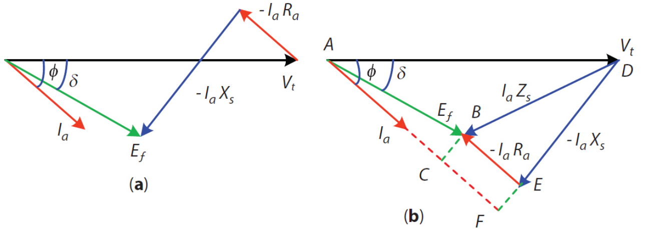


Fig. 10.7 Phasor diagram with a lagging power factor.

Again, draw the reactance drop first with the terminal voltage and then draw the resistive drop as shown in Fig. 10.7(b). In this case, the excitation voltage is less than the terminal voltage so the synchronous motor is underexcited. From Fig. 10.7(b), based on triangle *ABC*, the following relation is written as,

$$AB^2 = AC^2 + BC^2 \tag{10.13}$$

$$AB^2 = (AF - CF)^2 + (DF - ED)^2 \tag{10.14}$$

$$E_f{}^2 = (V_t \cos\phi - I_a R_a)^2 + (V_t \sin\phi - I_a X_s)^2 \tag{10.15}$$

$$E_f = \sqrt{(V_t \cos\phi - I_a R_a)^2 + (V_t \sin\phi - I_a X_s)^2} \tag{10.16}$$

10.6.3 Leading Power Factor

In this phasor diagram, the armature current leads the terminal voltage by an angle of ϕ. The resistive drop $I_a R_a$ is drawn from the end of the terminal voltage V_t, which is parallel and opposite to the armature current I_a. The reactive drop $I_a X_s$ is perpendicular to the resistive drop $I_a R_a$. The excitation voltage is the phasor sum of the terminal voltage, resistive and reactive drop. The phasor diagram with a leading power factor is shown in Fig. 10.8. In this case, the excitation voltage is higher than the terminal voltage. Therefore, it is overexcited under the leading power factor. From

Fig. 10.8(b), based on the triangle *ABF*, the following relation is written as,

$$AF^2 = AB^2 + BF^2 \tag{10.17}$$

$$AC^2 = (AC - AB)^2 + (CD + DE)^2 \tag{10.18}$$

$$E_f^{\ 2} = (V_t \cos\phi - I_a R_a)^2 + (V_t \sin\phi + I_a X_s)^2 \tag{10.19}$$

$$E_f = \sqrt{(V_t \cos\phi - I_a R_a)^2 + (V_t \sin\phi + I_a X_s)^2} \tag{10.20}$$

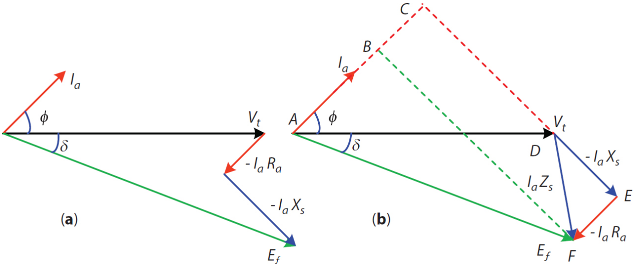


Fig. 10.8 Phasor diagram with a leading power factor.

10.7 Synchronous Impedance Diagrams

Consider the leading power factor diagram for understanding the synchronous impedance diagram. The negative signs for resistive and reactive drops are omitted for easy analysis. Fig. 10.9 shows the different forms of the impedance diagrams. In these diagrams, the value of the resultant voltage is,

$$E_r = I_a Z_s \tag{10.21}$$

In Fig. 10.9(a), β is the angle between the resistive and impedance drops and δ is the torque angle between the terminal voltage and the excitation voltage. The value of the armature resistance is very small compared to the synchronous reactance.

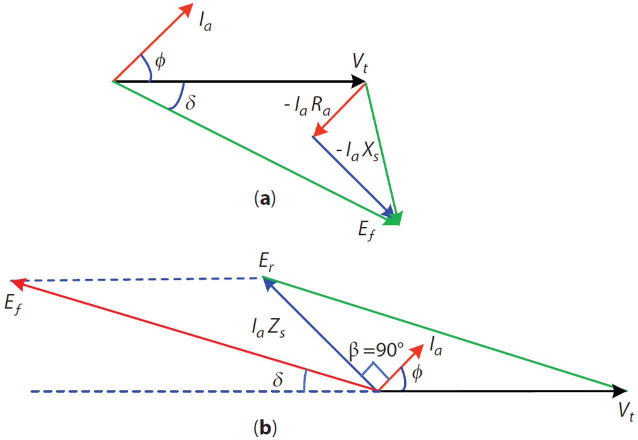


Fig. 10.9 Impedance phasor diagrams.

The synchronous impedance will be equal to the synchronous reactance if the armature resistance is neglected. The impedance diagram without a resistive drop is shown in Fig. 10.9(b). In this diagram, the impedance drop is perpendicular to armature resistance, i.e., $\beta = 90°$ and the expression of the resultant voltage is,

$$E_r = I_a X_s \tag{10.22}$$

10.8 Effect of Different Field Excitations

Let us consider that the synchronous motor is connected to an infinite bus. Therefore, its terminal voltage and frequency are constant. Normally, the speed and the frequency of the synchronous motor are constant. In this case, the excitation voltage per phase of the synchronous motor varies with the variation of the field excitation. The phasor diagrams with different field excitations are discussed below.

10.8.1 Critical Excitation

A synchronous motor is at the normal excitation when the field excitation is set at such a value that makes the relation $E_f = V_t$. The phasor diagram for normal excitation is shown in Fig. 10.10. At the unity power factor, the armature current is in phase with the terminal voltage. From Fig. 10.10, based on the right angle triangle, the excitation voltage per phase is written as,

$$E_f = \sqrt{V_t^{\,2} + E_r^{\,2}} \tag{10.23}$$

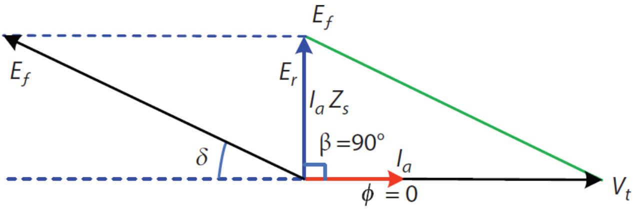


Fig. 10.10 Phasor diagram with normal excitation.

10.8.2 Under-Excitation

Under-excitation, the field excitation is set at such a value (less than 100%) that makes the relationship between the excitation voltage and the terminal voltage $E_f < V_t$. The phasor diagram under-excitation is shown in Fig. 10.11. In this phasor diagram, the armature current lags behind the terminal voltage by an angle of ϕ. Again the impedance drop $I_a Z_s$ is perpendicular to the armature current I_a. The excitation voltage per phase can be calculated as,

$$E_f = \sqrt{V_t^{\,2} + E_r^{\,2} - 2V_t E_r \cos(\beta - \phi)} \tag{10.24}$$

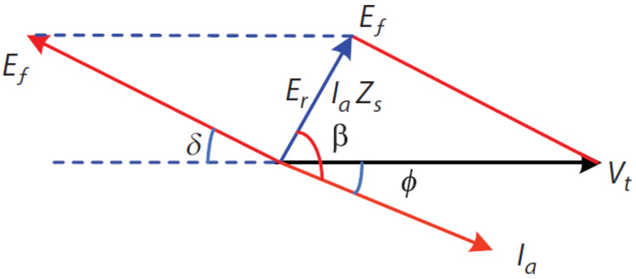


Fig. 10.11 Phasor diagram with under-excitation.

10.8.3 Over-Excitation

In over-excitation, the field current is adjusted at such a value that makes the relation between the excitation voltage and the terminal voltage, as $E_f > V_t$. The phasor diagram with over-excitation is shown in Fig. 10.12. In this phasor diagram, the armature current leads the terminal voltage by an angle of ϕ and the impedance drop I_aZ_s is perpendicular to the armature current I_a. From Fig. 10.12, the excitation voltage per phase can be calculated as,

$$E_f=\sqrt{V_t^2+E_r^2-2V_tE_r\cos(\beta+\phi)} \tag{10.25}$$

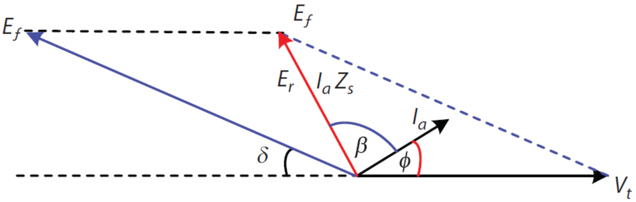

E_f
E_f
E_r
I_aZ_s
I_a
β
ϕ
δ
V_t

Fig. 10.12 Phasor diagram with over-excitation.

The phasor diagram with a constant load and variable excitation is shown in Fig. 10.13. The load is constant means that the power ($P = V_tI_a\cos\phi$) is constant. At the same time, the armature current and terminal voltage is also constant. Now vary the field excitation in such a way that the induced emf is equal to the terminal voltage is known as normal excitation ($E_f=V_t$) as shown in Fig. 10.13(a).

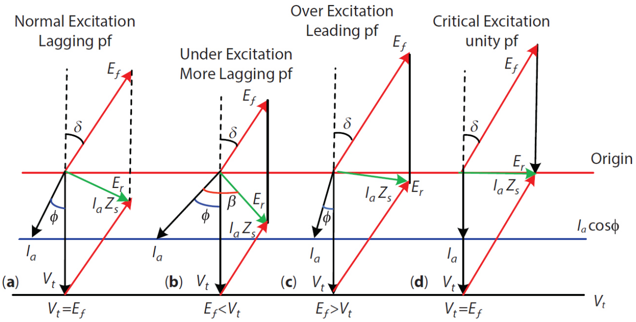

Normal Excitation
Lagging pf
Under Excitation
More Lagging pf
Over Excitation
Leading pf
Critical Excitation
unity pf
E_f
E_f
E_f
E_f
δ
δ
δ
δ
Origin
E_r
I_aZ_s
ϕ
β
E_r
I_aZ_s
ϕ
ϕ
I_aZ_s
E_r
E_r
I_aZ_s
$I_a\cos\phi$
I_a
I_a
I_a
I_a
(a)
V_t
(b)
V_t
(c)
V_t
(d)
V_t
V_t
$V_t=E_f$
$E_f<V_t$
$E_f>V_t$
$V_t=E_f$

Fig. 10.13 Phasor diagram with different field excitation.

Similarly, varying the field excitation in such a way that the induced emf is less than the terminal voltage is known as under-excitation ($E_f < V_t$) as shown in Fig. 10.13(b), whereas varying the field excitation to achieve the conditions $E_f > V_t$ and $E_f = V_t$ is known as over-excitation and critical excitation as shown in Fig.

10.13(c) and (d).

Example 10.1

The per-phase armature resistance and the synchronous reactance of a 2.5 kV, three-phase, star-connected synchronous motor are 0.12 Ω and 3.2 Ω, respectively. The motor runs with a line current of 180 A at a 0.8 leading power factor. Find the excitation voltage per phase.

Solution

The terminal voltage per phase is calculated as,

$$V_t = \frac{2.5 \times 1000}{\sqrt{3}} = 1443.4\,\text{V} \tag{10.26}$$

The impedance per phase is calculated as,

$$Z_s = 0.12 + j3.2 = 3.2\angle 87.8^\circ\,\Omega \tag{10.27}$$

The resultant voltage due to impedance is calculated as,

$$E_r = I_a Z_s = 185 \times 3.2 = 592\,\text{V} \tag{10.28}$$

The angle β is calculated as,

$$\beta = \angle 87.8^\circ \tag{10.29}$$

The angle ϕ is calculated as,

$$\phi = \cos^{-1}(0.8) = 36.87^\circ \tag{10.30}$$

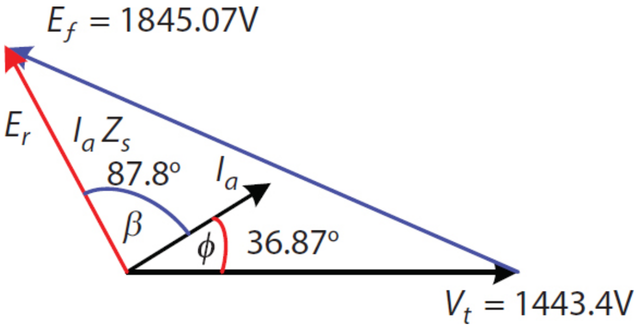


Fig. 10.14 Phasor diagram for example 10.1.

Fig. 10.14 shows the phasor diagram. The excitation voltage per phase can be calculated as,

$$E_f = \sqrt{V_t^2 + E_r^2 - 2V_t E_r \cos(\beta + \phi)} = \sqrt{1443.4^2 + 592^2 - 2 \times 1443.4 \times 592 \times \cos(87.8 + 36.8)} \tag{10.31}$$

$$E_f = 1845.07\,\text{V} \tag{10.32}$$

Example 10.2

The per-phase armature resistance and the synchronous reactance of a 1200 kVA, 14 kV, three-phase, wye-connected synchronous motor are 4.8 Ω and 35 Ω, respectively. Determine the excitation voltage per phase and the torque angle at a 0.95 power factor lagging.

Solution

The synchronous impedance per phase is calculated as,

$$Z_s = 4.8 + j35 = 35.3\underline{|82.2^\circ}\,\Omega \tag{10.33}$$

The value of the angle is written as,

$$\beta = 82.2^\circ \tag{10.34}$$

The armature current is calculated as,

$$I_a = \frac{1200 \times 1000}{\sqrt{3} \times 14 \times 1000} = 49.5\,\text{A} \tag{10.35}$$

The voltage due to impedance is calculated as,

$$E_r = I_a Z_s = 49.5 \times 35.3 = 1747.35\,\text{V} \tag{10.36}$$

The terminal voltage per phase is calculated as,

$$V_t = \frac{14 \times 1000}{\sqrt{3}} = 8082.9\,\text{V} \tag{10.37}$$

The power factor angle is calculated as,

$$pf = \cos\phi = 0.95 \tag{10.38}$$

$$\phi = 18.2^\circ \tag{10.39}$$

The difference in angle is determined as,

$$\beta - \phi = 82.2^\circ - 18.2^\circ = 64^\circ \tag{10.40}$$

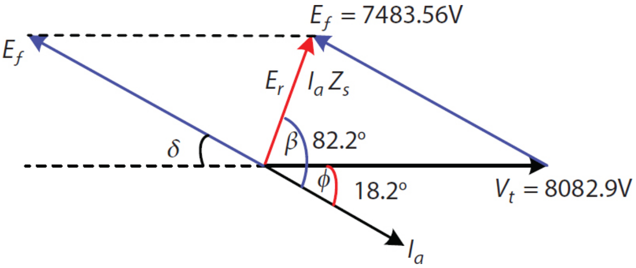


Fig. 10.15 Phasor diagram for example 10.2.

According to Fig. 10.15, the excitation voltage per phase is,

$$E_f = \sqrt{V_t^2 + E_r^2 - 2V_t E_r \cos(\beta - \phi)} = \sqrt{8082.9^2 + 1747.35^2 - 2 \times 8082.9 \times 1747.35 \times \cos 64} \tag{10.41}$$

$$E_f = 7483.56\,\text{V} \tag{10.42}$$

The torque angle can be calculated as,

$$(10.43)\ \frac{E_f}{\sin(\beta-\phi)} = \frac{E_r}{\sin\delta}$$

$$(10.44)\ \frac{7483.56}{\sin 64} = \frac{1747.35}{\sin\delta}$$

$$(10.45)\ \delta = 12.11^\circ$$

Practice Problem 10.1

The per-phase synchronous impedance of a 3.5 kV, three-phase, star-connected synchronous motor is 5 Ω per phase. The motor is running by a line current of 200 A at a unity power factor. Determine the excitation voltage per phase.

Practice Problem 10.2

An 1100 kVA, 12 kV, three-phase, wye-connected alternator has a resistance of 3 Ω per phase and a synchronous reactance of 7 Ω per phase. Determine the torque angle at a 0.8 power factor leading.

10.9 Power of Cylindrical Rotor

For deriving the expression of power, consider that the excitation voltage lags behind the terminal voltage by a torque angle δ and the terminal voltage is the reference vector. The following parameters can be defined as,

$$(10.46)\ V_t = V_t\angle 0^\circ$$

$$(10.47)\ E_f = E_f\angle -\delta$$

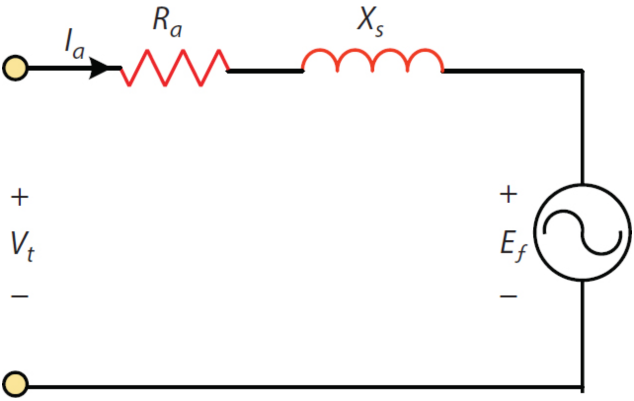


Fig. 10.16 Phasor diagram for example 10.2.

From Fig. 10.16, the expression of armature current is written as,

$$I_a = \frac{V_t - E_f\angle{-\delta}}{R_a + jX_s} \tag{10.48}$$

The conjugate of the armature current is expressed as,

$$I_a^* = \frac{V_t - E_f\angle{\delta}}{R_a - jX_s} \tag{10.49}$$

The power developed per phase by the synchronous motor is,

$$P_d = \mathrm{Re}[E_f . I_a^*] \tag{10.50}$$

Substituting equations (10.47) and (10.48) into equation (10.50) yields,

$$P_d = \mathrm{Re}\left[E_f\angle{-\delta} \times \frac{V_t - E_f\angle{\delta}}{R_a - jX_s}\right] \tag{10.51}$$

$$P_d = \mathrm{Re}\left[\frac{V_t E_f\angle{-\delta} - E_f^2}{R_a^2 + X_s^2} \times (R_a + jX_s)\right] \tag{10.52}$$

$$P_d = \mathrm{Re}\left[\frac{V_t E_f(\cos\delta - j\sin\delta) - E_f^{\,2}}{R_a^{\,2} + X_s^{\,2}} \times (R_a + jX_s) \right] \tag{10.53}$$

$$P_d = \frac{V_t E_f (R_a \cos\delta + X_s \sin\delta)}{Z_s^{\,2}} - \frac{E_f^{\,2} R_a}{Z_s^{\,2}} \tag{10.54}$$

Since the armature resistance is very small as compared to the synchronous reactance, therefore, the armature resistance R_a can be neglected for simplification of the power expression. Equation (10.54) is modified as,

$$P_d = \frac{V_t E_f X_s \sin\delta}{X_s^{\,2}} \tag{10.55}$$

$$P_d = \frac{V_t E_f}{X_s} \sin\delta \tag{10.56}$$

From equation (10.56), it is observed that the power developed by the synchronous motor is inversely proportional to the synchronous reactance and is directly proportional to the sine of the torque angle.

Let N_s be the synchronous speed of the motor in rpm. The expression of the torque can be derived as,

$$T_d = \frac{P_d}{\omega_s} = \frac{P_d}{2\pi N_s} \tag{10.57}$$

If we express the speed in rps to rpm, equation (10.57) becomes,

$$T_d = \frac{P_d}{2\pi \dfrac{N_s}{60}} \tag{10.58}$$

$$T_d = 9.55 \frac{P_d}{N_s} \tag{10.59}$$

Alternative approach: Consider the impedance diagram with a lagging power factor. Here, the armature current lags the terminal voltage by an angle of ϕ. Fig. 10.17 shows the impedance phasor diagram.

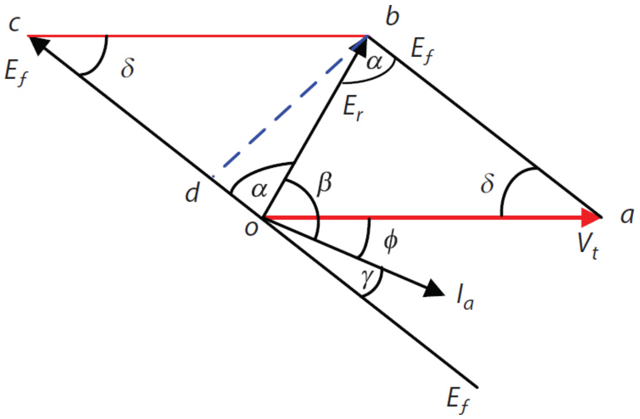


Fig. 10.17 Impedance phasor diagram.

From Fig. 10.17, the expression of the angle γ is,

$$\gamma = \pi - (\alpha + \beta) \tag{10.60}$$

The power developed per phase by the synchronous motor is,

$$P_d = E_f I_a \cos\gamma \tag{10.61}$$

Substituting equation (10.60) into equation (10.61) yields,

$$P_d = E_f I_a \cos[\pi - (\alpha + \beta)] \tag{10.62}$$

$$P_d = -E_f I_a \cos(\alpha + \beta) \tag{10.63}$$

$$P_d = -E_f I_a (\cos\alpha \cos\beta - \sin\alpha \sin\beta) \tag{10.64}$$

From the triangle *oab* of Fig. 10.17, the following equation can be written as,

$$\frac{V_t}{\sin\alpha} = \frac{E_r}{\sin\delta} \quad (10.65)$$

$$\sin\alpha = \frac{V_t \sin\delta}{E_r} \quad (10.66)$$

From the triangle *obc*, the following relations can be written as,

$$oc = E_f \quad (10.67)$$

$$cd = V_t \cos\delta \quad (10.68)$$

The length *od* is expressed as,

$$od = oc - cd = E_f - V_t \cos\delta \quad (10.69)$$

Substituting equations (10.67) and (10.68) into equation (10.69) yields,

$$od = E_f - V_t \cos\delta \quad (10.70)$$

From the triangle *obd*, the following relation can be written as,

$$\cos\alpha = \frac{od}{ob} \quad (10.71)$$

Substituting equation (10.70) and $ob = E_r$ into equation (10.71) yields,

$$\cos\alpha = \frac{E_f - V_t \cos\delta}{E_r} \quad (10.72)$$

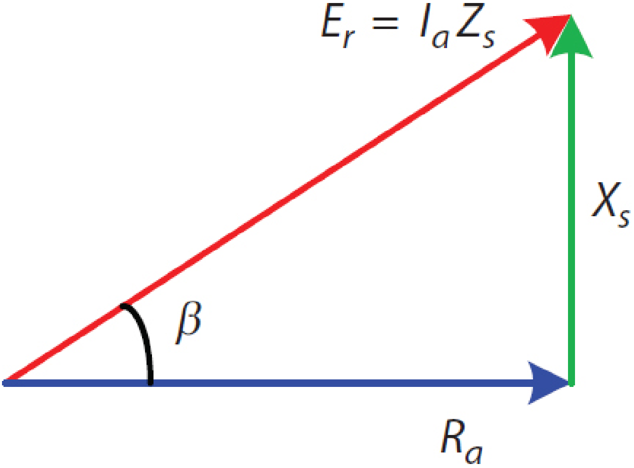


Fig. 10.18 Impedance diagram.

From Fig. 10.18, the following angles are calculated as,

$$\cos\beta = \frac{R_a}{Z_s} \tag{10.73}$$

$$\sin\beta = \frac{X_s}{Z_s} \tag{10.74}$$

Substituting equations (10.66), (10.72), (10.73) and (10.74) into equation (10.64) yields the expression of power developed per phase,

$$P_d = -E_f I_a \left[\frac{R_a}{Z_s} \times \frac{E_f - V_t \cos\delta}{E_r} - \frac{X_s}{Z_s} \times \frac{V_t \sin\delta}{E_r} \right] \tag{10.75}$$

$$P_d = -E_f I_a \left[\frac{R_a}{Z_s} \times \frac{E_f - V_t \cos\delta}{I_a Z_s} - \frac{X_s}{Z_s} \times \frac{V_t \sin\delta}{I_a Z_s} \right] \tag{10.76}$$

$$P_d = -E_f \left[\frac{R_a}{Z_s} \times \frac{E_f - V_t \cos\delta}{Z_s} - \frac{X_s}{Z_s} \times \frac{V_t \sin\delta}{Z_s} \right] \tag{10.77}$$

$$P_d = \frac{E_f V_t}{Z_s^2}(R_a \cos\delta + X_s \sin\delta) - \frac{E_f^2 R_a}{Z_s^2} \tag{10.78}$$

From equation (10.78), it is seen that the power developed per phase varies with the excitation voltage (E_f), impedance and cosine and sine of the torque angle δ.

If the armature resistance is neglected then $Z_s = X_s$, and the expression of power developed is,

$$P_d = \frac{E_f V_t}{X_s^2} X_s \sin\delta \tag{10.79}$$

$$P_d = \frac{E_f V_t}{X_s} \sin\delta \tag{10.80}$$

10.10 Various Conditions of Power

It is already observed that the power developed per phase varies with different parameters, especially excitation voltage E_f and the torque angle δ. The following conditions are used to find the new expressions of power:

(i) Here, the terminal voltage, excitation voltage and synchronous reactance are considered as fixed parameters. The power developed per phase varies only due to the torque angle δ. Under this condition, the maximum power developed occurs when $dP_d/d\delta = 0$. Differentiating equation (10.78) with respect to the torque angle δ and set equal to zero, it becomes,

$$\frac{dP_d}{d\delta} = \frac{V_t E_f(-R_a \sin\delta + X_s \cos\delta)}{Z_s^2} = 0 \tag{10.81}$$

$$-R_a \sin\delta + X_s \cos\delta = 0 \tag{10.82}$$

$$R_a \sin\delta = X_s \cos\delta \tag{10.83}$$

$$\tan\delta = \frac{X_s}{R_a} = \tan\beta \tag{10.84}$$

$$\delta = \beta \tag{10.85}$$

$$\delta = \tan^{-1}\frac{X_s}{R_a} \tag{10.86}$$

From equation (10.86), it is observed that the power developed per phase will be maximum for any given value of the terminal voltage and the excitation voltage when the torque angle $\delta = \tan^{-1}(X_s/R_a)$, i.e., $\delta = \beta$.

Substituting equation (10.85) into equations (10.73) and (10.74) yields,

$$\cos\delta = \frac{R_a}{Z_s} \tag{10.87}$$

$$\sin\delta = \frac{X_s}{Z_s} \tag{10.88}$$

Substituting equations (10.87) and (10.88) into equation (10.78) yields the maximum power developed as,

$$P_{dm} = \frac{E_f V_t}{Z_s^{\ 2}}\left(\frac{R_a R_a}{Z_s} + \frac{X_s X_s}{Z_s}\right) - \frac{E_f^{\ 2} R_a}{Z_s^{\ 2}} \tag{10.89}$$

$$P_{dm} = \frac{E_f V_t}{Z_s^{\ 2}}\left(\frac{R_a^{\ 2} + X_s^{\ 2}}{Z_s}\right) - \frac{E_f^{\ 2} R_a}{Z_s^{\ 2}} \tag{10.90}$$

$$P_{dm} = \frac{E_f V_t}{Z_s} - \frac{E_f^{\ 2} R_a}{Z_s^{\ 2}} \tag{10.91}$$

$$P_{dm} \times \frac{Z_s^{\,2}}{R_a} = \frac{E_f V_t}{Z_s} \times \frac{Z_s^{\,2}}{R_a} - \frac{E_f^{\,2} R_a}{Z_s^{\,2}} \times \frac{Z_s^{\,2}}{R_a} \tag{10.92}$$

$$E_f^{\,2} - \frac{Z_s V_t}{R_a} E_f + P_{dm} \times \frac{Z_s^{\,2}}{R_a} = 0 \tag{10.93}$$

$$E_f = -\frac{\left(-\frac{Z_s V_t}{R_a}\right)}{2} \pm \sqrt{\left(-\frac{Z_s V_t}{R_a}\right)^2 - 4 \times 1\left(P_{dm} \times \frac{Z_s^{\,2}}{R_a}\right)} \tag{10.94}$$

$$E_f = \frac{Z_s}{2R_a}\left[V_t \pm \sqrt{V_t^{\,2} - 4R_a P_{dm}}\right] \tag{10.95}$$

Since armature resistance R_a is very small it can be neglected. Putting R_a is equal to zero in equation (10.95) yields,

$$E_f = \frac{Z_s}{2R_a}\left[V_t \pm \sqrt{V_t^{\,2} - 4 \times 0 \times P_{dm}}\right] \tag{10.96}$$

Considering the positive sign of equation (10.96) yields,

$$E_f = \frac{Z_s}{2R_a}\left[V_t + \sqrt{V_t^{\,2}}\right] \tag{10.97}$$

$$E_f = \frac{Z_s}{2R_a}[V_t + V_t] \tag{10.98}$$

$$R_a = \frac{V_t Z_s}{2E_f} \tag{10.99}$$

Substituting equation (10.99) into equation (10.91) yields,

$$P_{dm} = \frac{E_f V_t}{Z_s} - \frac{E_f^{\,2}}{Z_s^{\,2}} \times \frac{V_t Z_s}{2E_f} \tag{10.100}$$

$$P_{dm} = \frac{E_f V_t}{Z_s} - \frac{E_f V_t}{2Z_s} \tag{10.101}$$

$$P_{dm} = \frac{E_f V_t}{X_s} \tag{10.102}$$

(ii) Here, terminal voltage, synchronous reactance and armature resistance are considered fixed parameters. Differentiating equation (10.91) with respect to E_f and setting is equal to zero. The relation is,

$$\frac{dP_{dm}}{dE_f} = \frac{V_t}{Z_s} - \frac{2E_f R_a}{Z_s^{\,2}} = 0 \tag{10.103}$$

$$\frac{V_t}{Z_s} = \frac{2E_f R_a}{Z_s^{\,2}} \tag{10.104}$$

$$E_f = \frac{V_t Z_s}{2R_a} \tag{10.105}$$

Substituting equation (10.105) into equation (10.91) and the expression of maximum power becomes,

$$P_{dm}{}' = \frac{V_t Z_s V_t}{2R_a Z_s} - \frac{V_t^{\,2} Z_s^{\,2} R_a}{4R_a^{\,2} Z_s^{\,2}} \tag{10.106}$$

$$P_{dm}{}' = \frac{V_t^{\,2}}{2R_a} - \frac{V_t^{\,2}}{4R_a} \tag{10.107}$$

$$P_{dm}' = \frac{V_t^2}{4R_a} \quad (10.108)$$

From equations (10.105) and (10.108), it is seen that the excitation voltage and maximum power developed would be very large due to a smaller value of the armature resistance.

(iii) The maximum power for given values of R_a, X_s and E_f will occur when the torque angle (δ) is equal to β. By applying this condition, draw the impedance diagram as shown in Fig. 10.19.

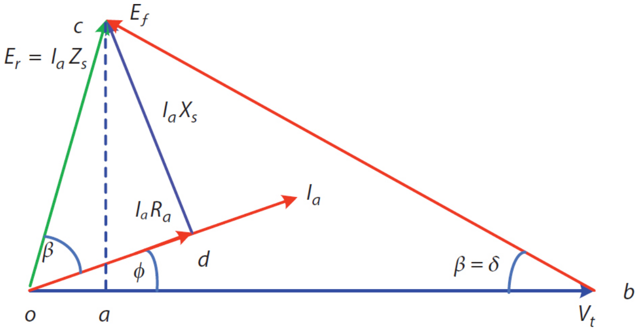


Fig. 10.19 Impedance diagram with δ= β.

From the triangle *oac*, the following length can be determined as,

$$\cos(\beta + \phi) = \frac{oa}{oc} = \frac{oa}{I_a Z_s} \quad (10.109)$$

$$oa = I_a Z_s \cos(\beta + \phi) \quad (10.110)$$

Again from the triangle *abc*, the length *ab* is determined as,

$$\cos\beta = \frac{ab}{bc} = \frac{ab}{E_f} \quad (10.111)$$

$$ab = E_f \cos\beta \quad (10.112)$$

The length *ob* can be determined as,

$$ob = oa + ab \tag{10.113}$$

Substituting equations (10.110) and (10.112) into equation (10.113),

$$V_t = I_a Z_s \cos(\beta + \phi) + E_f \cos\beta \tag{10.114}$$

From the triangle *odc*, the following relation is written as,

$$\cos\beta = \frac{I_a R_a}{I_a Z_s} = \frac{R_a}{Z_s} \tag{10.115}$$

$$\sin\beta = \frac{I_a X_s}{I_a Z_s} = \frac{X_s}{Z_s} \tag{10.116}$$

Substituting equations (10.105) and (10.115) into equation (10.114) yields,

$$V_t = I_a Z_s \cos(\beta + \phi) + \frac{V_t Z_s}{2R_a} \times \frac{R_a}{Z_s} \tag{10.117}$$

$$V_t = I_a Z_s \cos(\beta + \phi) + \frac{V_t}{2} \tag{10.118}$$

$$\frac{V_t}{2} = I_a Z_s \cos(\beta + \phi) \tag{10.119}$$

$$\frac{V_t}{2} = I_a Z_s (\cos\beta \cos\phi - \sin\beta \sin\phi) \tag{10.120}$$

$$\frac{V_t}{2} = I_a (Z_s \cos\beta \cos\phi - Z_s \sin\beta \sin\phi) \tag{10.121}$$

Substituting equations (10.115) and (10.116) into equation (10.121) yields,

$$\frac{V_t}{2} = I_a (R_a \cos\phi - X_s \sin\phi) \tag{10.122}$$

For a unity power factor (ϕ = 0), equation (10.122) is modified as,

$$\frac{V_t}{2} = I_a R_a \times 1 - 0 \tag{10.123}$$

$$\frac{V_t}{2} = I_a R_a \tag{10.124}$$

$$I_a = \frac{V_t}{2R_a} \tag{10.125}$$

Substituting equation (10.125) in equation (10.105) yields,

$$E_f = \frac{V_t Z_s}{2R_a} = I_a Z_s \tag{10.126}$$

The expression of the input power is written as,

$$P_{in} = V_t I_a \tag{10.127}$$

Substituting equation (10.125) into equation (10.127) yields,

$$P_{in} = V_t \frac{V_t}{2R_a} = \frac{V_t^2}{2R_a} \tag{10.128}$$

The expression of the power loss can be determined as,

$$P_{loss} = P_{in} - P_{dm}, \tag{10.129}$$

Substituting equations (10.128) and (10.108) into equation (10.129) yields,

$$P_{loss} = \frac{V_t^2}{2R_a} - \frac{V_t^2}{4R_a} \tag{10.130}$$

$$P_{loss} = \frac{V_t^{2}}{4R_a} \tag{10.131}$$

The efficiency can be determined as,

$$\eta = \frac{P_{dm}'}{P_{in}} \times 100 \tag{10.132}$$

$$\eta = \frac{\frac{V_t^{2}}{4R_a}}{\frac{V_t^{2}}{2R_a}} \times 100 = 50\% \tag{10.133}$$

Example 10.3

The per-phase armature resistance and the synchronous reactance of a 440 V, 50 Hz, 4-pole, three-phase, wye-connected synchronous motor are 1.5 Ω and 8 Ω, respectively. The motor takes a current of 50 A from the line at a leading power factor of 0.9 on its full excitation. Calculate the maximum power per phase.

Solution

The terminal voltage per phase is calculated as,

$$V_t = \frac{440}{\sqrt{3}} = 254.03\,\text{V} \tag{10.134}$$

The value of the angle is determined as,

$$\phi = \cos^{-1} 0.9 = 25.8^\circ \tag{10.135}$$

The value of the synchronous impedance is calculated as,

$$Z_s = 1.5 + j8 = 8.13\angle 79^\circ\ \Omega \tag{10.136}$$

The resultant voltage due to impedance is determined as,

$$E_r = I_a Z_s = 50 \times 8.13 = 406.5\,\text{V} \tag{10.137}$$

The excitation voltage per phase is calculated using the following formula,

$$E_f = \sqrt{V_t^{\,2} + E_r^{\,2} - 2V_t E_r \cos(\beta + \phi)} \tag{10.138}$$

$$E_f = \sqrt{254.03^2 + 406.5^2 - 2 \times 254.03 \times 406.5 \times \cos(79 + 25.8)} \tag{10.139}$$

$$E_f = 513.5\,\text{V} \tag{10.140}$$

The value of the maximum power per phase can be calculated as,

$$P_{dm} = \frac{E_f V_t}{Z_s} - \frac{E_f^{\,2} R_a}{Z_s^{\,2}} = \frac{513.5 \times 254}{8.13} - \frac{513.5^2 \times 1.5}{8.13^2} = 10059\,\text{W} \tag{10.141}$$

Example 10.4

The terminal voltage per phase of a 4-pole, 50 Hz, synchronous motor is 1500 V and the excitation voltage per phase is 1000 V. The motor has an armature resistance of 1.5 Ω per phase and a synchronous impedance of 12 Ω per phase. Determine the maximum power developed and the maximum torque.

Solution

Here, the terminal voltage per phase is given as,

$$V_t = 1500\,\text{V} \tag{10.142}$$

The excitation voltage is given as,

$$E_f = 1000\,\text{V} \tag{10.143}$$

The maximum power is calculated as,

$$P_{dm} = \frac{E_f V_t}{Z_s} - \frac{E_f^{\,2} R_a}{Z_s^{\,2}} = \frac{1500 \times 1000}{12} - \frac{1000^2 \times 1.5}{12^2} = 114583\,\text{W} \tag{10.144}$$

The maximum torque can be calculated as,

$$T_{dm} = 9.55 \frac{P_{dm}}{N_s} = \frac{9.55 \times 114583}{1500} = 729.5\,\text{N-m} \tag{10.145}$$

Practice Problem 10.3

A 4-pole, 415 V, 50 Hz, wye-connected synchronous motor has an armature resistance of 2.5 Ω per phase and a synchronous reactance of 8 Ω per phase. At full load running condition, the motor takes a current of 100 A at a 0.85 power factor of lagging. Calculate the maximum power per phase.

Practice Problem 10.4

A 4-pole, 11 kV, 50 Hz, the star-connected synchronous motor has an armature resistance of 2.6 Ω per phase and a synchronous impedance of 10 Ω per phase. The excitation voltage per phase is 900 V. Determine the maximum power developed and maximum torque.

10.11 Phasor Diagrams of Salient Pole Motor

Like the salient pole synchronous generator, the synchronous motor has a large air gap and the armature current is divided into two components. These components of current are the direct axis current I_d and the quadrature axis current I_q. The terminal voltage of a synchronous motor is expressed as,

$$V_t = E_f + I_a R_a + jI_d X_d + jI_q X_q \tag{10.146}$$

$$I_a = I_d + jI_q \tag{10.147}$$

The phasor diagrams of a synchronous motor are drawn based on the power factor. The field current is 90° out of phase from the excitation voltage E_f.

The armature current I_a lags the excitation voltage by an internal angle ψ. As shown in Fig. 10.20.

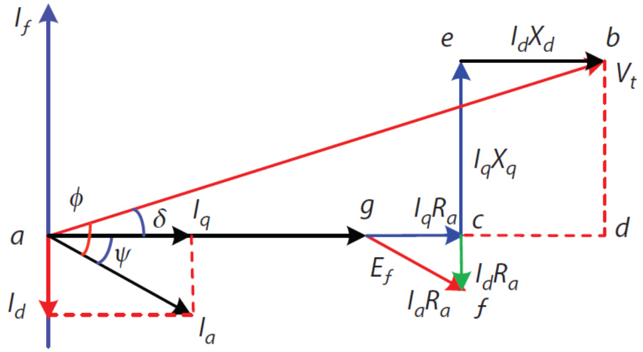


Fig. 10.20 Phasor diagram with lagging power factor.

From *abd* triangle, the following relationships are written as,

$$ad = V_t \cos\delta \tag{10.148}$$

$$bd = V_t \sin\delta \tag{10.149}$$

$$cd = be = I_d X_d \tag{10.150}$$

The length of *ad* is expressed as,

$$ad = ag + gc + cd \tag{10.151}$$

Substituting equations (10.148), (10.149) and (10.150) into equation (10.151) yields,

$$V_t \cos\delta = E_f + I_q R_a + I_d X_d \tag{10.152}$$

From Fig. 10.19, the following relation can be written as,

$$ef = ec + cf \tag{10.153}$$

Substituting $ef = bd = V_t \sin\delta$, $ec = I_q X_q$ and $cf = I_d R_a$ into equation (10.153) yields,

$$I_q X_q = V_t \sin\delta + I_d R_a \tag{10.154}$$

Internal power factor angle is written as,

$$\psi = \phi - \delta \tag{10.155}$$

The d-axis and q-axis components of the current are written as,

$$I_d = I_a \sin\psi \tag{10.156}$$

$$I_q = I_a \cos\psi \tag{10.157}$$

Substituting equation (10.155) into equations (10.156) and (10.157) yields,

$$I_d = I_a \sin(\phi - \delta) \tag{10.158}$$

$$I_q = I_a \cos(\phi - \delta) \tag{10.159}$$

Substituting equations (10.158) and (10.159) into equation (10.154) yields,

$$X_q I_a \cos(\phi - \delta) = V_t \sin\delta + R_a I_a \sin(\phi - \delta) \tag{10.160}$$

$$X_q I_a \cos\delta \cos\phi + X_q I_a \sin\delta \sin\phi = V_t \sin\delta + R_a I_a \sin\phi \cos\delta - R_a I_a \sin\delta \cos\phi \tag{10.161}$$

$$[V_t - X_q I_a \sin\phi - R_a I_a \cos\phi]\sin\delta = [X_q I_a \cos\phi - R_a I_a \sin\phi]\cos\delta \tag{10.162}$$

$$\frac{\sin\delta}{\cos\delta} = \frac{X_q I_a \cos\phi - R_a I_a \sin\phi}{V_t - X_q I_a \sin\phi - R_a I_a \cos\phi} \tag{10.163}$$

$$\tan\delta = \frac{X_q I_a \cos\phi - R_a I_a \sin\phi}{V_t - X_q I_a \sin\phi - R_a I_a \cos\phi} \tag{10.164}$$

The excitation voltage is determined by the following expression,

$$ag = ad - gc - cd \tag{10.165}$$

Substituting $ag = E_f$, $gc = I_q R_a$, $ad = V_t \cos\delta$ and $cd = bc = I_d X_d$ into equation (10.164) yields,

$$E_f = V_t \cos\delta - I_q R_a - I_d X_d \tag{10.166}$$

For a leading power factor, the armature current I_a leads the excitation voltage E_f by an internal power factor angle ψ as shown in Fig. 10.21. The I_aR_a drop resolves into two components I_qR_a and I_dR_a.

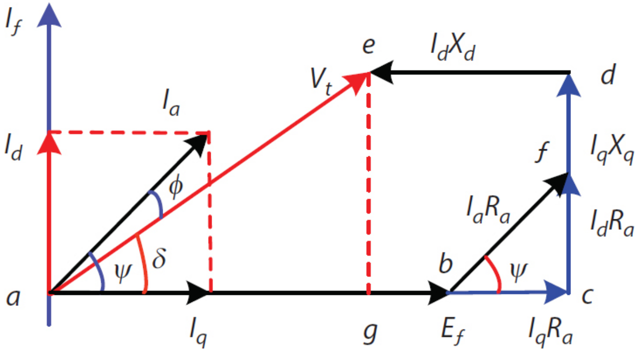


Fig. 10.21 Phasor diagram with leading power factor.

From Fig. 10.21, the following expressions are written as,

$$(10.167)\quad ab = E_f$$

$$(10.168)\quad \psi = \delta + \phi$$

$$(10.169)\quad bc = I_q R_a = \cos\psi = \cos(\delta + \phi)$$

From *age* triangle, the following are written as follows,

$$(10.170)\quad eg = V_t \sin\delta$$

$$(10.171)\quad ag = V_t \cos\delta$$

Again, the length of *cd* is written as,

$$(10.172)\quad cd = cf + fd = I_d R_a + I_q X_q$$

The d-axis and q-axis components of the armature current are expressed as,

$$(10.173)\quad I_d = I_a \sin\psi = I_a \sin(\delta + \phi)$$

$$I_q = I_a \cos\psi = I_a \cos(\delta + \phi) \tag{10.174}$$

The length of *eg* and *cd* are equal and it is written as,

$$eg = cd \tag{10.175}$$

Substituting equations (10.170) and (10.172) into equation (10.175) yields,

$$V_t \sin\delta = I_d R_a + I_q X_q \tag{10.176}$$

Substituting equations (10.173) and (10.174) into equation (10.176) yields,

$$V_t \sin\delta = I_a \sin(\delta + \phi) R_a + I_a \cos(\delta + \phi) X_q \tag{10.177}$$

$$V_t \sin\delta = I_a R_a \sin\delta \cos\phi + I_a R_a \sin\phi \cos\delta + I_a X_q \cos\delta \cos\phi - I_a X_q \sin\delta \sin\phi \tag{10.178}$$

$$(V_t - I_a R_a \cos\phi + I_a X_q \sin\phi) \sin\delta = (I_a R_a \sin\phi + I_a X_q \cos\phi) \cos\delta \tag{10.179}$$

$$\frac{\sin\delta}{\cos\delta} = \frac{I_a R_a \sin\phi + I_a X_q \cos\phi}{V_t - I_a R_a \cos\phi + I_a X_q \sin\phi} \tag{10.180}$$

$$\tan\delta = \frac{I_a R_a \sin\phi + I_a X_q \cos\phi}{V_t - I_a R_a \cos\phi + I_a X_q \sin\phi} \tag{10.181}$$

The excitation voltage is determined by the following expression,

$$ab = ag - bc + ed \tag{10.182}$$

Substituting $ab = E_f$, $bc = I_q R_a$, $ag = V_t \cos\delta$ and $ed = I_d X_d$ into equation (10.164) yields,

$$E_f = V_t \cos\delta - I_q R_a + I_d X_d \tag{10.183}$$

For a unity power factor, the armature current I_a is in phase with the terminal voltage V_t as shown in Fig. 10.22. Due to this condition, the power factor is 1, i.e., $\cos\phi = 1$ and $\sin\phi = 0$.

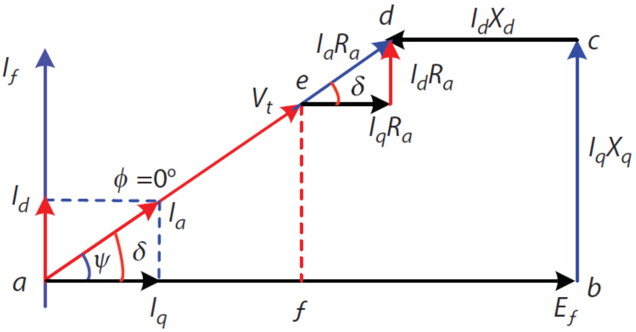


Fig. 10.22 Phasor diagram with a unity power factor.

For a unity power factor, substituting cos ϕ = 1 and sin ϕ = 0 in equation (10.181) yields,

$$\tan\delta = \frac{I_aR_a \times 0 + I_aX_q \times 1}{V_t - I_aR_a \times 1 + I_aX_q \times 0} \tag{10.184}$$

$$\tan\delta = \frac{I_aX_q}{V_t - I_aR_a} \tag{10.185}$$

For a unity power factor, the excitation voltage is expressed as,

$$E_f = V_t\cos\delta + I_aR_a\cos\delta + I_dX_d \tag{10.186}$$

Example 10.5

A three-phase, 20 MVA, 8-pole, 60 Hz, 11 kV, wye-connected synchronous motor has $X_q = 6\ \Omega$, $X_d = 4\ \Omega$, and $R_a = 0.6\ \Omega$. Find the torque angle and excitation voltage with a 0.95 lagging power factor.

Solution

Here, the terminal voltage per phase is determined as,

$$V_t = \frac{11000}{\sqrt{3}} = 6350.85\,\mathrm{V} \tag{10.187}$$

The armature current per phase is calculated as,

$$I_a = \frac{20000}{\sqrt{3} \times 11} = 1049.73\,\text{A} \tag{10.188}$$

$$\cos\phi = 0.95 \tag{10.189}$$

$$\sin\phi = \sin(\cos^{-1} 0.95) = 0.31 \tag{10.190}$$

The torque angle for a lagging power factor is calculated as,

$$\tan\delta = \frac{X_q I_a \cos\phi - R_a I_a \sin\phi}{V_t - X_q I_a \sin\phi - R_a I_a \cos\phi} = \frac{6 \times 1049.73 \times 0.95 - 0.8 \times 1049.73 \times 0.31}{6350.85 - 6 \times 1049.73 \times 0.31 - 0.8 \times 1049.73 \times 0.95} \tag{10.191}$$

$$\tan\delta = 1.59 \tag{10.192}$$

$$\delta = 57.83^\circ \tag{10.193}$$

The d-axis and q-axis components of the current are determined as,

$$I_d = I_a \sin(\phi - \delta) = 1049.79 \sin(18.19^\circ - 57.83^\circ) = -669.69\,\text{A} \tag{10.194}$$

$$I_q = I_a \cos(\phi - \delta) = 1049.79 \cos(18.19^\circ - 57.83^\circ) = 808.42\,\text{A} \tag{10.195}$$

The excitation voltage is calculated as,

$$E_f = V_t \cos\delta - I_q R_a - I_d X_d = 6350.85 \cos 57.83^\circ - 808.42 \times 0.8 + 669.69 \times 4 = 5413.43\,\text{A} \tag{10.196}$$

Practice Problem 10.5

A three-phase, 15 MVA, 10-pole, 60 Hz, 11 kV, wye-connected synchronous motor has $X_q = 5\ \Omega$, $X_d = 3\ \Omega$, and $R_a = 0.6\ \Omega$. Find the torque angle and excitation voltage for a 0.95 lagging and leading power factor.

10.12 Power Expression of Salient Pole Motor

A leading power factor phasor diagram is considered to derive the power expression of a salient pole synchronous motor. In this diagram, the armature resistance is neglected as shown in Fig.

10.22(b). The following relations are written as,

$$ae = V_t \cos\delta \tag{10.197}$$

$$de = V_t \sin\delta \tag{10.198}$$

$$dc = I_d X_d \tag{10.199}$$

The expression of the input power in terms of direct axis and quadrature axis components of current is,

$$P_i = I_q V_t \cos\delta + I_d V_t \sin\delta \tag{10.200}$$

The d-axis component of the current I_d is determined as,

$$ae = ab - be \tag{10.201}$$

Substituting equations (10.198), (10.200) and $ab = E_f$ into equation (10.201) yields,

$$V_t \cos\delta = E_f - I_d X_d \tag{10.202}$$

$$I_d = \frac{E_f - V_t \cos\delta}{X_d} \tag{10.203}$$

From Fig. 10.23(b), the q-axis component of the current I_q is determined as,

$$ab = cd \tag{10.204}$$

Substituting equation (10.198) and $cd = I_q X_q$ into equation (10.204) yields,

$$V_t \sin\delta = I_q X_q \tag{10.205}$$

$$I_q = \frac{V_t \sin\delta}{X_q} \tag{10.206}$$

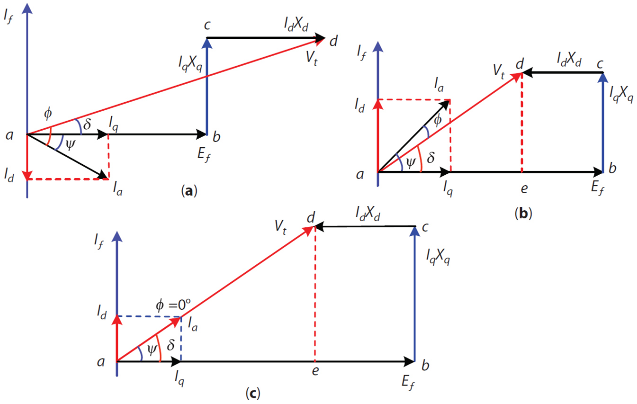

I_f
c
I_dX_d
d
I_qX_q
V_t
ϕ
δ
I_q
a
ψ
b
E_f
I_d
I_a
(a)
I_f
V_t
d
I_dX_d
c
I_a
I_qX_q
I_d
ϕ
a
ψ
δ
b
I_q
e
E_f
(b)
I_f
V_t
d
I_dX_d
c
I_qX_q
ϕ =0°
I_d
I_a
ψ
δ
a
I_q
e
b
E_f
(c)

Fig. 10.23 Modified phasor diagrams.

Substituting equations (10.203) and (10.206) into equation (10.200) yields,

$$P_i = V_t\cos\delta\frac{V_t\sin\delta}{X_q} + V_t\sin\delta\frac{E_f - V_t\cos\delta}{X_d} \tag{10.207}$$

$$P_i = \frac{V_t^2\sin\delta\cos\delta}{X_q} - \frac{V_t^2\sin\delta\cos\delta}{X_d} + \frac{E_f V_t\sin\delta}{X_d} \tag{10.208}$$

$$P_i = \frac{E_f V_t\sin\delta}{X_d} + \frac{V_t^2(X_d - X_q)\sin 2\delta}{2X_dX_q} \tag{10.209}$$

The armature copper loss is expressed as,

$$P_{cu} = \left(I_d^2 + I_q^2\right)R_a \tag{10.210}$$

The expression of the power developed is equal to the input power minus the armature copper loss. Hence, the expression of power developed is,

$$P_d = P_i - P_{cu} \tag{10.211}$$

Substituting equations (10.209) and (10.210) into equation (10.211) yields,

$$P_d = \frac{E_f V_t \sin\delta}{X_d} + \frac{V_t^2 (X_d - X_q)\sin 2\delta}{2X_d X_q} - (I_d^2 + I_q^2)R_a \quad (10.212)$$

If we neglect the effect of armature resistance then the expression of power developed would be the same as in equation (10.209).

$$P_d = \frac{E_f V_t \sin\delta}{X_d} + \frac{V_t^2 (X_d - X_q)\sin 2\delta}{2X_d X_q} \quad (10.213)$$

The first part of equation (10.213) is the power due to the field excitation and the second part is the reluctance power due to the saliency.

Example 10.6

A three-phase, 20 MVA, 10-pole, 60 Hz, 11 kV, wye-connected synchronous motor has $X_q = 4\ \Omega$, $X_d = 2\ \Omega$, and $R_a = 0.4\ \Omega$. Find the excitation voltage and mechanical power developed with a 0.85 lagging power factor.

Solution

Here, the terminal voltage per phase is determined as,

$$V_t = \frac{11000}{\sqrt{3}} = 6350.85\,\mathrm{V} \quad (10.214)$$

The armature current per phase is calculated as,

$$I_a = \frac{20000}{\sqrt{3} \times 11} = 1049.73\,\mathrm{A} \quad (10.215)$$

$$\cos\phi = 0.85 \quad (10.216)$$

$$\phi = 31.79^\circ \quad (10.217)$$

$$\sin\phi = \sin(31.79^\circ) = 0.53 \quad (10.218)$$

The torque angle for a lagging power factor is calculated as,

$$\tan\delta = \frac{X_q I_a \cos\phi - R_a I_a \sin\phi}{V_t - X_q I_a \sin\phi - R_a I_a \cos\phi} = \frac{4\times 1049.73\times 0.85 - 0.4\times 1049.73\times 0.53}{6350.85 - 4\times 1049.73\times 0.53 - 0.4\times 1049.73\times 0.85} \tag{10.219}$$

$$\tan\delta = 0.89 \tag{10.220}$$

$$\delta = 41.61^\circ \tag{10.221}$$

The d-axis and q-axis components of the current are determined as,

$$I_d = I_a \sin(\phi - \delta) = 1049.79 \sin(31.79^\circ - 41.61^\circ) = -179.04\,\text{A} \tag{10.222}$$

$$I_q = I_a \cos(\phi - \delta) = 1049.79 \cos(31.79^\circ - 41.61^\circ) = 1034.35\,\text{A} \tag{10.223}$$

The excitation voltage is calculated as,

$$E_f = V_t \cos\delta - I_q R_a - I_d X_d = 6350.85 \cos 41.61^\circ - 1034.35\times 0.4 + 179.04\times 2 = 4692.76\,\text{A} \tag{10.224}$$

The mechanical power developed is calculated using equation (10.213) as,

$$P_d = \frac{4692.76\times 6350.85 \sin 41.61^\circ}{2} + \frac{6350.85^2 (2-4)\sin(2\times 41.61^\circ)}{2\times 2\times 4} = 14901850.62\,\text{W} \tag{10.225}$$

$$P_d = 14901850.62\,\text{W} = 14.9\,\text{MW} \tag{10.226}$$

Practice Problem 10.6

A three-phase, 20 MVA, 6-pole, 60 Hz, 11 kV, wye-connected synchronous motor has $X_q = 5\ \Omega$, $X_d = 2\ \Omega$, and $R_a = 0.3\ \Omega$. Find the excitation voltage and mechanical power developed with a 0.90 leading power factor.

10.13 V-Curves of a Synchronous Motor

The V-curves of a synchronous motor are obtained by plotting the armature current versus the field current while the motor input power is kept constant. To draw the v-curves, the motor runs by a constant voltage and constant frequency. The field current is increased in steps and the corresponding armature currents are recorded. If the armature currents are plotted against the field currents, then the v-curves are obtained for a particular load. Consider the general equation of the induced voltage as

$$E_f = V_t - I_a R_a - jI_a X_s \tag{10.227}$$

$$E_f = V_t + (-I_a R_a) + (-jI_a X_s) \tag{10.228}$$

Now draw a phasor diagram with a lagging power factor and consider the terminal voltage as a reference phasor as shown in Fig. 10.24.

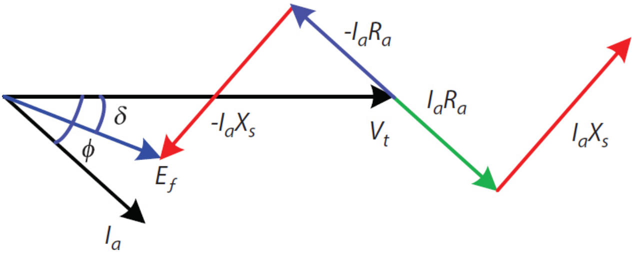


Fig. 10.24 Lagging power factor phasor diagram for v-curve.

From Fig. 10.24, it is seen that the induced voltage E_f is less than the terminal voltage V_t which is known as under-excited. Therefore, the armature current I_a will be higher (maximum) and the rotor field current I_f will be minimum based on the following equations.

$$I_a \uparrow = \frac{V_t - E_f \downarrow}{Z_s} \tag{10.229}$$

$$E_f \propto I_f \tag{10.230}$$

In this condition, the motor will deliver the power to an inductive load. Again, draw the phasor diagram with a resistive load as shown in Fig. 10.25. In this phasor diagram, the armature current I_a is in phase with the terminal voltage V_t. The induced emf E_f is again increased which in turn increases the field current I_f based on equation (10.230). The armature current I_a will again decrease (minimum) based on equation (10.231).

$$I_a \downarrow = \frac{V_t - E_f \uparrow}{Z_s} \tag{10.231}$$

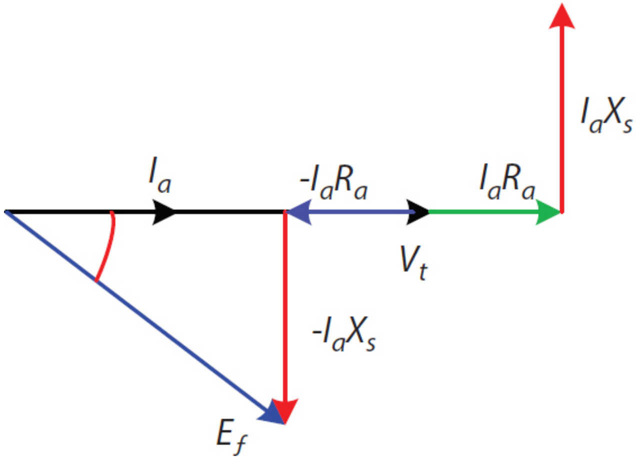


Fig. 10.25 Unity power factor phasor diagram for v-curve.

Finally, the synchronous motor delivers power to a capacitive load which is the motor operating at a leading power factor. The phasor diagram for a leading power factor is shown in Fig. 10.26.

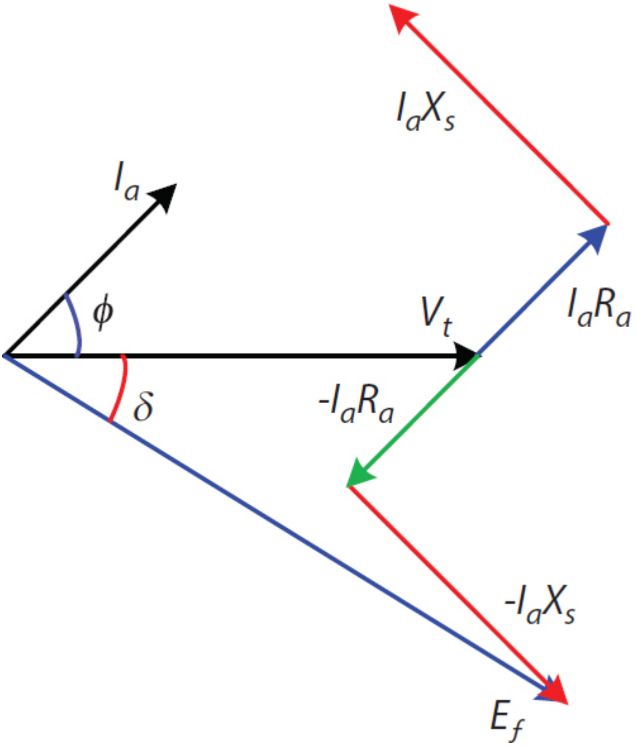


Fig. 10.26 Leading power factor phasor diagram for v-curve.

From Fig. 10.27, it is again seen that the induced emf E_f increased, which increases the field current I_f further. For a higher field current, the motor behaves like a capacitive load or delivers power to the capacitive load. In the capacitive load, the armature current I_a again will be maximum.

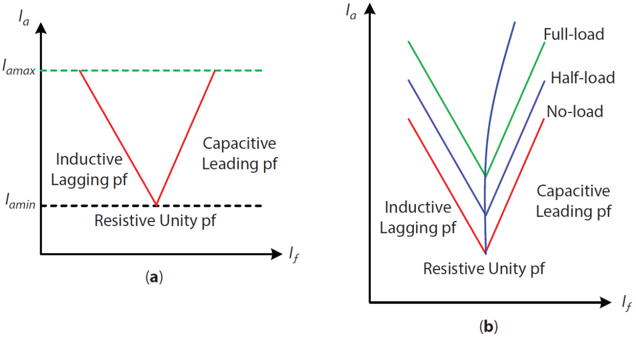


Fig. 10.27 V-curves of a synchronous motor.

When the load increases the armature current increases, which increases the field current. Therefore, the V-curves are shifted upward for no-load, half-load and full-load as shown in Fig. 10.27(b). The inverted V-curve is obtained by plotting the power factor versus the field current. The power factor for an inductive load is less than one. Whereas the power factor for a resistive load is one and the power factor for the capacitive load is also less than one. If these three points are plotted versus the field current, then the inverted V-curve is shown in Fig. 10.28(a).

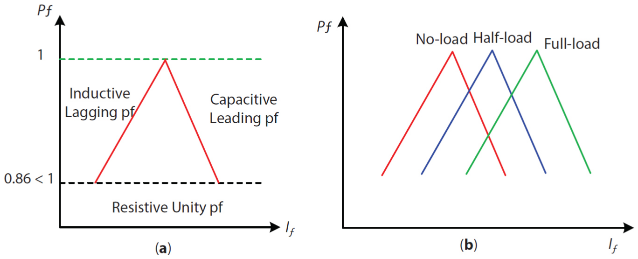


Fig. 10.28 Inverted v-curves.

For different loads, the inverted V-curves are shown in Fig. 10.28(b).

10.14 Power Factor Correction

In the industry, induction motors are used to derive the mechanical machines. That is why the induction motors are known as the workhorse in the industry. At light loads, the induction motor takes a lagging current from the supply and the power taken by the induction motor has a large magnitude of the reactive component. The overall power factor of the supply system is then reduced due to this large component of the reactive power. The over-excited synchronous motor is connected in parallel with the load, which delivers the leading component of the power. This leading power partly neutralizes the lagging power of the load. As a result, the overall power factor of the system is improved. An over-excited synchronous motor draws a leading current from the supply system so that it behaves like a capacitor. Therefore, an over-excited synchronous motor running on no-load is known as a synchronous condenser as shown in Fig. 10.29.

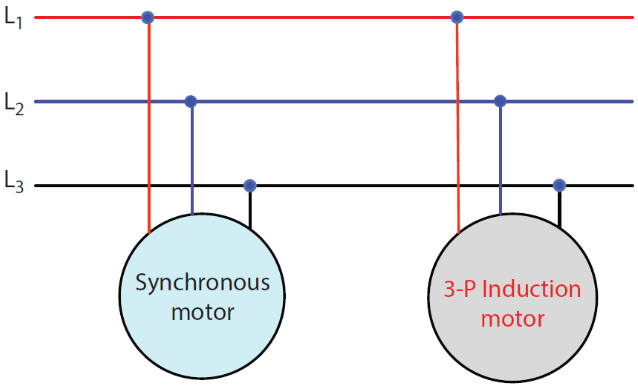

L1
L2
L3
Synchronous motor
3-P Induction motor

Fig. 10.29 Synchronous motor with load.

Consider a synchronous motor and induction motor load is connected to the source as shown in Fig. 10.30(a). Again, consider the terminal voltage V_t as a reference phasor. The load current (induction motor) I_{im} lags the terminal voltage by an angle θ_{im}. A synchronous motor draws a current I_{sm} that leads to the terminal voltage V_t by an angle θ_{sm} as shown in Fig. 10.30(b).

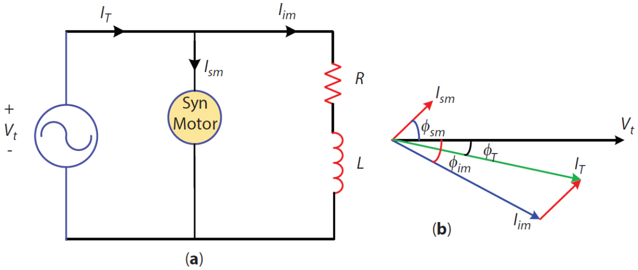

I_T
I_{im}
I_{sm}
Syn Motor
R
L
+
V_t
-
(a)
I_{sm}
ϕ_{sm}
V_t
ϕ_T
ϕ_{im}
I_T
I_{im}
(b)

Fig. 10.30 Equivalent circuit and phasor diagram of synchronous motor with a load.

Draw the phasor diagram of reactive power versus real or active power for both the induction motor and synchronous motor as shown in Fig. 10.31. The synchronous motor draws the leading current and the induction motor draws the lagging current.

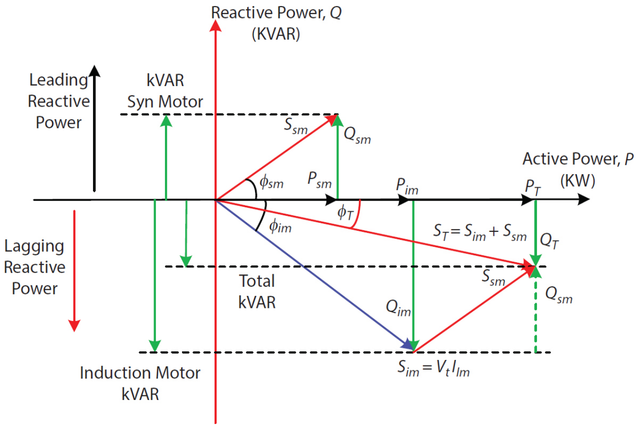

Reactive Power, Q
(KVAR)
Leading Reactive Power
kVAR Syn Motor
S_{sm}
Q_{sm}
Active Power, P
(KW)
ϕ_{sm}
P_{sm}
P_{im}
P_T
ϕ_T
ϕ_{im}
$S_T = S_{im} + S_{sm}$
Q_T
Lagging Reactive Power
Total kVAR
S_{sm}
Q_{sm}
Q_{im}
Induction Motor kVAR
$S_{im} = V_t I_{lm}$

Fig. 10.31 Reactive power versus real power for power factor correction.

From Fig. 10.31, the following relations for induction motor and synchronous motor are written as,

$$\tan\phi_{im} = \frac{Q_{im}}{P_{im}} \tag{10.232}$$

$$Q_{im} = P_{im}\tan\phi_{im} \tag{10.233}$$

$$\tan\phi_{sm} = \frac{Q_{sm}}{P_{sm}} \tag{10.234}$$

$$Q_{sm} = P_{sm}\tan\phi_{sm} \tag{10.235}$$

The total reactive power is determined by the following relation,

$$\tan\phi_T = \frac{Q_T}{P_T} \tag{10.236}$$

$$Q_T = P_T\tan\phi_T \tag{10.237}$$

But the total power is written as,

$$P_T = P_{im} + P_{sm} \tag{10.238}$$

Substituting equation (10.238) into equation (10.237) yields,

$$Q_T = (P_{im} + P_{sm})\tan\phi_T \tag{10.239}$$

From Fig. 10.30, the total reactive power is expressed as,

$$Q_T = Q_{im} - Q_{sm} \tag{10.240}$$

Substituting equations (10.233) and (10.235) into equation (10.240) yields,

$$Q_T = P_{im}\tan\phi_{im} - P_{sm}\tan\phi_{sm} \tag{10.241}$$

Again substituting equation (10.239) into equation (10.241) yields,

$$(P_{im} + P_{sm})\tan\phi_T = P_{im}\tan\phi_{im} - P_{sm}\tan\phi_{sm} \tag{10.242}$$

$$\tan\phi_{sm} = \frac{1}{P_{sm}}[P_{im}\tan\phi_{im} - (P_{im} + P_{sm})\tan\phi_T] \tag{10.243}$$

The power factor of the synchronous motor is calculated as,

$$\cos\phi_{sm} = \cos\left(\tan^{-1}\left[\frac{P_{im}\tan\phi_{im} - (P_{im} + P_{sm})\tan\phi_T}{P_{sm}}\right]\right) \tag{10.244}$$

Example 10.7

A 200 kW, 0.75 lagging power factor industrial load is connected to a three-phase system. A synchronous motor is used to improve the power factor to 0.85 lagging and the synchronous motor carries a load of 90 kW. Calculate the leading reactive power, apparent power and power factor of the synchronous motor.

Solution

The total real power supplied by the load and the synchronous motor is,

$$P_T = 200\,\text{kW} + 90\,\text{kW} = 290\,\text{kW} \tag{10.245}$$

The total power factor is given by,

$$\cos\phi_T = 0.85 \tag{10.246}$$

$$\phi_T = \cos^{-1}(0.85) = 31.79^\circ \tag{10.247}$$

The total reactive power is calculated as,

$$Q_T = P_T \tan\phi_T = 290 \tan 31.79^\circ = 179.73\,\text{kVar} \tag{10.248}$$

The total reactive power is calculated as,

$$S_T = \frac{P_T}{\cos\phi_T} = \frac{290}{0.85} = 341.18\,\text{kVA} \tag{10.249}$$

The load power factor is given by,

$$\cos\phi_L = 0.75 \tag{10.250}$$

$$\phi_L = \cos^{-1}(0.75) = 41.41^\circ \tag{10.251}$$

The reactive power of the load is calculated as,

$$Q_L = P_L \tan\phi_L = 200 \tan 41.41^\circ = 176.39\,\text{kVar} \tag{10.252}$$

The apparent power of the load is calculated as,

$$S_L = \sqrt{P_L^2 + Q_L^2} = \sqrt{200^2 + 176.39^2} = 266.67\,\text{kVA} \tag{10.253}$$

The reactive power supply by the synchronous motor is calculated as,

$$Q_{sm} = Q_T - Q_L = 179.73 - 176.39 = 3.34\,\text{kVar} \tag{10.254}$$

The apparent power supplied by the synchronous motor is determined as,

$$S_{sm} = S_T - S_L = 341.18 - 266.67 = 74.51\,\text{kVA} \tag{10.255}$$

$$\tan\phi_{sm} = \frac{Q_{sm}}{P_{sm}} = \frac{3.34}{90} = 0.037 \tag{10.256}$$

The power factor of the synchronous motor is calculated as,

$$\cos\phi_{sm} = \cos\left(\tan^{-1}(0.037)\right) = 0.99\,\text{lead} \tag{10.257}$$

Example 10.8

A three-phase induction motor draws a current of 15 A with a 0.70 power factor lagging from the supply whose line voltage is 220 V, 60 Hz supply. A synchronous motor is used to improve the power factor to 0.90 lagging. An 8 kW load is connected to a synchronous motor and the efficiency of the synchronous motor is 85%. Calculate the leading reactive power, apparent power and power factor of the synchronous motor.

Solution

The input power of the induction motor is calculated as,

$$P_{im} = \sqrt{3}V_L I_L \cos\phi_1 = \sqrt{3}\times 220\times 15\times 0.70 = 4\,\text{kW} \tag{10.258}$$

The input power of the synchronous motor is calculated as,

$$P_{sm} = \frac{8}{0.85} = 9.41\,\text{kW} \tag{10.259}$$

The total real power supplied by the load and the synchronous motor is,

$$P_T = 4\,\text{kW} + 9.41\,\text{kW} = 13.41\,\text{kW} \tag{10.260}$$

The total power factor is given by,

$$\cos\phi_T = 0.90 \tag{10.261}$$

$$\phi_T = \cos^{-1}(0.90) = 25.84^\circ \tag{10.262}$$

The total reactive power is calculated as,

$$Q_T = P_T\tan\phi_T = 13.41\tan 25.84^\circ = 6.49\,\text{kVar} \tag{10.263}$$

The total reactive power is calculated as,

$$S_T = \frac{P_T}{\cos\phi_T} = \frac{13.41}{0.90} = 14.9\,\text{kVA} \qquad (10.264)$$

The load power factor is given by,

$$\cos\phi_{im} = 0.70 \qquad (10.265)$$

$$\phi_{in} = \cos^{-1}(0.70) = 45.57^\circ \qquad (10.266)$$

The reactive power of the load is calculated as,

$$Q_{im} = P_{im}\tan\phi_{im} = 4\tan 45.57^\circ = 4.08\,\text{kVar} \qquad (10.267)$$

The apparent power of the load is calculated as,

$$S_{im} = \sqrt{P_{im}{}^2 + Q_{im}{}^2} = \sqrt{4^2 + 4.08^2} = 5.71\,\text{kVA} \qquad (10.268)$$

The reactive power supply by the synchronous motor is calculated as,

$$Q_{sm} = Q_T - Q_{im} = 6.49 - 4.08 = 2.41\,\text{kVar} \qquad (10.269)$$

The apparent power supplied by the synchronous motor is calculated as,

$$S_{sm} = S_T - S_{im} = 14.9 - 5.71 = 9.19\,\text{kVA} \qquad (10.270)$$

$$\tan\phi_{sm} = \frac{Q_{sm}}{P_{sm}} = \frac{2.41}{9.41} = 0.256 \qquad (10.271)$$

The power factor of the synchronous motor is calculated as,

$$\cos\phi_{sm} = \cos\left(\tan^{-1}(0.256)\right) = 0.96\,\text{lead} \qquad (10.272)$$

Practice Problem 10.7

A 350 kW, 0.65 lagging power factor industrial load is connected to a three-phase system. A synchronous motor is used to improve the power factor to 0.90 lagging and the synchronous motor carries a load of 120 kW. Find the leading reactive power, apparent power and power factor of the synchronous motor.

Practice Problem 10.8

A three-phase induction motor draws a current of 20 A with a 0.75 power factor lagging from the supply whose line voltage is 440 V, 50 Hz supply. A synchronous motor is used to improve the power factor to 0.85 lagging. A 40 kW load is connected to a synchronous motor and the efficiency of the synchronous motor is 80%. Determine leading reactive power, apparent power and power factor of the synchronous motor.

10.15 Hunting of a Synchronous Motor

The magnetic field rotor of the synchronous motor usually coincides with the stator magnetic field under no-load conditions. Under this condition, the rotor magnetic field locks with the stator magnetic field and rotates at synchronous speed. In addition to that, the electromagnetic torque produced by the motor will be equal to the load torque and the torque angle will remain constant. When a load is applied to a synchronous motor, the rotor field falls back to the stator field which changes the torque angle δ as shown in Fig. 10.32. Therefore, the motor needs to produce more torque to cope with the increased load to hold the equilibrium position. As the load increases gradually, the power or torque angle also increases gradually. When the load is disconnected suddenly, the torque angle will be zero and the rotor field will be pulled into the stator field. However, due to the inertia constant, the rotor field will fall back again and will oscillate to its equilibrium position. Therefore, the term hunting of synchronous motor is an undesirable phenomenon where the rotor of the synchronous motor oscillates, swings or hunts around its equilibrium position. This oscillation is typically characterized by small, rapid movements of the rotor back and forth. Synchronous motors may lose synchronism due to hunting. The hunting of synchronous machines occurs when sudden changes in the load, changes in the field current and faults occur in the system.

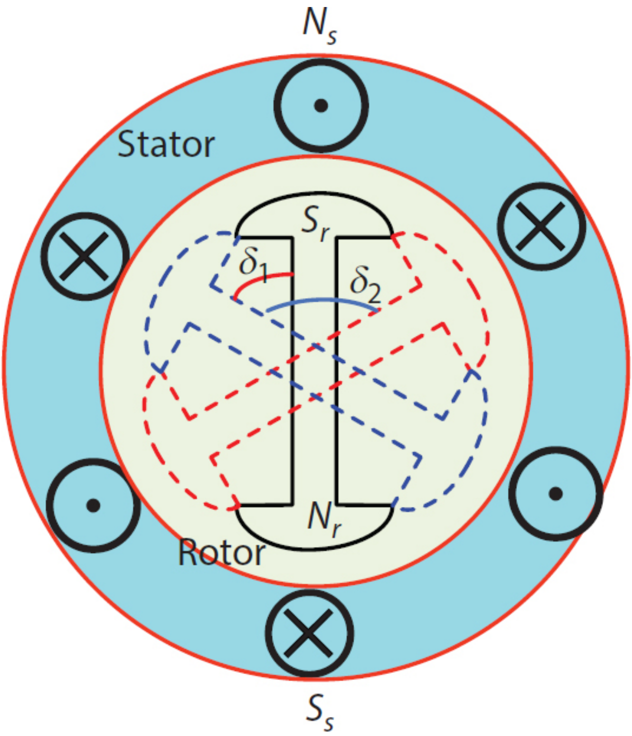


Fig. 10.32 Position of a rotor with different loads.

10.16 Applications of Synchronous Motor

The synchronous motor is used for the following purposes:

1. **Power factor correction:** An over-excited synchronous motor is used to improve the power factor of the supply system.
2. **Voltage regulation of transmission lines:** The synchronous motor either with condenser (over-excited) or reactor (under-excited) is used to regulate the magnitude of the voltage at the receiving end of the transmission lines.
3. **Constant speed and constant load drives:** The synchronous motor has high efficiency and constant speed. Because of these unique characteristics, the synchronous

motor is suitable for running compressors, blowers, centrifugal pumps, paper mills, etc.

10.17 Comparison of Synchronous Motor and Induction Motor

The main comparison between the synchronous motor and the induction motor is given below:

1. The speed of the synchronous motor remains constant from no load to full load and its speed is independent of the load. In the induction motor, the speed decreases with an increase in the load.
2. The synchronous motor is not self-starting and it needs to run up to synchronous speed by other devices. Whereas the induction motor is a self-starting machine.
3. The field excitation is applied to the rotor of the synchronous motor. In the induction motor, the field excitation is not necessary.
4. The construction of the synchronous motor is more complicated. The construction of the induction motor is very simple.
5. The synchronous motor can operate in a wide range of power factors such as from lagging to leading power factors. The induction motor can operate only with a lagging power factor.
6. The synchronous motor has a higher efficiency of the same voltage and power rating as the induction motor. However, the induction motor has less efficiency.

Exercise Problems

10.1 The per-phase impedance of a 2200 V, three-phase, star-connected synchronous motor is $2 + j10\Omega$. The motor draws a line current of 150 A at a 0.85 lagging power factor from the supply system. Determine the excitation voltage per phase.

10.2 A 1100 kVA, 11 kV, three-phase, star-connected synchronous motor has a synchronous reactance of 11 Ω per phase and a negligible armature resistance. Determine the excitation voltage per phase for the 0.95 lagging power factor and 0.95 leading power factor.

10.3 A 1100 kVA, 12 kV, three-phase, wye-connected synchronous motor has a synchronous impedance of $1.5 + j8\Omega$ per-phase. Calculate the excitation voltage per phase and the torque angle at a unity power factor.

10.4 The per-phase excitation voltage of a single-phase synchronous motor is 1000 V. The synchronous impedance per phase is $0.5 + j4.5\Omega$ and the terminal voltage per phase is 1100 V. Calculate the per phase maximum power developed.

10.5 The per-phase excitation voltage of a 6-pole, 50 Hz synchronous motor is 1500 V. The motor has a synchronous impedance of $2.5 + j12\Omega$ per phase and a terminal voltage of 1300 V per phase. Calculate the per-phase maximum developed power and the maximum torque.

10.6 The line current of a 6-pole, three-phase, 415 V, 50 Hz, the wye-connected synchronous motor is 120 A and the power factor is 0.95 lagging. Calculate the maximum torque while neglecting all kinds of losses.

10.7 The maximum torque of a 12-pole synchronous motor is 1165 N-m and the output power is 60966 W. Determine the supply frequency.

10.8 The maximum torque of a 6-pole, three-phase, 50 Hz, 440 V, star-connected synchronous motor is 700 N-m. Determine the input power of the synchronous motor by neglecting all kinds of losses.

10.9 The synchronous impedance of a three-phase, 2200 V, star-connected synchronous motor is $3 + j15\Omega$. If the line-to-line excitation voltage is 3 kV, find the per-phase maximum power developed.

10.10 A three-phase, delta-connected, 440 V, synchronous motor has an excitation voltage of 600 V per phase and a synchronous impedance of $1.5 + j6\Omega$ per phase. Calculate the efficiency for maximum power output if the friction and iron losses are 8000 W.

10.11 A three-phase, 11 kV, wye-connected synchronous motor has a synchronous reactance of 15 Ω per phase and a negligible armature resistance. The motor input power is 5000 kW per phase and the excitation voltage is 8000 V per phase. Find the power factor and the line current.

10.12 The synchronous reactance of a three-phase, 1200 V, star-connected synchronous motor is 12 Ω per phase. The input power of the synchronous motor is 500 kW and the excitation line voltage is 6000 V. Determine the line current. Neglect the effect of the armature resistance.

10.13 The synchronous reactance of a three-phase, 440 V, star-connected synchronous motor is 5 Ω per phase. The excitation voltage per phase is 415 V. Find the power developed per phase if the power angle is 30°.

10.14 A synchronous motor is connected across 420 V per phase supply and the excitation voltage per phase is 500 V. The power developed per phase is 12000 W. Determine the synchronous reactance if the torque angle is 45°.

10.15 The terminal voltage of a synchronous motor is 400 V per phase and the excitation voltage is 450 V per phase. The power developed by the motor is found to be 12500 W. Find the torque angle if the synchronous reactance is 10 Ω.

10.16 A 500 kW, 0.60 lagging power factor industrial load is connected to a three-phase system. A synchronous motor is used to improve the power factor to 0.85 lagging and the synchronous motor carries a load of 150 kW. Calculate the leading reactive power, apparent power and power factor of the synchronous motor.

10.17 A three-phase induction motor draws a current of 12 A with a 0.70 power factor lagging from the supply whose line voltage is 240 V, 60 Hz supply. A synchronous motor is used to improve the power factor to 0.95 lagging. A 60 kW load is connected to a synchronous motor and the efficiency of the synchronous motor is 85%. Find the leading reactive power, apparent power and power factor of the synchronous motor.

11

Single-Phase Induction Motors

11.1 Introduction

The three-phase induction motor has already been discussed in Chapter 7. Here, you will learn about a single-phase motor. The motors operated by a single-phase supply are known as single-phase motors. Single-phase motors are smaller in size and are mostly made in the fractional horsepower range. They are often known as fractional horsepower motors. Single-phase motors are extensively used in home appliances, offices, shops and industries. For example, these motors are used to operate washing machines, refrigerators, air-conditioning units, blenders, coffee makers, record players, etc. When the stator winding of a single-phase motor is fed from the single-phase supply, its stator winding produces pulsating flux. This pulsating flux cannot produce rotation as the synchronously revolving flux of a polyphase induction motor produces rotation. That is why single-phase motors are not self-starting. An extra winding is added with the main winding to run the single-phase motor, which is known as auxiliary/ starting winding. After starting, the main winding is enough to keep the motor in running condition. In this chapter, the details of the single-phase motors are discussed.

11.2 Construction and Synchronous Speed

The construction of a single-phase motor is almost identical to a polyphase induction motor. The main components are the stator, rotor, end ring and rotor bar or conductor as shown in Fig. 11.1.

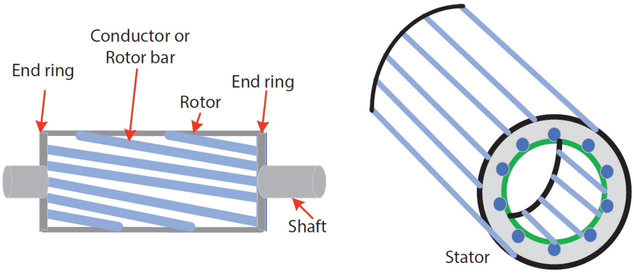


Fig. 11.1 Components of a single-phase motor.

The stator is the stationary part of the motor. It is made up of high-grade silicon steel to minimize losses and it provides slots to its inner periphery. Then the single-phase windings are placed into the stator slots which are energized by a single-phase supply, whereas the rotor is the rotating part of the motor. The rotor is made up of a highly laminated cylindrical core. The rotor provides the slots to its outer periphery where aluminum bar conductors are placed. Two end rings are used to short the rotor conductors permanently. The two end rings are often known as short-circuiting rings. A hook is used to carry the motor to the required position. A terminal box is used to connect the end terminals of the stator coils and a single-phase supply is connected to the terminal box. These components are shown in a schematic view of a single-phase induction motor in Fig. 11.2. Similar to a three-phase induction motor, there is no electrical connection between the stator and the rotor. A single-phase induction motor has a squirrel cage rotor and a single-phase distributed stator winding. The stator contains a main winding, which creates the north and south poles. The stator also contains an auxiliary winding, which operates only the starting period of the motor. The air gap between the stator and the rotor is uniform.

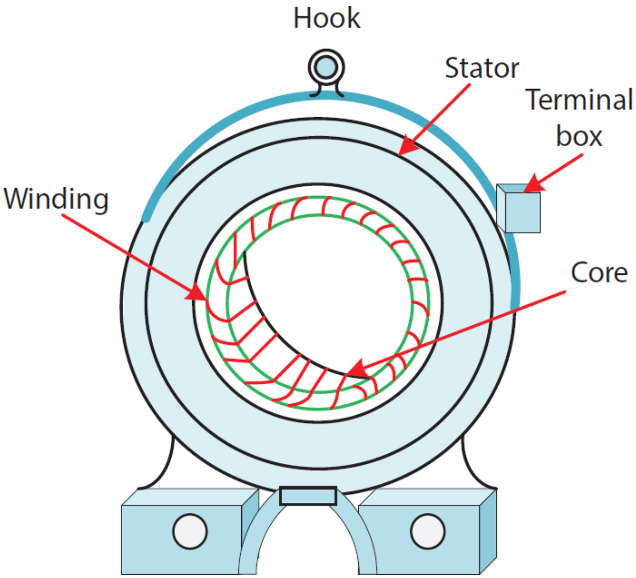


Fig. 11.2 Schematic view of a single-phase motor.

According to the three-phase induction motor, the synchronous speed of all the single-phase induction motors can be expressed as,

$$N_s = \frac{120f}{P} \tag{11.1}$$

The rotor of the motor rotates at a speed less than the synchronous speed. Therefore, the full load slip of a single-phase motor varies from 3% to 5%.

11.3 Double Revolving Field Theory

The operation of the single-phase induction motor is based on the principle of the double-revolving field theory. This theory can be stated as, “*A pulsating stationary field can be resolved into two equal magnitudes but rotates at a synchronous speed in opposite directions*”.

Let the total magnetomotive force (mmf) along the position of the angle of θ is,

$$\Im_t = Ni\sin\theta \tag{11.2}$$

Let the expression of the stator current in sinusoidal form be,

$$i = I_m \sin \omega t \tag{11.3}$$

The total mmf can be rearranged by substituting equation (11.3) into equation (11.2) as,

$$\Im_t = N I_m \sin \omega t \sin \theta \tag{11.4}$$

$$\Im_t = \frac{N I_\mathrm{m}}{2} [2 \sin \omega t \sin \theta] \tag{11.5}$$

$$\Im_t = \frac{N I_\mathrm{m}}{2} \cos(\omega t - \theta) - \frac{N I_\mathrm{m}}{2} \cos(\omega t + \theta) \tag{11.6}$$

$$\Im_t = \Im_f + \Im_b \tag{11.7}$$

where the expressions of the forward mmf and backward mmf are,

$$\Im_f = \frac{N I_m}{2} \cos(\omega t - \theta) \tag{11.8}$$

$$\Im_b = -\frac{N I_m}{2} \cos(\omega t + \theta) \tag{11.9}$$

The alternating flux or magnetic field that is created in the stator passes to the air gap. This magnetic field consists of two components of the same magnitude which are revolving at the same speed; one is rotating in a clockwise direction and the other in an anticlockwise direction. In Fig. 11.3(a), two magnetic fields are in parallel or in the phase with the reference that produces maximum flux, ϕ_m, whereas in Fig. 11.3(b), two magnetic fields make an angle θ with the reference that produces the resultant field $\phi_m \cos \theta$. In Fig. 11.3(c), two fields are in opposite directions, making a 180-degree phase shift that produces zero flux. Fig. 11.3(d) is just opposite to Fig. 11.3(a) and Fig. 11.3(e) is the opposite of Fig. 11.3(c). The resultant flux waveform with the different angles is shown in Fig. 11.4.

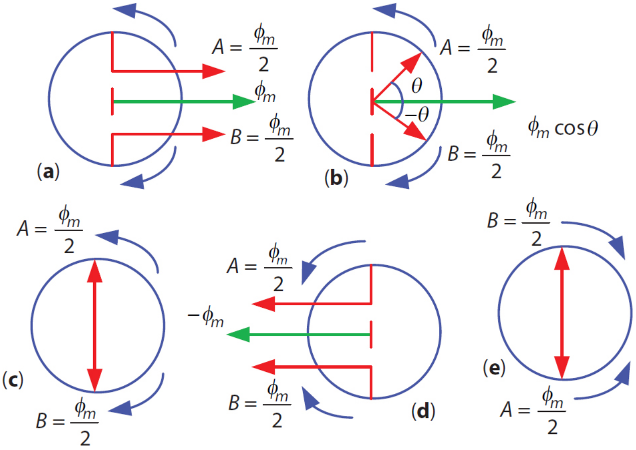

$A = \frac{\phi_m}{2}$
ϕ_m
$B = \frac{\phi_m}{2}$
(a)
$A = \frac{\phi_m}{2}$
θ
$-\theta$
$B = \frac{\phi_m}{2}$
$\phi_m \cos\theta$
(b)
$A = \frac{\phi_m}{2}$
$B = \frac{\phi_m}{2}$
(c)
$A = \frac{\phi_m}{2}$
$-\phi_m$
$B = \frac{\phi_m}{2}$
(d)
$B = \frac{\phi_m}{2}$
$A = \frac{\phi_m}{2}$
(e)

Fig. 11.3 Sequence operation of two magnetic fields.

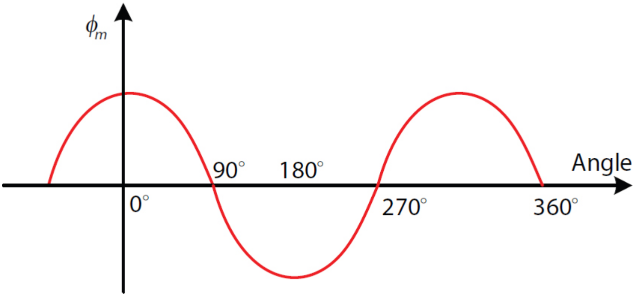


Fig. 11.4 Resultant flux waveform.

11.4 Working Principle

The single-phase supply voltage is connected to the main winding that produces an alternating magnetic field or flux as shown in Fig. 11.5. According to the double field revolving theory, this magnetic flux is represented by two opposite rotating fluxes of half of the magnitude as shown in Fig. 11.6. These opposite rotating fluxes induce a current in the closed rotor circuit which in turn produces the rotor flux.

The main flux and the rotor flux produce two torques in opposite directions, hence the net torque is zero.

Therefore, the rotor remains in a standstill condition. Therefore, a single-phase induction motor is not self-starting.

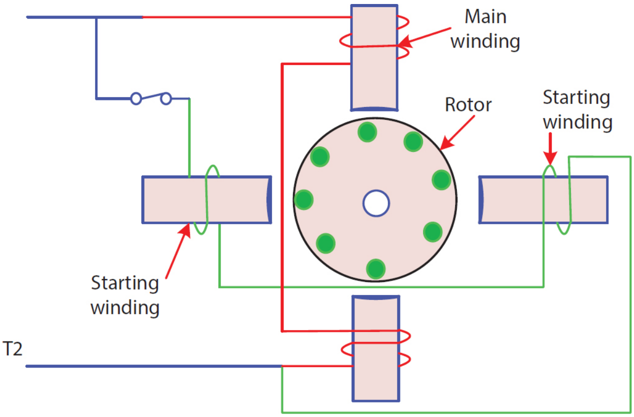

Main
winding
Rotor
Starting
winding
Starting
winding
T2

Fig. 11.5 Main and starting winding.

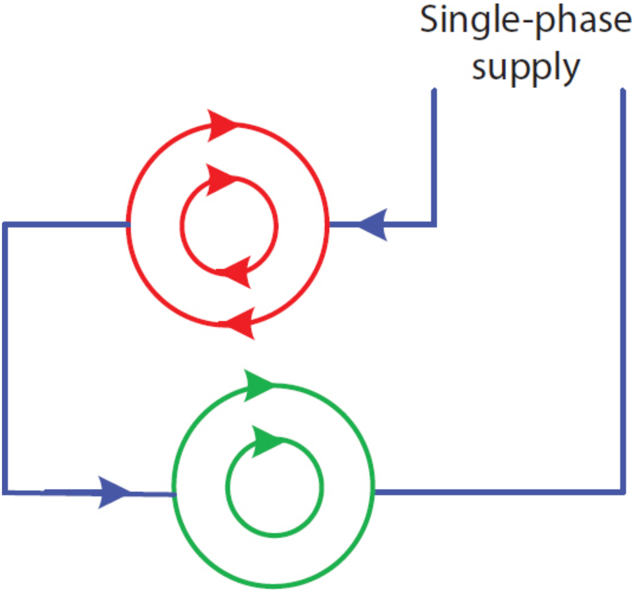

Single-phase
supply

Fig. 11.6 Two fluxes oppose each other.

The stator of a single-phase motor consists of the main winding and starting winding as shown in Fig. 11.5. A single-phase alternating voltage is applied to the stator winding which creates the alternating flux. This alternating flux revolves with a synchronous speed and is known as a revolving magnetic field (RMF). According to Faraday's laws, the relative speed between the stator RMF and rotor conductors creates the emf across the rotor circuit. Due to this emf, the current will flow in the rotor conductors which creates the rotor flux. This rotor flux lags behind the stator flux. The rotor current will create the force that tends to rotate the rotor. The rotor will rotate in the same direction of stator flux or revolving magnetic field to minimize the relative speed between them.

11.5 Slip

The slip is defined as the ratio of relative speed to the synchronous speed. Let the rotor run at a speed of N_l rpm and the synchronous speed is N_s. The slip of the rotor with respect to the forward field is expressed as,

$$s_f = \frac{N_s - N_l}{N_s} \tag{11.10}$$

The slip of the rotor with respect to the backward field is,

$$s_b = \frac{N_s - (-N_l)}{N_s} = \frac{N_s + N_l}{N_s} \tag{11.11}$$

$$s_b = \frac{2N_s - (N_s - N_l)}{N_s} \tag{11.12}$$

$$s_b = 2 - \frac{N_s - N_l}{N_s} \tag{11.13}$$

$$s_b = 2 - s \tag{11.14}$$

Therefore, the rotor equivalent circuits due to the forward and the backward fields are shown in Fig. 11.7.

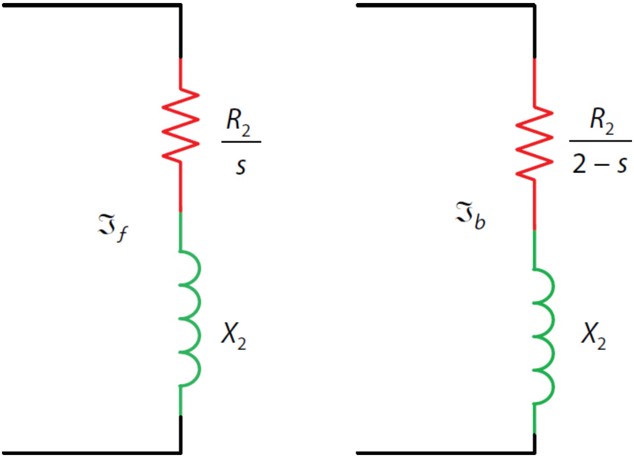


Fig. 11.7 Rotor equivalent circuit due to forward and backward fields.

Example 11.1

The speed of a 220 V, 50 Hz, 4-pole single-phase induction motor is 1450 rpm. Determine the slip due to the forward field, slip due to the backward field and effective rotor resistance in each branch if the rotor resistance at a standstill is 10 Ω.

Solution

The synchronous speed is determined as,

$$N_s = \frac{120\times 50}{4} = 1500\text{rpm} \tag{11.15}$$

The slip due to the forward field is calculated as,

$$s_f = \frac{N_s - N_l}{N_s} = \frac{1500-1450}{1500} = 0.03 \tag{11.16}$$

The slip due to the backward field is calculated as,

$$s_b = 2 - s = 2 - 0.03 = 1.97 \tag{11.17}$$

The effective rotor resistance due to the forward slip is determined as,

$$R_f = \frac{R_2}{s} = \frac{10}{0.03} = 333.3\Omega \tag{11.18}$$

The effective rotor resistance due to the backward slip is calculated as,

$$R_b = \frac{R_2}{2-s} = \frac{10}{1.97} = 5.08\Omega \tag{11.19}$$

Practice Problem 11.1

The slip of a 120 V, 50 Hz, 4-pole single-phase induction motor is 4%. Determine the (i) slip due to the forward field, (ii) slip due to the backward field, and (iii) effective rotor resistance in each branch if the rotor resistance at a standstill is 6 Ω.

11.6 Equivalent Circuit

The equivalent circuit of a single-phase induction motor consists of a series connection between the forward field circuit and the backward field circuit of the rotor with the stator. That means two rotors rotate opposite to each other with a common stator and the values of the rotor and magnetizing parameters are half of the actual values. Each circuit is similar to that of the polyphase induction motor. In the backward-rotating field circuit, the parameter slip *s* is replaced by 2-*s*. The equivalent circuit at standstill conditions with and without magnetizing resistance are shown in Fig. 11.8(a) and (b), respectively. The equivalent circuit at any slip *s* i.e., running condition is shown in Fig. 11.8(c). The impedance due to the forward field is expressed as,

$$Z_f = \frac{j\frac{X_m}{2}\left(\frac{R_2}{2s} + j\frac{X_2}{2}\right)}{\frac{R_2}{2s} + j\left(\frac{X_2}{2} + \frac{X_m}{2}\right)} = \frac{j0.5X_m\left(\frac{R_2}{s} + jX_2\right)}{\frac{R_2}{s} + j(X_2 + X_m)} \tag{11.20}$$

$$Z_f = R_f + jX_f \tag{11.21}$$

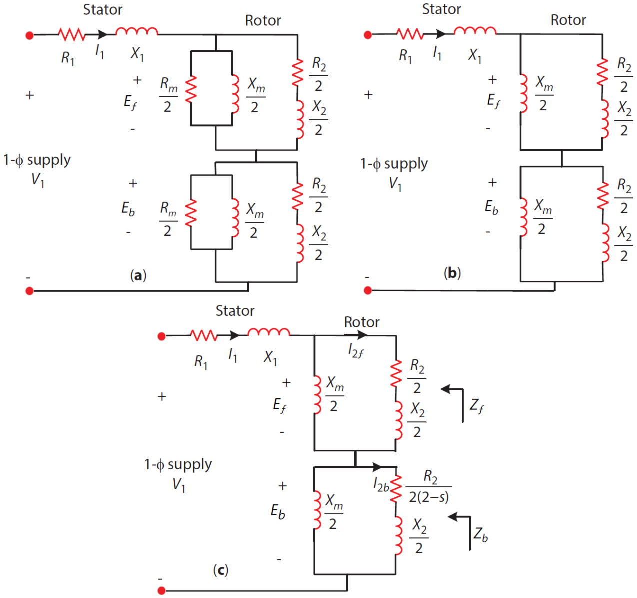
The impedance due to the backward field is expressed as,

$$Z_b = \frac{j\dfrac{X_m}{2}\left(\dfrac{R_2}{2(2-s)} + j\dfrac{X_2}{2}\right)}{\dfrac{R_2}{2(2-s)} + j\left(\dfrac{X_2}{2} + \dfrac{X_m}{2}\right)} = \frac{j0.5X_m\left(\dfrac{R_2}{2(2-s)} + jX_2\right)}{\dfrac{R_2}{2(2-s)} + j(X_2 + X_m)} \quad (11.22)$$

$$Z_b = R_b + jX_b \quad (11.23)$$

The impedance of the stator winding is expressed as,

$$Z_1 = R_1 + jX_1 \quad (11.24)$$



Stator
Rotor
R_1
I_1
X_1
+
E_f
-
$\frac{R_m}{2}$
$\frac{X_m}{2}$
$\frac{R_2}{2}$
$\frac{X_2}{2}$
1-ϕ supply
V_1
+
E_b
-
(a)
(b)
(c)
I_{2f}
I_{2b}
$\frac{R_2}{2(2-s)}$
Z_f
Z_b

Fig. 11.8 Equivalent circuit of a single-phase motor.

The total impedance of the circuit is then written as,

$$Z_t = Z_1 + Z_f + Z_b \quad (11.25)$$

The value of the current through the stator winding is determined as, (11.26) $$I_1 = \frac{V_1}{Z_t}$$

The stator copper loss is calculated as,

(11.27) $$P_{scu} = I_1^2 R_1$$

The current through the forward field is determined as,

(11.28) $$I_{2f} = I_1 \times \frac{jX_m}{\frac{R_2}{s} + j(X_2 + X_m)}$$

The current through the backward field is calculated as,

(11.29) $$I_{2b} = I_1 \times \frac{jX_m}{\frac{R_2}{2-s} + j(X_2 + X_m)}$$

The air gap powers due to forward and backward fields are calculated as,

(11.30) $$P_{agf} = I_1^2 R_f$$

(11.31) $$P_{agb} = I_1^2 R_b$$

In a polyphase induction motor, the expression of the power developed is already derived as,

(11.32) $$P_d = P_{ag}(1-s)$$

Therefore, the power developed in the single-phase induction motor is,

(11.33) $$P_d = I_1^2 \operatorname{Re}(Z_f)(1-s) + I_1^2 \operatorname{Re}(Z_b)[1-(2-s)]$$

(11.34) $$P_d = I_1^2 R_f(1-s) - I_1^2 R_b(1-s)$$

$$P_d = (I_1^2 R_f - I_1^2 R_b)(1-s) \tag{11.35}$$

$$P_d = (P_{agf} - P_{agb})(1-s) \tag{11.36}$$

The torque due to the forward field is expressed as,

$$T_f = \frac{P_{agf}}{\omega_s} \tag{11.37}$$

The torque due to the backward field is written as,

$$T_b = \frac{P_{agb}}{\omega_s} \tag{11.38}$$

The expression of the torque developed is expressed as,

$$T_d = T_f - T_b \tag{11.39}$$

In an alternative way, the expression of the torque can be expressed as,

$$T_d = 9.55\frac{P_d}{N_s} \tag{11.40}$$

It is seen that there are two revolving fields the forward field rotates in a clockwise direction and the backward field rotates in an anticlockwise direction. The forward field produces the torque in a clockwise direction whereas the backward field produces torque in an anticlockwise direction as shown in Fig. 11.6. When the rotor rotates, the impedance due to the forward field is higher than the impedance due to the backward field, i.e., $Z_f > Z_b$ and consequently the voltage due to the forward field is higher than the backward field, i.e., $E_f > E_b$. Therefore, as the speed increases, the resultant forward air gap field increases while the resultant backward air gap field decreases. Hence, the forward torque increases and the backward torque decreases. At standstill conditions, the slip is one for both torques. However, at synchronous speed, the slip for the forward torque is zero, whereas the slip for the backward torque is two. The resultant torque is calculated by the combination of the forward field torque and the backward field torque. This resultant torque developed by the motor is drawn that passes through the origin as shown in Fig. 11.9. The resultant torque has the magnitude except for the slip at $s = 1$.

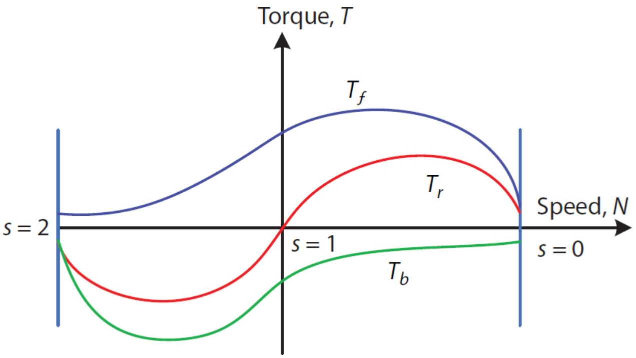


Fig. 11.9 Torque-speed characteristics.

Example 11.2

The equivalent circuit parameters of a 220 V, single-phase induction motor are $R_1 = R_2 = 6\Omega$, $X_1 = X_2 = 10\Omega$, $X_m = 150\Omega$. Calculate the input current, input power, power developed and torque developed. Consider the slip is 3% and the motor speed is 1500 rpm.

Solution

The impedance due to the forward field is calculated as,

$$Z_f = \frac{j75\left(\frac{3}{0.03} + j5\right)}{\frac{3}{0.03} + j75 + j5} = \frac{7509.4\angle 92.8^\circ}{128.06\angle 38.6^\circ} \tag{11.41}$$

$$Z_f = 58.6\angle 38.6^\circ = 34.3 + j47.5\Omega \tag{11.42}$$

The impedance due to the backward field is determined as,

$$Z_b = \frac{j75\left(\frac{3}{1.97}+j5\right)}{\frac{3}{1.97}+j75+j5} = \frac{392\angle 163^\circ}{139.4\angle 35^\circ} \tag{11.43}$$

$$Z_b = 2.8\angle 128^\circ = -1.7 + j2.2\,\Omega \tag{11.44}$$

The total impedance is then calculated as,

$$Z_t = R_1 + jX_1 + Z_f + Z_b = 6 + j10 + 34.3 + j47.5 - 1.7 + j2.2 \tag{11.45}$$

$$Z_t = 38.6 + j59.7 = 71.09\angle 57^\circ\,\Omega \tag{11.46}$$

The input current is determined as,

$$I_1 = \frac{V_t}{Z_t} = \frac{220}{71.09\angle 57^\circ} = 3.1\angle -57^\circ\ \mathrm{A} \tag{11.47}$$

The power developed is calculated as,

$$P_d = I_1^2(R_f - R_b)(1-s) = 3.1^2(34.3+1.7)(1-0.03) \tag{11.48}$$

$$P_d = 335.6\,\mathrm{W} \tag{11.49}$$

The torque developed is determined as,

$$T_d = 9.55\frac{P_d}{N} = \frac{335.6\times 9.55}{1500} = 2.14\,\mathrm{N\text{-}m} \tag{11.50}$$

Practice Problem 11.2

The equivalent circuit parameters of a 230 V, single-phase induction motor are $R_1 = R_2 = 10\Omega$, $X_1 = X_2 = 15\Omega$, $X_m = 150\Omega$. Calculate the input current, input power and power developed. Consider the slip is 4%.

11.7 Classification of a Single-Phase Motor

Depending on the method of starting, the single-phase motors are classified into four types. These are discussed below.

- Split-phase or resistance-start induction motor
- Capacitor start induction motor
- Capacitor run or permanent split capacitor induction motor
- Capacitor start and capacitor run induction motor
- Shaded pole motor
- Single-phase series or universal induction motor
- Single-phase synchronous motor

Single-phase series or universal motor: This type of motor can run either by the DC or the single-phase AC supply. The AC series motor normally runs at a high speed and provides high starting torque.

Single-phase synchronous motors: These types of motors normally run at a synchronous speed and are used in different types of clocks. The most common types of single-phase synchronous motors are the reluctance type and the hysteresis type.

11.8 Starting of Single-Phase Motors

A single-phase motor has one winding known as main/stator winding. This winding does not produce any rotating field. Two magnetomotive forces are necessary to produce the rotating field. Some methods for starting single-phase motors are discussed below.

11.8.1 Split-Phase Motor

The resistance split-phase motor is often called a split-phase motor. Main and auxiliary windings are connected in parallel to the AC supply and displaced 90° apart from each other as shown in Fig. 11.10(a).

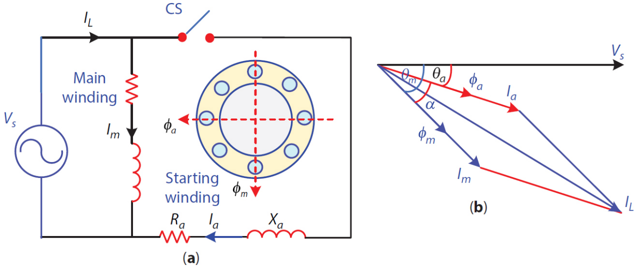


Fig. 11.10 Connection and vector diagrams of a split-phase motor.

The auxiliary coil has a fine wire with a small number of turns. The main winding has a thick wire with a large number of turns. The starting winding resistance is higher than the main winding resistance, whereas the main winding reactance is higher than the starting winding reactance. Therefore, the ratio of R_a/X_a is higher than the ratio of R_m/X_m. The phase difference between the main and starting winding currents is obtained at 30º by inserting variable resistance to the starting winding and the motor will start to run. When the motor reaches about 75 to 80% of the rated speed, the centrifugal switch will activate and disconnect the starting winding. Then the motor is kept running by the main winding. The starting current

Let, I_m and *Is* be the currents of the main and the starting windings. The expression of the starting torque and the line current is given by,

$$(11.51)\quad T_s = kI_mI_s\sin\alpha$$

$$(11.52)\quad I_L = \sqrt{I_s^2 + I_m^2 + 2I_sI_m\cos\alpha}$$

where the standard value of k is,

$k = 0.4$ in N.m/A^2.

In the vector diagram, the starting current I_a lags the source voltage V_s by θ_a and the main winding current I_m by θ_m as shown in Fig. 11.10(b). The minimum value of the angle between the main and starting windings is 30º. This minimum value of phase angle will produce a small value of the starting torque. The angle between the main winding and the auxiliary winding is calculated as,

$$\alpha = \theta_m - \theta_s \tag{11.53}$$

The starting current is usually 5 to 7 times higher than the rated current and starting torque is 1.5 to 2 times higher than the rated torque. The torque-speed characteristics of this motor with and without starting winding are shown in Fig. 11.11. Initially, the value of the torque due to starting winding is higher than the main winding. This type of motor is used in fans, small lathe machines, centrifugal pumps, blowers, washing machines, etc.

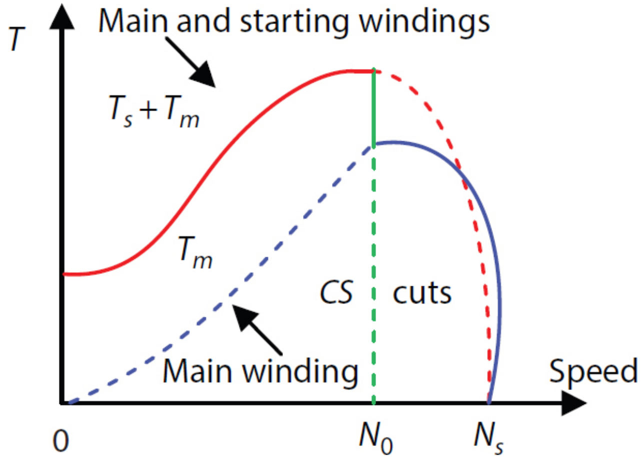


Fig. 11.11 Torque-speed characteristics of a split-phase motor.

Example 11.3

The rating of a resistance-start single-phase induction motor is 0.5 HP, 1600 rpm, 60Hz, 110V, k = 0.04 N.m/A^2. When the rotor is blocked by applying 40V, the other parameters are recorded I_m = 3.5 A, P_m = 75 W, I_s = 1.6 A, P_s = 45 W. Calculate the phase angle between the main winding current and auxiliary winding current, α and torque developed at 120V.

Solution

For the main winding, the apparent power is calculated as,

$$S_m = V_m I_m = 40 \times 3.5 = 140\ \mathrm{kVA} \tag{11.54}$$

The power factor and the angle are calculated as,

(11.55) $$\cos\theta_m = \frac{P_m}{S_m} = \frac{75}{140} = 0.54$$

(11.56) $$\theta_m = \cos^{-1}(0.54) = 57.32^\circ$$

Again, for the auxiliary winding, the apparent power is determined as,

(11.57) $$S_a = V_a I_a = 40 \times 1.6 = 64\,\text{kVA}$$

The power factor and the angle are calculated as,

(11.58) $$\cos\theta_a = \frac{P_a}{S_a} = \frac{45}{64} = 0.70$$

(11.59) $$\theta_a = \cos^{-1}(0.70) = 45.32^\circ$$

The angle α is calculated as,

(11.60) $$\alpha = \theta_m - \theta_s = 57.32^\circ - 45.32^\circ = 12^\circ$$

The new values of the main winding and the auxiliary winding currents at 120 V are calculated as,

(11.61) $$I_m = \frac{120}{40} \times 3.5 = 10.5\,\text{A}$$

(11.62) $$I_a = \frac{120}{40} \times 1.6 = 4.8\,\text{A}$$

The torque is calculated as,

(11.63) $$T_s = kI_m I_s \sin\alpha = 0.04 \times 10.5 \times 4.8 \sin 12^\circ = 0.42\,\text{N-m}$$

Example 11.4

The impedances of a 220 V, 50 Hz, split-phase induction motor are $Z_m = 3 + j5\ \Omega$ and $Zs = 5 + j3\ \Omega$. Find the starting current, main winding current and line current.

Solution

The starting winding impedance is given as,

$$Z_s = 5 + j3\Omega = 5.83\underline{|30.96^\circ}\ \Omega \tag{11.64}$$

The value of the angle is,

$$\theta_s = 30.96^\circ \tag{11.65}$$

The starting current is calculated as,

$$I_s = \frac{V_t}{Z_s} = \frac{220}{5.83\underline{|30.96^\circ}} = 37.9\underline{|-30.96^\circ}\ \text{A} \tag{11.66}$$

The main winding impedance is given as,

$$Z_m = 3 + j5\Omega = 5.83\underline{|59^\circ}\ \Omega \tag{11.67}$$

The value of the angle is,

$$\theta_m = 59^\circ \tag{11.68}$$

The main winding current is calculated as,

$$I_m = \frac{V_t}{Z_m} = \frac{220}{3 + j5} = 37.9\underline{|-59^\circ}\ \text{A} \tag{11.69}$$

The value of the angle of the line current is,

$$\alpha = \theta_m - \theta_s = 59^\circ - 30.9^\circ = 28.1^\circ \tag{11.70}$$

The value of the line current is,

$$I_L = \sqrt{I_s^2 + I_m^2 + 2I_sI_m\cos\alpha} = \sqrt{37.9^2 + 37.9^2 + 2\times 37.9\times 37.9\cos 28.1^\circ} \tag{11.71}$$

$$I = 73.5\ \text{A} \tag{11.72}$$

Practice Problem 11.3

The rating of a resistance-start single-phase induction motor is 0.75 HP, 1700 rpm, 60Hz, 100V, k = 0.04 N.m/A^2. When the rotor is blocked by applying 40V, the other parameters are recorded $I_m = 4.5$ A, $P_m = 65$ W, $I_s = 1.4$ A, $P_s = 55$ W. Find the phase angle between the main winding current and auxiliary winding current, α and torque developed at 110V.

Practice Problem 11.4

The impedances of a 120 V, 60 Hz, split-phase induction motor are $Z_m = 2 + j6\ \Omega$ and $Z_s = 6 + j2\ \Omega$. Calculate the starting current, main winding current and line current.

11.8.2 Capacitor Start Motor

A capacitor start motor means that the single-phase induction motor starts with the help of a capacitor. This capacitor is connected in series with starting winding to increase the phase difference between the starting and main winding currents as shown in Fig. 11.12(a). The phase difference is obtained around 80º to 90º by proper selection of the capacitor.

The approximate value of the capacitor for a 1/4 HP motor is 135 μF and 175 μF for a 1/3 HP motor. The value of the capacitor for the motors is around 30 μF or higher. Because of this capacitor, the phase angle between the starting and main windings currents will be increased. Therefore, the motor will develop a higher starting torque. This starting torque is about 3 to 4 times higher than the full load torque. The centrifugal switch will disconnect the capacitor when the motor reaches about 80% of its rated speed as shown in Fig. 11.13. When the auxiliary winding is disconnected by the centrifugal switch only the main winding is connected with the source and the motor is running at the rated speed. Some capacitor motors do not have a centrifugal switch. In the vector diagram, the starting current I_a leads the source voltage Vs by θ_a and the main winding current I_m by θ_m as shown in Fig. 11.12(b). The angle between the main winding and the auxiliary winding is calculated as,

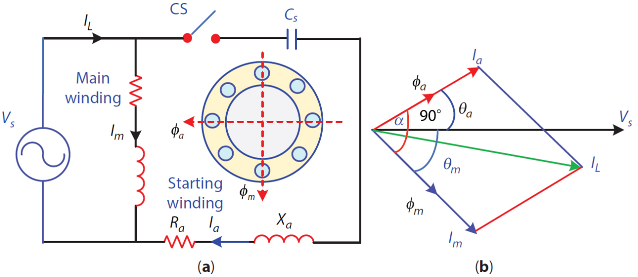

CS
C_s
I_L
Main winding
V_s
I_m
ϕ_a
Starting winding
ϕ_m
R_a
I_a
X_a
(a)
I_a
ϕ_a
θ_a
90°
α
V_s
I_L
θ_m
ϕ_m
I_m
(b)

Fig. 11.12 Connection and vector diagrams of a capacitor start motor.

$$\alpha = \theta_m + \theta_s \tag{11.73}$$

The capacitor is connected permanently for those types of motors. The torque-speed characteristics of the capacitor start motor are shown in Fig. 11.13. These types of motors are used in pumps, compressors, air-conditioners, machine tools, conveyors, refrigerators, etc.

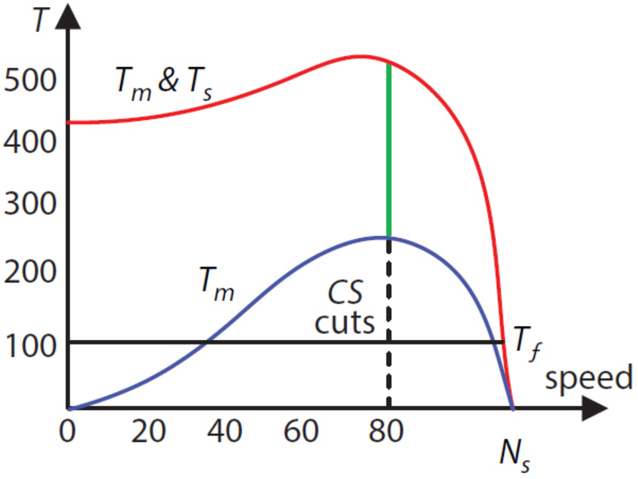


Fig. 11.13 Torque-speed characteristics of a capacitor start motor.

Example 11.5

The rating of a capacitor-start motor is 0.85 HP, 1750 rpm, 60Hz, 110V, k = 0.04 N.m/A^2. When the rotor is blocked by applying 45V, the main and auxiliary winding currents and powers are recorded $I_m = 5$ A, $P_m = 80$ W, $Is = 2$ A, $P_s = 50$ W. Determine the phase angle between the main winding current and auxiliary winding current, α and torque developed at 115V.

Solution

For the main winding, the apparent power is calculated as,

$$S_m = V_m I_m = 45 \times 5 = 225\,\mathrm{kVA} \tag{11.74}$$

The power factor and the angle are calculated as,

$$\cos\theta_m = \frac{P_m}{S_m} = \frac{80}{225} = 0.36 \tag{11.75}$$

$$\theta_m = \cos^{-1}(0.36) = 69.17^\circ \tag{11.76}$$

Again, for the auxiliary winding, the apparent power is determined as,

$$S_a = V_a I_a = 45 \times 2 = 90\,\mathrm{kVA} \tag{11.77}$$

The power factor and the angle are calculated as,

$$\cos\theta_a = \frac{P_a}{S_a} = \frac{60}{90} = 0.67 \tag{11.78}$$

$$\theta_a = \cos^{-1}(0.56) = 48.19^\circ \tag{11.79}$$

The angle α is calculated as,

$$\alpha = \theta_m + \theta_s = 49.17^\circ + 48.19^\circ = 97.36^\circ \tag{11.80}$$

The new values of the main winding and the auxiliary winding currents at 115 V are calculated as,

$$I_m = \frac{115}{45} \times 5 = 12.78\,\mathrm{A} \tag{11.81}$$

$$I_a = \frac{115}{45} \times 2 = 5.11\,\text{A} \tag{11.82}$$

The torque is calculated as,

$$T_s = kI_m I_s \sin\alpha = 0.04 \times 12.78 \times 5.11 \sin 97.36^\circ = 2.59\,\text{N-m} \tag{11.83}$$

Example 11.6

The main and starting windings impedances of a 220 V, 50 Hz capacitor-start motor are $Z_m = 4 + j3.5\ \Omega$ and $Z_s = 5 + j3\ \Omega$, respectively. Find the value of the starting capacitance for getting maximum torque.

Solution

The main winding impedance and angle are given as,

$$Z_m = 4 + j3.5 = 5.31\underline{|41.2^\circ}\,\Omega \tag{11.84}$$

$$\theta_m = 41.2^\circ \tag{11.85}$$

Let *Xc* be the reactance of the capacitor that is connected in series with the starting winding. Then the total impedance of the starting winding is expressed as,

$$Z_s = 5 + j3 - jX_c = 5 + j(3 - X_c) \tag{11.86}$$

The torque will be maximum when the angle between the starting winding and main winding currents is 90º i.e., $\theta_m - \theta_s = 90^\circ$. The value of the angle of the starting winding current is calculated as,

$$\theta_m - \theta_s = 90^\circ \tag{11.87}$$

$$41.2^\circ - \theta_s = 90^\circ \tag{11.88}$$

$$\theta_s = 41.2^\circ - 90^\circ = -48.8^\circ \tag{11.89}$$

The value of the capacitive reactance of the starting or auxiliary winding is determined as,

$$\tan(-48.8^\circ) = \frac{3 - X_c}{5} \quad (11.90)$$

$$X_c = 8.71 \quad (11.91)$$

$$\frac{1}{2\pi fC} = 8.71 \quad (11.92)$$

$$C = \frac{1}{2\pi \times 50 \times 8.71} = 365\mu\text{F} \quad (11.93)$$

Practice Problem 11.5

The rating of a capacitor-start motor is 1 HP, 1725 rpm, 60Hz, 115V, k = 0.04 N.m/A^2. When the rotor is blocked by applying 35V, the main and auxiliary winding currents and powers are recorded $I_m = 4$ A, $P_m = 66$ W, $I_s = 1.2$ A, $P_s = 39$ W. Calculate the phase angle between the main winding current and auxiliary winding current, α and torque developed at 120V.

Practice Problem 11.6

The main and starting windings impedances of a 240 V, 50 Hz capacitor-start motor are $Z_m = 2 + j4.5\ \Omega$ and $Z_s = 4 + j3.5\ \Omega$, respectively. Find the value of the starting capacitance for getting maximum torque.

11.8.3 Capacitor Start and Capacitor Run Motor

The capacitor start motor has a low starting torque but it is much better than the split-phase motor. However, in an application where a high starting torque is necessary, a capacitor start and capacitor run motor need to be employed. For a higher starting torque, a large current is required. In this case, the starting capacitive reactance ($X_s = 1/\omega C_s$) must be low. To achieve low reactance, the value of the starting capacitance must be large. After starting, when the motor operates under running conditions, it requires less current. For this less running current, the reactance

($X_r = 1/\omega C_r$) of the running capacitor must be higher. The capacitor known as a running capacitor is permanently connected to the motor. The other one known as the starting capacitor will be disconnected later when the speed increases to 80% of the rated speed. The capacitor C_s is used for a short time during starting and capacitor C_r is used for continuous running as shown in Fig. 11.14.

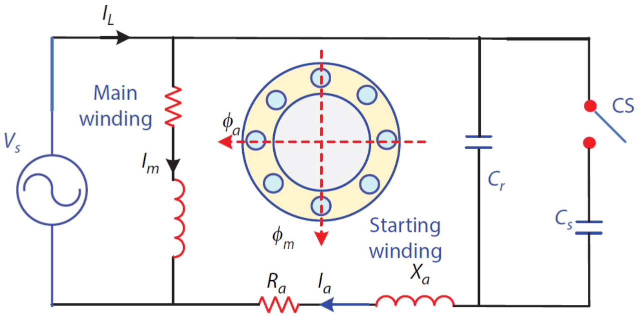


Fig. 11.14 Schematic of a capacitor start and capacitor run motor.

The starting capacitor is always connected in series with the starting winding. The starting capacitor has a higher value (500-600μF) than the running capacitor (10-15μF). At the time of starting, the phase difference between two currents is $\phi_m + \phi_a >$ 90° as shown in phasor diagram Fig. 11.15(a), while in the running time, the phase difference between two currents is $\phi_m + \phi_a = 90°$ as can be seen in Fig. 11.15(b).

In the second case, the motor is running in a balanced condition. In this condition, the number of turns for both windings must be the same. Therefore, only forward forward-rotating field is present. In this condition, the efficiency of the motor is higher compared to the motors running only with the main winding and provide noiseless and quiet operation. This motor of course is expensive compared to others. However, it provides better performance.

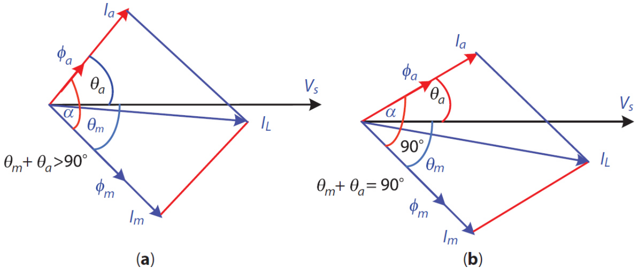


Fig. 11.15 Vector diagrams of a capacitor start and capacitor run motor.

This motor has starting torque, low starting current and a higher power factor. This type of motor is used in compressors, air compressors, refrigerators, etc. Fig. 11.16 shows the torque-speed characteristics of a capacitor start and capacitor run motor.

11.8.4 Capacitor Run Motor

In a capacitor run motor, the capacitor is permanently connected in series with the starting winding without a centrifugal switch as shown in Fig. 11.17. The power factor is very high in this motor as the capacitor is permanently connected in series with the starting winding. This motor operates as a balanced two-phase motor and produces a noise-free uniform torque. The value of the run capacitor varies between 20 μF to 30 μF. The power factor of this motor is very high as the capacitor is permanently connected to the starting winding. This permanent capacitor is usually chosen to get a speed of about 80% of its rated speed. This capacitor provides a small phase angle between the main and starting currents at the time of starting. The capacitor run motor is more expensive than the capacitor start motor.

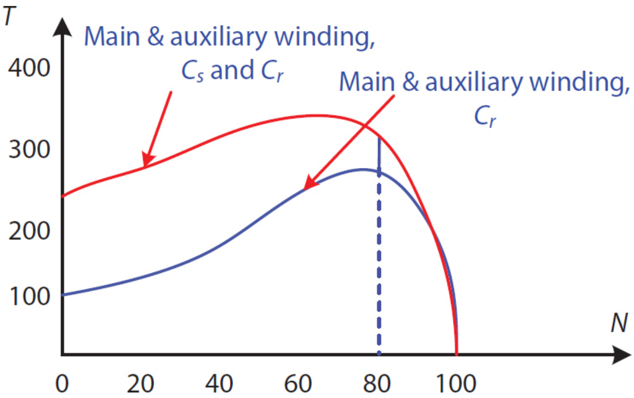


Fig. 11.16 Torque-speed characteristics of a capacitor start and capacitor run motor.

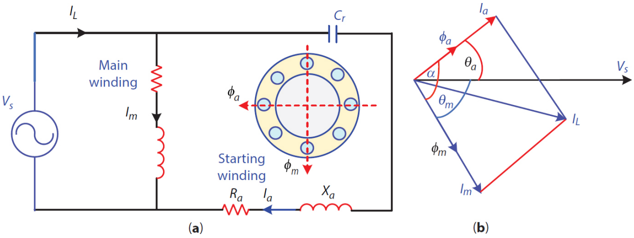

I_L
C_r
Main winding
V_s
I_m
ϕ_a
Starting winding
ϕ_m
R_a
I_a
X_a
(a)
I_a
ϕ_a
θ_a
α
V_s
θ_m
I_L
ϕ_m
I_m
(b)

Fig. 11.17 Schematic of a capacitor run motor.

This is due to the starting winding works as the main winding. This motor has a high efficiency and low starting torque. The torque-speed characteristics of a capacitor run motor are shown in Fig. 11.18.

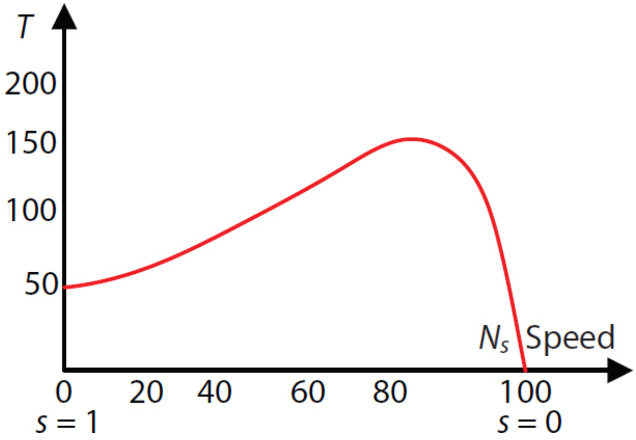


Fig. 11.18 Torque-speed characteristics of a capacitor run motor.

11.8.5 Shaded Pole Motor

A shaded pole motor has one winding known as a main winding. The shaded pole motor has two or four poles. The pole of the shaded pole motor is a salient type which is projected towards the armature of the motor. One portion of the poles is surrounded by a short-circuited copper turn, which is known as a shading coil. This shading coil acts as a secondary for the motor. When a single-phase supply is given to the stator winding it produces a time-varying flux, which induces a voltage in the shading coil. As a result of this voltage, the current flows through the shading coil. The flux due to the shading coil current opposes the original core flux. The flux in the shading coil lags behind the flux of an un-shading coil. As a result, a periodic shift of flux from the un-shading coil to the shading coil is obtained which produces a low starting torque. The connection diagram of the shaded pole motor is shown in Fig. 11.19 and the torque-speed characteristics are shown in Fig. 11.20.

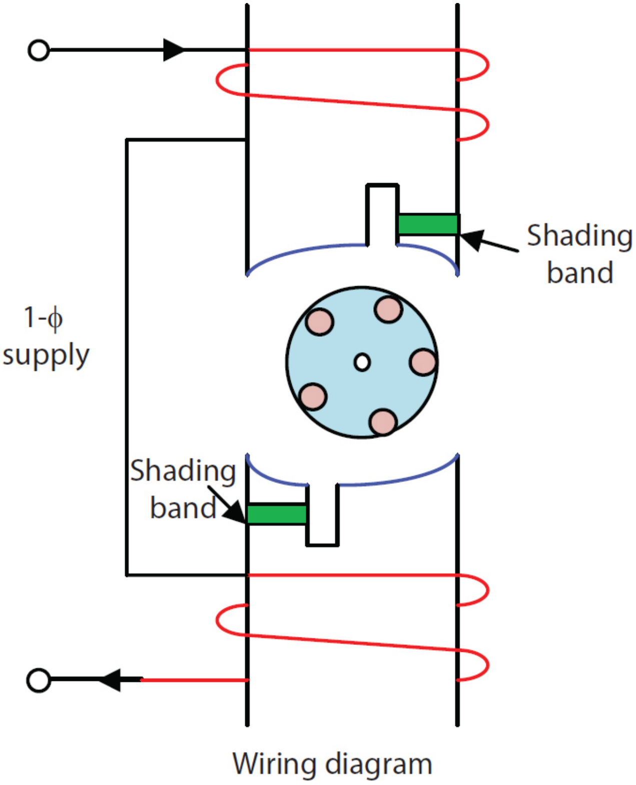

Shading
band
1-ϕ
supply
Shading
band
Wiring diagram

Fig. 11.19 Connection of a shaded pole motor.

The motor rotates in only one direction and reverse direction is not possible. The starting torque of this motor is very small, which is around 50% of the full load torque as shown in Fig. 11.17. The motor has poor efficiency due to the continuous power loss in the copper shading coil. Due to a small torque, shaded pole motors are used in clocks, car seats, gramophones, etc.

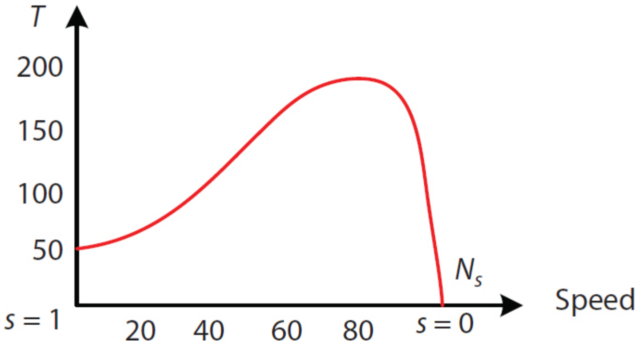


Fig. 11.20 Torque-speed characteristics of a shaded pole motor.

11.9 AC Series Motor

The AC series motor is often known as a universal motor. According to the name, this motor may operate by either a DC supply or a single-phase AC supply at the same speed and output. Its operating principle is the same as the DC series motor, i.e., the same current will flow through the armature and the field winding. The current in the field winding produces an electromagnetic field. When a current-carrying armature conductor is placed in a magnetic field it produces the Lorentz force as well as the torque. Again, when a single-phase AC supply is fed to a DC series motor, in the positive half-cycle of the supply voltage, the same current will flow through the series field winding and the armature winding as shown in Fig. 11.21. This current will generate a unidirectional torque which is represented as $T \propto \phi_f I_a$.

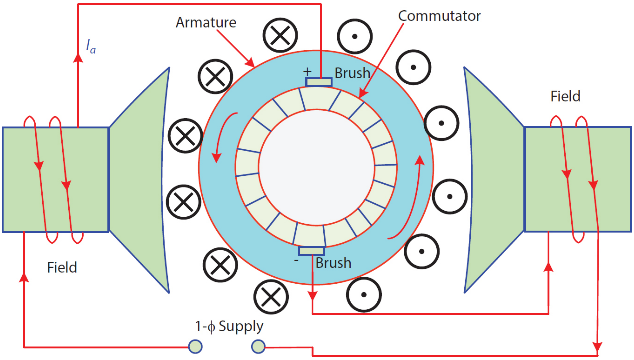


Fig. 11.21 Unidirectional torque for positive half-cycle.

Similarly, in the negative half-cycle, the current flow will be reversed in the field winding as well as the armature as shown in Fig. 11.22.

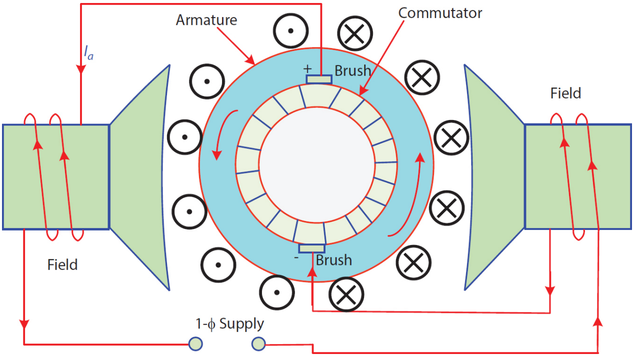

Armature
Commutator
I_a
Brush
+
Field
Field
-
Brush
1-ϕ Supply

Fig. 11.22 Unidirectional torque for a negative half-cycle.

This current will generate a unidirectional torque which is represented as T ∞ $(-\phi_f)(-I_a)$. Due to this torque, the motor will rotate continuously and the direction of the rotation is determined by Fleming's left-hand rule. The drawbacks of the universal motor are eddy current loss appears that will create heat in the field and yoke and the large value of the field and armature reactance will create a low power factor. In addition to that, sparks may occur across the bush of the motor. The universal motor is used for vacuum cleaners, hair dryers, food mixers, portable drill machines, etc. An AC series motor operates at the speed of 1500 rpm to 10000 rpm. The simple connection diagram of the AC series motor is shown in Fig. 11.23.

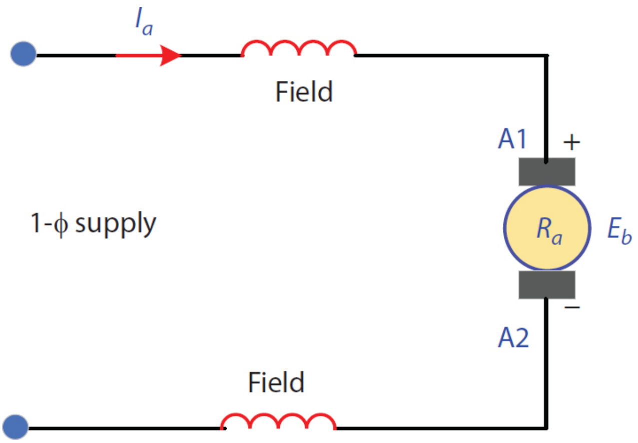

I_a
Field
A1
+
1-ϕ supply
R_a
E_b
−
A2
Field

Fig. 11.23 Schematic of an AC series motor.

Some important facts need to be known about the universal motor. When a DC series motor is fed by a single-phase AC supply the core loss and eddy current loss will increase, which reduces the efficiency of the motor. In this case, the stator and rotor of this motor are made of laminated steel to reduce these losses. It also needs to use a smaller number of turns of the field winding. In addition to that, when a single-phase AC supply is given to the DC series motor, a sparking commutation takes place during operation. This drawback can be reduced by increasing the number of commutation segments. The power factor of the single-phase AC series motor is less because of the armature and the series field reactance. The power factor can be increased by decreasing the total reactance. For performance analysis, the expression of the back emf of a DC series motor is,

$$E_b = \frac{PZ\phi N}{60A} \tag{11.94}$$

The number of poles P, number of conductors Z, and parallel paths A are constant then equation (11.94) is modified as,

$$N \infty \frac{E_b}{\phi} \infty \frac{E_b}{I_a} \tag{11.95}$$

The torque equation of a DC series motor is expressed as,

$$T \infty \phi I_a \infty I_a^2 \tag{11.96}$$

If the load increases, the armature current increases, which reduces the speed and torque. However, in the case of an AC motor, if frequency increases, the reactance and the reactive drop (IX) will increase. Due to this, $E_{ac} = V_s - I_a X$ will decrease and the corresponding speed ($N_{ac} \infty E_{ac}/\phi$) will decrease. When a DC motor is fed by a DC supply, the speed is higher than the speed of the motor when fed by a single-phase AC supply as shown in Fig. 11.24.

The current I_a drawn by the motor is a 180° phase shift from induced voltage E_{ac}. This current will create the flux ϕ which will be in phase with it. Then draw the series field resistive drop in phase with the armature current and the reactive drop perpendicular to it.

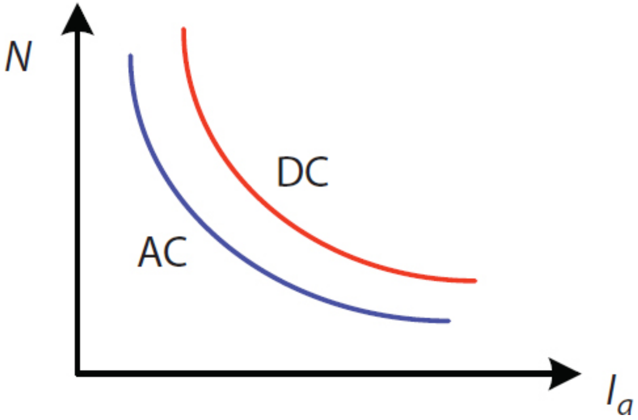


Fig. 11.24 Performance of a universal motor.

Finally, add up the AC-induced voltage E_{ac} to obtain the supply voltage V_s vectorically as shown in Fig. 11.25(a). For further analysis, the resistive and total reactive drop are drawn separately as shown in Fig. 11.25(b).

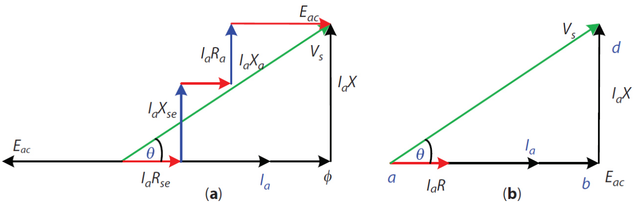

E_{ac}
I_aR_a
I_aX_a
V_s
I_aX
I_aX_{se}
E_{ac}
θ
I_aR_{se}
(a)
I_a
ϕ
V_s
d
I_aX
θ
I_a
a
I_aR
(b)
b
E_{ac}

Fig. 11.25 Performance of a universal motor.

The power factor is calculated as,

$$\sin\theta = \frac{I_a X}{V_s} \qquad (11.97)$$

$$\cos\theta = \sqrt{1 - \left(\frac{I_a X}{V_s}\right)^2} \qquad (11.98)$$

$$\cos\theta = \frac{E_{ac} + I_a R_a}{V_s} \tag{11.99}$$

From Fig. 11.23(b), the following equation is written as,

$$(E_{ac} + I_a R_a)^2 + (I_a X)^2 = (V_s)^2 \tag{11.100}$$

$$(E_{ac} + I_a R_a)^2 = (V_s)^2 - (I_a X)^2 \tag{11.101}$$

$$(E_{ac} + I_a R_a) = \sqrt{(V_s)^2 - (I_a X)^2} \tag{11.102}$$

$$E_{ac} = \sqrt{(V_s)^2 - (I_a X)^2} - I_a R_a \tag{11.103}$$

The ratio speed due to AC and DC supply is written as,

$$\frac{N_{ac}}{N_{dc}} = \frac{E_{ac}}{E_b} \tag{11.104}$$

Substituting back emf expression $E_b = V_s - I_a R_a$ for DC and $E_b = V_s \cos\theta - I_a R_a$ for AC supply in equation (11.104) yields,

$$\frac{N_{ac}}{N_{dc}} = \frac{V_s \cos\theta - I_a R_a}{V_s - I_a R_a} \tag{11.105}$$

The $I_a R_a$ drop is very small compared to the supply voltage, so it is neglected. Then equation (11.104) is modified as,

$$\frac{N_{ac}}{N_{dc}} = \frac{V_s \cos\theta_a}{V_s} \tag{11.106}$$

$$\frac{N_{ac}}{N_{dc}} = \cos\theta \tag{11.107}$$

$$N_{ac} = N_{dc} \cos\theta \tag{11.108}$$

The torque is calculated from the expression as,

$$P_{ac} = \omega_{ac} T_{ac} \quad (11.109)$$

$$E_{ac} I_a = \frac{2\pi N_{ac}}{60} T_{ac} \quad (11.110)$$

$$E_{ac} I_a = 9.55 N_{ac} T_{ac} \quad (11.111)$$

$$T_{ac} = \frac{E_{ac} I_a}{9.55 N_{ac}} \quad (11.112)$$

From equation (11.112), it is seen that torque can be calculated if the other parameters are given.

Example 11.7

The armature resistance and reactance of a universal motor are 20 Ω and 100 Ω, respectively. The motor draws a current of 0.5 A when connected to a 120 V DC which runs at a speed of 2000 rpm. When connected to a 120V AC, find the speed, torque and power factor.

Solution

The back emf due to a DC supply is calculated as,

$$E_b = V_s - I_a R_a = 120 - 0.5 \times 20 = 110\,\mathrm{V} \quad (11.113)$$

The value of the DC speed is given by,

$$N_{dc} = 2000\,\mathrm{rpm} \quad (11.114)$$

The back emf due to AC supply is calculated as,

$$E_{ac} = \sqrt{(V_s)^2 - (I_a X)^2} - I_a R_a = \sqrt{(120)^2 - (0.5 \times 100)^2} - 0.5 \times 20 = 99.09\,\mathrm{V} \quad (11.115)$$

From equation (11.104), the speed of the motor due to AC supply is calculated as,

$$N_{ac} = \frac{E_{ac}}{E_b} \times N_{dc} = \frac{99.09}{110} \times 2000 = 1801.63\,\mathrm{rpm} \quad (11.116)$$

The torque is calculated as,

$$T_{ac} = \frac{E_{ac} I_a}{9.55 N_{ac}} = \frac{99.09 \times 0.5}{9.55 \times 1801.64} = 0.0028\,\text{N-m} \tag{11.117}$$

The power factor is calculated as,

$$\cos\theta = \frac{E_{ac} + I_a R_a}{V_s} = \frac{99.09 + 0.5 \times 20}{120} = 0.91(\text{lag}) \tag{11.118}$$

Practice Problem 11.7

The armature resistance and reactance of a universal motor are 30 Ω and 120 Ω, respectively. The motor draws a current of 0.9 A when connected to a 110 V DC which runs at a speed of 2200 rpm. When connected to a 110V AC, calculate the speed, torque and power factor.

11.10 Reluctance and Hysteresis Motors

A single-phase synchronous motor is classified as a reluctance motor and hysteresis motor, which are very popular for some special purposes. The descriptions of these types of motors are given below.

11.10.1 Reluctance Motor

A single-phase induction with a main winding and auxiliary winding with a variable air gap rotor without a DC excitation is known as a reluctance motor. The rotor is made of ferromagnetic material with a salient pole structure. The reluctance motor is a single-phase motor whose working principle depends on the reluctance torque. The reluctance torque is the torque that is acting on the magnetic material to align with the main field axis. The construction of a reluctance motor is simple such as no DC field winding in the rotor, no brushes, or sliprings. The manufacturing cost is low and easy to maintain.

The stator winding is energized by the single-phase AC source. A revolving field is produced in the stator winding which rotates at a synchronous speed. Initially, the motor runs as a squirrel cage induction motor and later on as a synchronous motor. The slip is very small when the rotor reaches near synchronous speed. In this condition, the rotating field produces the reluctance torque on the salient type of rotor poles which have smaller reluctance (Low air gap). This reluctance torque tries to line up the rotor pole axis with the axis of the main rotating field.

As a result, the rotor poles are locked with the main rotating field and the motor runs at the speed of the rotating field, i.e., the synchronous speed as shown in Fig. 11.26.

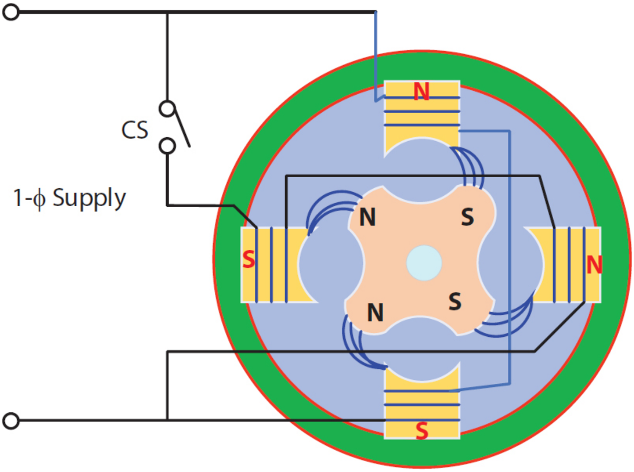


Fig. 11.26 Schematic of a reluctance motor.

The torque-speed characteristics of a reluctance motor are shown in Fig. 11.27. The motor will start between 200-300% of its full load. When the motor achieves 75% of its rated speed, the centrifugal switch cuts the auxiliary winding and the motor runs with the main winding. When the speed increases more that is nearer to the synchronous speed, the reluctance torque at that time is enough to pull the rotor into synchronism with the stator field. This torque is known as pull-in torque. This motor is used in recording devices, signaling devices, and electric clock timers,

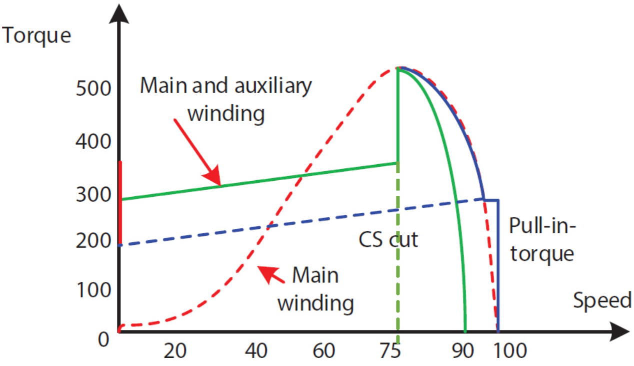
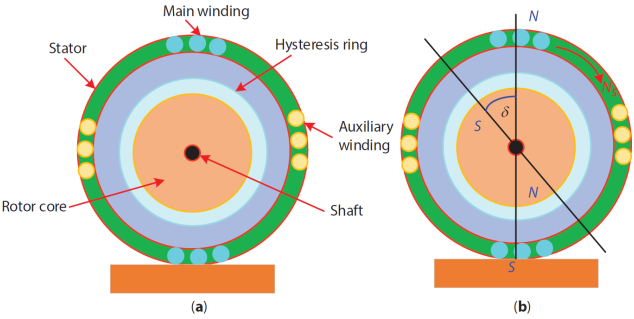


Fig. 11.27 Performance of a reluctance motor.

11.10.2 Hysteresis Motor

The hysteresis motor is the second type of single-phase synchronous motor. The hysteresis motor also does not require DC excitation for starting purposes. The stator winding can be either single-phase or three-phase. The rotor of this motor is surrounded by a special magnetic ring known as a hysteresis ring and its inside has a non-magnetic material as shown in Fig. 11.28. The hysteresis ring is made either of hard chrome cobalt or steel which has a very large hysteresis loop as shown in Fig. 11.29(a). This hysteresis ring is also laminated to reduce the eddy current loss. The remaining inside part of the rotor is made of non-magnetic material such as aluminum to obtain light weight.



(a) (b)

Fig. 11.28 Schematic of a hysteresis motor.

The operation of this type of motor depends on the hysteresis effect. The stator of this motor is similar to the induction motor.

For a single-phase stator, a permanent capacitor is connected to the starting winding to produce enough phase difference. If the stator winding is energized by a single-phase supply, a stator current will flow through the stator windings. Because of this current, a rotating flux will be created within the air gap, which rotates at synchronous speed. This stator-rotating flux magnetizes the rotor and creates the rotor poles. Due to the hysteresis effect, the flux is not in the same phase as the magnetizing component of the current. The rotor magnetization axis will lag the stator field axis by an angle of δ as can be seen in Fig. 11.29(b). This angle is known as a hysteresis angle. The hysteresis loop is the relationship between the magnetic flux density B and the magnetic field intensity H as shown in Fig. 11.29(a). At point a, the value of B is negative which is shown in Fig. 11.29(b). At point b, H is positive and B is zero. Similarly, at point c, the B is positive and has the maximum value. After point d, B will reduce and at point e, it will become zero. After continuing, the complete curve for B will be obtained as shown in Fig. 11.29(b). From Fig. 11.29(b), it is seen that the variation of H with respect to t is sinusoidal whereas the variation of B is a flat top. Therefore, it is seen that the magnetic flux density B lags the magnetic field intensity by the angle δ. When the motor reaches a synchronous speed, the stator flux will move with the rotor flux at the same speed. In this case, the expression of the starting torque becomes,

$$T_s = \phi_m \phi_s \sin\delta \tag{11.119}$$

where ϕ_s is the flux due to starting winding and ϕ_m is the flux due to main winding.

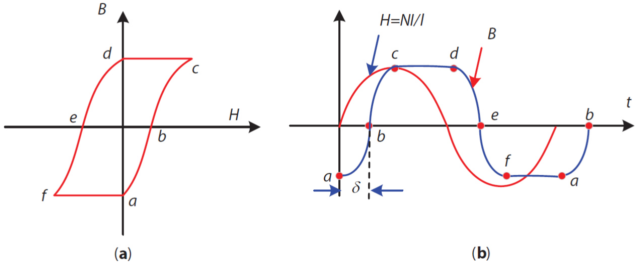


Fig. 11.29 Schematic of a hysteresis motor.

There will be some eddy current and hysteresis losses due to the iron and hysteresis ring. The power loss due to the eddy current is calculated as,

(11.120)$$P_e = k_e f_2^{\,2} B_m^{\,2} t^2$$

The rotor frequency $f_2 = sf$ substituting in equation (11.120) yields,

(11.121)$$P_e = k_e s^2 f^2 B_m^{\,2} t^2$$

From a three-phase induction motor, the relationship between the air gap power and the copper loss is,

(11.122)$$P_{cu} = sP_g$$

But the copper loss is equal to the eddy current power loss. Substituting equation (11.121) into equation (11.122) yields,

(11.123)$$k_e s^2 f^2 B_m^{\,2} t^2 = sP_g$$

(11.124)$$P_g = \frac{k_e s^2 f^2 B_m^{\,2} t^2}{s}$$

(11.125)$$P_g = s k_e f^2 B_m^{\,2} t^2$$

The torque due to eddy current loss is calculated as,

(11.126)$$T_e = \frac{P_g}{\omega_s}$$

Substituting equation (11.125) into equation (11.126) yields,

(11.127)$$T_e = \frac{s k_e f^2 B_m^{\,2} t^2}{\omega_s}$$

The frequency, magnetic flux density, angular synchronous speed and thickness are constant and equation (11.127) becomes,

$$T_e \propto s \tag{11.128}$$

The power loss due to the hysteresis effect is calculated as,

$$P_h = k_h f_2 B_m{}^n \tag{11.129}$$

Where n is the Steinmetz constant and its values vary from 1.5 to 2.5 depending on the material properties. Again, the rotor frequency $f_2 = sf$ substituting in equation (11.129) yields,

$$P_h = k_h sf B_m{}^n \tag{11.130}$$

But the copper loss is equal to the hysteresis power loss. Substituting equation (11.130) into equation (11.122) yields,

$$k_h sf B_m{}^n = sP_g \tag{11.131}$$

$$P_g = k_h f B_m{}^n \tag{11.132}$$

The torque due to hysteresis loss is calculated as,

$$T_h = \frac{P_g}{\omega_s} \tag{11.133}$$

Substituting equation (11.132) into equation (11.133) yields,

$$T_h = \frac{k_h f B_m{}^n}{\omega_s} \tag{11.134}$$

The frequency, magnetic flux density and angular synchronous speed are constant and equation (11.134) becomes,

$$T_h \propto \text{constant} \tag{11.135}$$

Due to smooth and noiseless operation, the hysteresis motor is used in the clock, timer circuit audio and video recording equipment. The torque characteristics of a hysteresis motor are shown in Fig. 11.30.

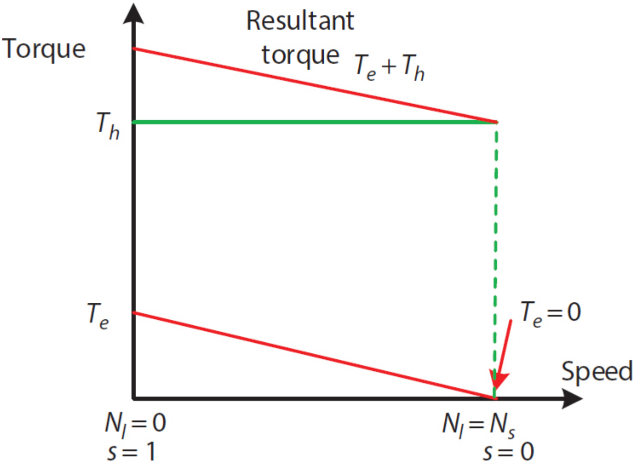


Fig. 11.30 Torque characteristics of a hysteresis motor.

11.11 Test of a Single-Phase Motor

There are two types of tests normally conducted in a single-phase motor. These are the blocked rotor test and the no-load test. The equivalent circuit parameters of the single-phase motor are determined by the blocked rotor test as well as by the no-load test. These tests are discussed below.

11.11.1 Blocked Rotor Test

In a blocked rotor test, the rotor is blocked by any means such that the speed of the motor at any load is zero ($N_l = 0$) and slip is equal to one ($s = 1$). This test is conducted by the application of reduced voltage. Under blocked rotor conditions, the slip of the motor is equal to one. The connection diagram for the blocked rotor test is shown in Fig. 11.31.

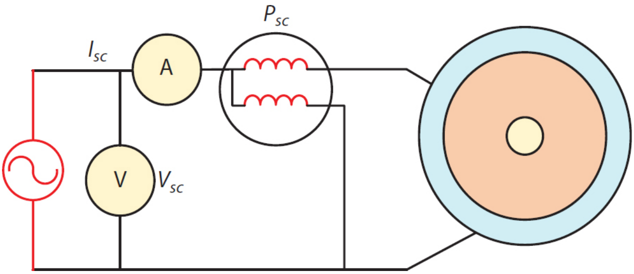


Fig. 11.31 Connection diagram of a blocked rotor test.

In a blocked rotor test, a smaller voltage known as short-circuit voltage (V_{sc}) or blocked rotor voltage (V_{br}) to get the short-circuit current (I_{sc} or I_{br}) and the short-circuit power (P_{sc} or P_{br}). The short-circuit impedance is determined as,

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} \tag{11.136}$$

The total resistance in the circuit can be calculated as,

$$R_{sc} = \frac{P_{sc}}{I_{sc}^{\ 2}} \tag{11.137}$$

The total reactance is determined as,

$$X_{sc} = \sqrt{Z_{sc}^{\ 2} - R_{sc}^{\ 2}} \tag{11.138}$$

The approximate equivalent circuit with ignoring magnetizing reactance is shown in Fig. 11.32. From Fig. 11.32(b), the following equations can be written as,

$$R_{sc} = R_1 + \frac{R_2}{2} + \frac{R_2}{2} = R_1 + R_2 \tag{11.139}$$

$$X_{sc} = X_1 + \frac{X_2}{2} + \frac{X_2}{2} = X_1 + X_2 \tag{11.140}$$

The stator circuit resistance R_1 is normally determined by the DC test or can be measured by an Ohmmeter across the main winding of the motor. The rotor circuit resistance is determined as,

$$R_2 = R_{sc} - R_1 \tag{11.141}$$

Generally, the reactances are determined as,

$$X_1 = X_2 = 0.5X_{sc} \tag{11.142}$$

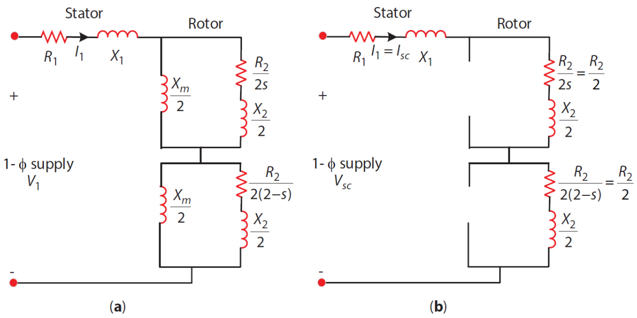

Stator
Rotor
R_1
I_1
X_1
$\frac{X_m}{2}$
$\frac{R_2}{2s}$
$\frac{X_2}{2}$
+
1-ϕ supply
V_1
$\frac{R_2}{2(2-s)}$
-
(a)
Stator
Rotor
R_1
$I_1 = I_{sc}$
X_1
$\frac{R_2}{2s} = \frac{R_2}{2}$
$\frac{X_2}{2}$
+
1-ϕ supply
V_{sc}
$\frac{R_2}{2(2-s)} = \frac{R_2}{2}$
-
(b)

Fig. 11.32 Approximate equivalent circuit for blocked rotor test.

11.11.2 No-Load Test

Fig. 11.33 shows the connection diagram for the no-load test of a single-phase induction motor. In this test, the motor runs by the rated voltage. The no-load test is conducted by running the motor without load. i.e., the motor is running nearer to the synchronous speed. Since the no-load slip is very small, $s \approx 0$, then the following expressions are written as,

$$\frac{R_2}{2s} = \frac{R_2}{2\times 0} = \infty \tag{11.143}$$

$$\frac{R_2}{2(2-s)} = \frac{R_2}{4} \tag{11.144}$$

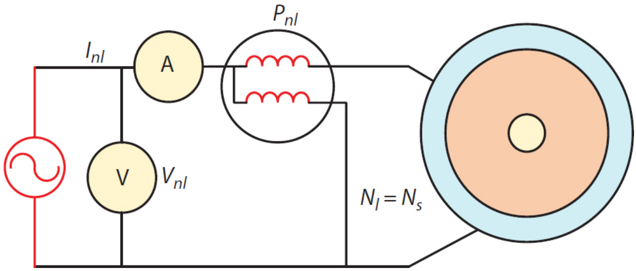

P_{nl}
I_{nl}
A
V
V_{nl}
$N_l = N_s$

Fig. 11.33 Connection diagram of a no-load test.

Based on equation (11.143), in a no-load test equivalent circuit diagram, the resistor $R_2/2s$ is considered an open circuit as shown in the forward circuit in Fig. 11.34(b). Again, based on equation

(11.144), $R_2/4 << X_m/2$. Then the term $X_m/2$ is considered an open circuit in the backward field of Fig. 11.34(b). Let V_{nl}, I_{nl} and P_{nl} be the measured values of the voltage, current and power, respectively. The input impedance is determined as,

$$Z_{nl} = \frac{V_{nl}}{I_{nl}} \quad (11.145)$$

The no-load circuit angle is calculated as,

$$\phi_{nl} = \cos^{-1}\left(\frac{P_{nl}}{V_{nl} I_{nl}}\right) \quad (11.146)$$

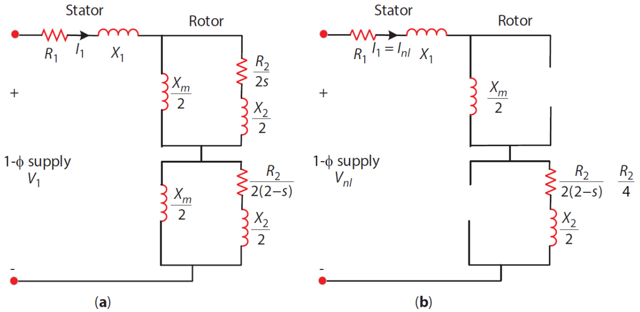

Stator
Rotor
R_1
I_1
X_1
+
1-ϕ supply
V_1
-
$\frac{X_m}{2}$
$\frac{R_2}{2s}$
$\frac{X_2}{2}$
$\frac{R_2}{2(2-s)}$
(a)
Stator
Rotor
R_1
$I_1 = I_{nl}$
X_1
+
1-ϕ supply
V_{nl}
-
$\frac{X_m}{2}$
$\frac{R_2}{2(2-s)}$
$\frac{R_2}{4}$
$\frac{X_2}{2}$
(b)

Fig. 11.34 Approximate equivalent circuit for no-load test.

From the theory of impedance triangle, the value of the no-load reactance is calculated as,

$$X_{nl} = Z_{nl} \sin\phi_{nl} \quad (11.147)$$

The no-load resistance can be determined as,

$$R_{nl} = \sqrt{Z_{nl}^2 - X_{nl}^2} \quad (11.148)$$

In an alternative way, it can be determined as,

$$R_{nl} = \frac{P_{nl}}{I_{nl}^2} \quad (11.149)$$

From the equivalent circuit, as shown in Fig. 11.31(b), the following equations can be written as,

$$X_{nl} = X_1 + \frac{1}{2}X_2 + \frac{1}{2}X_m \tag{11.150}$$

$$X_m = 2(X_{nl} - X_1 - 0.5X_2) \tag{11.151}$$

The rotational losses can be calculated from the following equation,

$$P_{rot} = P_{nl} - I_{nl}^2(R_1 + \frac{1}{4}R_2) \tag{11.152}$$

Example 11.8

A split-phase, 50 Hz, single-phase induction motor is tested and the following data are obtained.

No load test: 100 V, 2.5 A, 60 W

Blocked rotor test: 60 V, 3 A, 130 W

The main winding resistance is 2 Ω. Calculate the equivalent circuit parameters of the motor.

Solution

The impedance due to the blocked rotor test can be determined as,

$$Z_{br} = \frac{V_{br}}{I_{br}} = \frac{60}{3} = 20\Omega \tag{11.153}$$

The resistance due to the blocked rotor test is calculated as,

$$R_{br} = \frac{130}{3^2} = 14.4\Omega \tag{11.154}$$

The reactance under blocked rotor condition is determined as,

$$X_{br} = \sqrt{20^2 - 14.4^2} = 13.8\Omega \tag{11.155}$$

The values of the reactances are calculated as,

$$X_1 = X_2 = 0.5 \times 13.8 = 6.9\,\Omega \tag{11.156}$$

The value of the rotor resistance is calculated as,

$$R_2 = R_{br} - R_1 = 14.4 - 2 = 12.4\,\Omega \tag{11.157}$$

The value of the impedance due to the no-load test is determined as,

$$Z_{nl} = \frac{V_{nl}}{I_{nl}} = \frac{100}{2.5} = 40\,\Omega \tag{11.158}$$

The value of the resistance due to the no-load test is calculated as,

$$R_{nl} = \frac{P_{nl}}{I_{nl}^{\;2}} = \frac{60}{2.5^2} = 9.6\,\Omega \tag{11.159}$$

The value of the reactance due to the no-load test is determined as,

$$X_{nl} = \sqrt{Z_{nl}^{\;2} - R_{nl}^{\;2}} = \sqrt{40^2 - 9.6^2} = 38.8\,\Omega \tag{11.160}$$

The value of the magnetizing reactance is calculated as,

$$X_m = 2(38.8 - 6.9 - 0.5 \times 6.9) = 56.9\,\Omega$$

The rotational loss is determined as,

$$P_{rot} = P_{nl} - I_{nl}^{\;2}(R_1 + \frac{1}{4}R_2) = 60 - 2.5^2(2 + \frac{1}{4} \times 12.4) = 28\,\mathrm{W} \tag{11.161}$$

Practice Problem 11.8

A split-phase, 50 Hz, single-phase induction motor is tested and the following data are obtained.

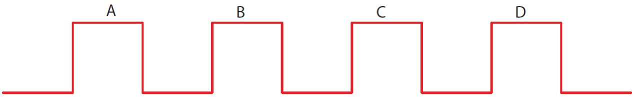
No load test: 130 V, 3.5 A, 80 W

Blocked rotor test: 80 V, 5 A, 230 W

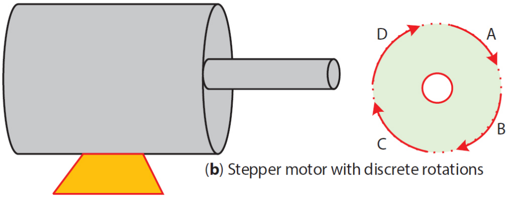
The main winding resistance is 4 Ω. Calculate the equivalent circuit parameters of the motor.

11.12 Stepper Motor

As the name of a stepper motor, it rotates in angular or discrete steps. A stepper motor is an electromechanical device, which converts electrical input pulses into non-continuous or discrete mechanical movements. Depending on the sequence of the applied pulses, the stepper motor can turn clockwise (CW) or counterclockwise (CCW). In general, the stepper motors rotate in discrete steps and each step depends on the pulse that is supplied to one of its stator windings. A series of discrete (Pulse) input supplies are used to explain the step movements of the stepper motor as shown in Fig. 11.35. When a pulse input A is applied, the motor runs a step as shown in Fig. 11.35(b). This step rotation is known as a step angle which will be discussed later. Similarly, when the pulse inputs B, C and D are applied, the motor completes the full rotation, i.e., 360°.



(**a**) Pulse Input



(**b**) Stepper motor with discrete rotations

Fig. 11.35 Pulse input with discrete rotation of the stepper motor.

In modern industries, stepper motors are used when both the motion and position of an object need to be controlled precisely. The rotation of the stepper motor is directly related to the frequency of the input pulses and the duration of the rotation is directly related to the number of applied input pulses. A stepper motor can advance by 1.5°, 2°, 2.5°, 5°, 15° etc., per pulse. The motor can be made to advance very slowly, one step at a time, or to rotate stepwise at speeds as high as 4000 r/min by varying the pulse rate.

Some advantages and disadvantages of a stepper motor are mentioned below:

Advantages

1. Precise position and motion can be obtained.
2. Excellent response for starting, reversing and stopping.
3. A wide range of rotational speeds can be obtained as the speed is directly related to the input pulses.
4. It is more reliable and the life of the motor is dependent on the life of the bearing.
5. It is simpler and less costly to control.

Disadvantages

1. It is really hard to operate at high speed.
2. Resonance might occur due to poor control.

The stepper motors mainly consist of the stator and the rotor. The stator contains the field poles which are energized by the AC supply. The rotor is made of by permanent magnet. The NEMA 14, NEMA 17, NEMA 23, and NEMA 24 stepper motors are used in 3-D printers. The stepper motors are also used on tape drives, disk drives, machine tools, CNC machines, process control systems, X–Y recorders, robotics, toys, etc.

Stepper motors are classified as permanent magnet type, variable-reluctance (VR) type and hybrid type.

11.13 Step Angle of Stepper Motor

The step size or step angle is very important for specific applications. The step angle depends on the construction of the stator phase and/or stator and rotor teeth. However, it is inconvenient and difficult to make stator phase more than five and the rotor teeth more than 50-100. The step angle of the stepper motor is defined as the angle at which the rotor moves from its initial position to the current position when the input pulse is applied to the stator as shown in Fig. 11.36.

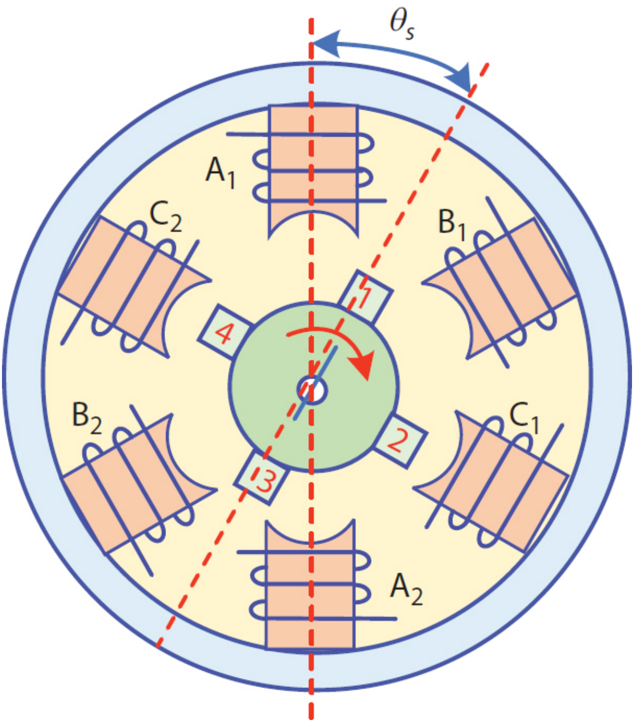


Fig. 11.36 Pulse input with discrete rotation of the stepper motor.

The range of the step angle is 0.72° to 90°. The most common step angles are 1.8°, 2.7° and 15°. The expression of step angle (θ_s) with rotor teeth (T_r) and the number of phases (m) is expressed as,

$$\theta_s = \frac{360^\circ}{m \times T_r} \tag{11.162}$$

The phase angle in terms of the stator teeth and the rotor teeth is expressed as,

$$\theta_s = \frac{(T_s - T_r)360^\circ}{T_s \times T_r} \tag{11.163}$$

The expression of tooth pitch is calculated as,

$$T_p = \frac{360^\circ}{T_r} \tag{11.164}$$

From Fig. 11.36, it is seen that the number of the stator phase is 3 (6-pole) and the number of the rotor teeth is 4. The step angle is calculated as,

$$\theta_s = \frac{360^\circ}{m \times T_r} = \frac{360^\circ}{3 \times 4_r} = 30^\circ \tag{11.165}$$

The step angle is also calculated as,

$$\theta_s = \frac{(T_s - T_r)360^\circ}{T_s \times T_r} = \frac{(6 - 4_r)360^\circ}{6 \times 4} = 30^\circ \tag{11.166}$$

Example 11.9

A stepper motor has 36 rotor teeth and 4 stator phases. Determine the tooth pitch and step angle.

Solution

The tooth pitch is determined as,

$$T_p = \frac{360^\circ}{T_r} = \frac{360^\circ}{36} = 10^\circ \tag{11.167}$$

The step angle is calculated as,

$$\theta_s = \frac{360^\circ}{mT_r} = \frac{360^\circ}{4 \times 36} = 2.5^\circ \tag{11.168}$$

Practice Problem 11.9

The step angle and stator phases are 0.9° and 8, respectively. Calculate the number of rotor teeth.

11.14 Permanent Magnet Stepper Motor

The permanent magnet stepper motor (PMSM) uses the permanent magnet rotor and works on the principle of attraction and repulsion towards the stator depending on the pulses applied to the stator phases. The stator poles are wound, and the rotor poles are made of by permanent magnet. The coils of the stator on diametrically opposite poles are connected in series. The rotor is made of high-grade steel and is in a cylindrical shape. PMSM has a step angle of 45° to 90° and produces high starting torque.

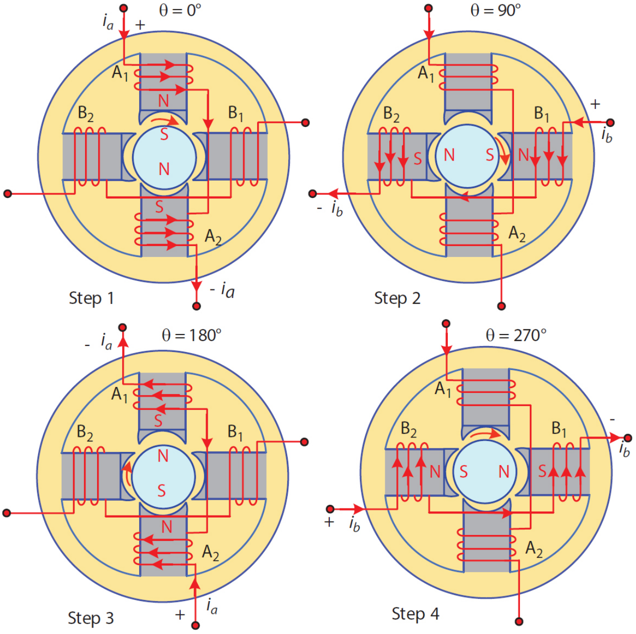


Fig. 11.37 Permanent magnet stepper motor operation steps.

Two coils A_1 and A_2 are connected in series to form phase A and the two coils B_1 and B_2 are connected in series to form phase B as shown in Fig. 11.37. There are three modes of operation used in the stepper motor operation. These are full-step, half-step and micro-step. The full step is again classified as a wave stepping one-phase ON and two-phase ON. The operation is explained here based on the wave drive method. When energized phase A creates the north pole (N-pole) in the first coil A_1 and the south pole (S-pole) in the second coil A_2 of the stator pole as shown in.

The N-pole of the stator attracts the S-pole of the rotor, and the S-pole of the stator attracts the N-pole of the rotor which aligns the rotor along the energized stator poles A_1 and A_2. Then turn OFF phase A and turn ON phase B, the rotor rotates 90° as shown in Fig. 11.37(b). Again, the current enters through coil A2 (positive) and comes out from coil A1(negative) which creates coil A_2 as an N-pole and coil A_1 as an S-pole as shown in Fig. 11.37(c). The stator N-pole will attract the S-pole of the rotor and the stator S-pole will attract the rotor N-pole and the rotor will move 180°. Similarly, in step 4, the current direction is opposite which rotates the rotor 270° and in step 5, the motor will rotate 360°.

The switching circuit for the PMSM is shown in Fig. 11.38 and the steps are summarized as in the truth Table 11.1.

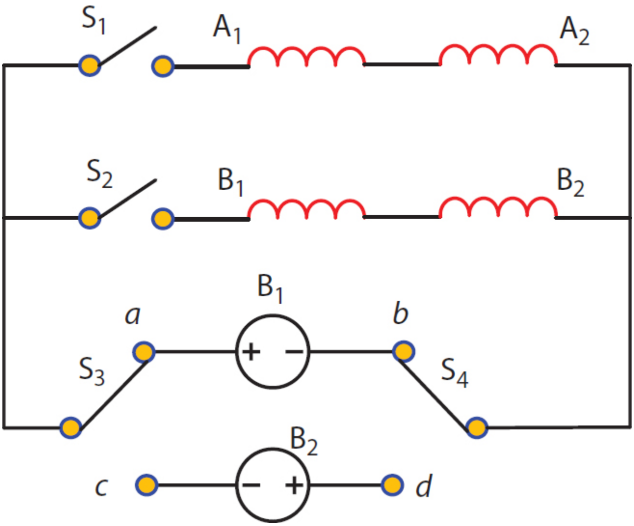


Fig. 11.38 Permanent magnet stepper motor switching circuit.

Table 11.1 Truth table for wave stepping with one-phase ON of PMSM.

S_1	S_2	S_3 and S_4	Step angle (θ)
1	0	*a*, *b*	0°
0	1	*a*, *b*	90°
1	0	*c*, *d*	180°
0	1	*c*, *d*	270°
1	0	*a*, *b*	360°

Advantages

- It needs less power to operate.
- The PMSM has a higher inertia constant due to the permanent magnet rotor.
- Produces more torque due to the two magnetic fields in the rotor (Permanent magnet) and the stator Electromagnet).
- No additional circuits are required to run the permanent magnet rotor.
- Shows better response during switching such as start, stop and reverse conditions.
- It is more reliable as there are no brushes in this motor.
- The motor provides complete torque when the stator coil is energized.

Disadvantages

- A smaller size (smaller step angle) motor is not possible due to the complexity of the rotor (Increasing teeth).
- The life of the permanent is limited as such it cannot be used for a long time.
- It cannot be used for a high-power application.

11.15 Variable-Reluctance Stepper Motor

The variable-reluctance motor has several stator poles and the salient or projected type rotor poles. The stators are smooth cylinder or non-salient pole type and constructed of highly laminated steel. The rotor is a salient or projected pole time and is made of ferromagnetic material. The rotor is made of laminated soft steel and does not have permanent magnets (PMs) or windings as shown in Fig. 11.39. A variable-reluctance stepper motor can be of the single-stack type or the multiple-stack type.

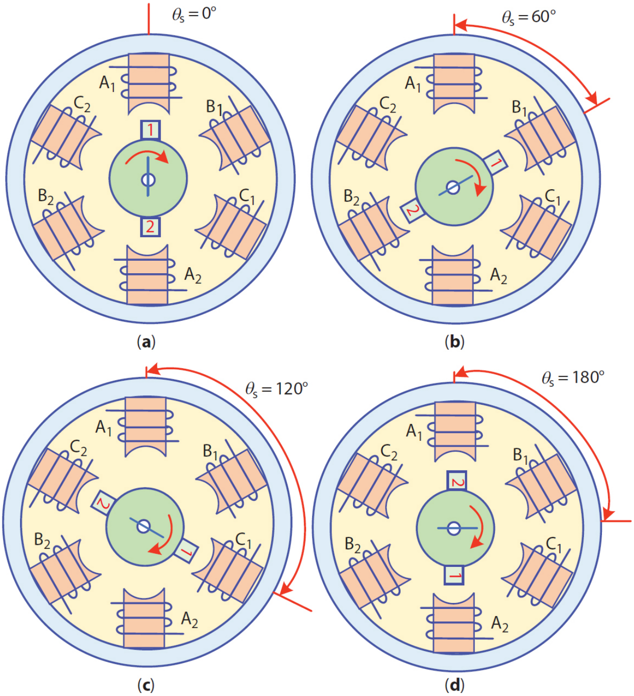


Fig. 11.39 Steps of a 6-pole stator and 2-pole rotor variable-reluctance stepper motor.

Working Principle

The working principle of a variable reluctance motor is based on the magnetic flux lines occupying or aligning the low reluctance path. Therefore, the stator and the rotor get aligned where the magnetic reluctance path is minimal.

The variable-reluctance stepper motor with six stator poles and two rotor poles is considered to explain the working principle. The six coils provide the three phases such as phase A, phase B, and phase C. These three phases are energized by a DC source and controlled by the semiconductor switches S_1, S_2, and S_3 as shown in Fig. 11.40. The step angle for this type of motor is calculated as 60º (360º/3×2).

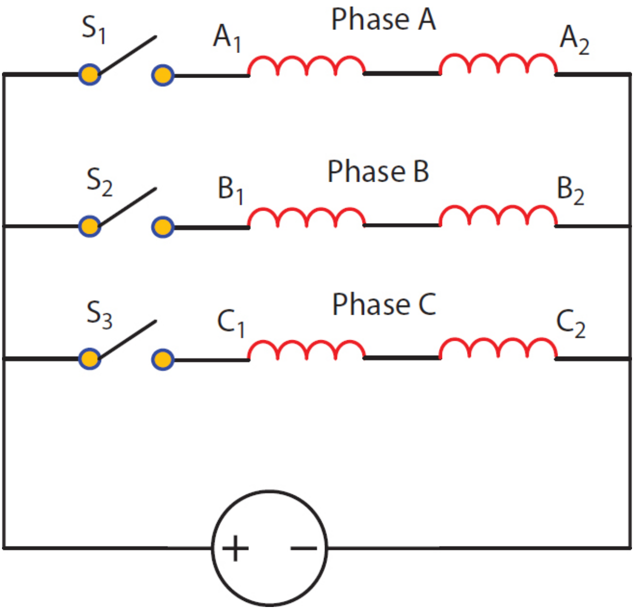


Fig. 11.40 Switching circuit of variable-reluctance stepper motor.

One-Phase ON

First, energize phase A by closing the switch S_1 which creates the magnetic field. As a result, the rotor teeth 1 and 3 will be aligned with the minimum reluctance of the stator poles A_1 and A_2. Again, de-energize phase A and energize phase B by closing the switch S_2. As a result, the rotor will rotate through a full step of 60° as shown in Fig. 11.41(b). If the sequence A, B, C, and A is continued by closing and opening the respective switches, the motor will complete one full rotation up to an angle of 180° as shown in Fig. 11.41(d). The truth table for one-phase ON is shown in Table 11.2.

Two-Phase ON

In this case, a 6-pole, three-phase stator and 4-pole rotor variable reluctance stepper motor is considered. In the two-phase ON mode operation, the two stator phases are excited simultaneously. When energizes phase A and phase B by closing the switches S1 and S2, the rotor experiences torque by both phases. As a result, the rotor will move and stay at the mid-position of phase A and phase B which advances 30º as shown in Fig. 11.41(b). Similarly, if the phase energizes sequence is maintained as AB, BC, CA, AB, then the rotor will rotate one complete rotation and the truth table is shown in Table 11.3.

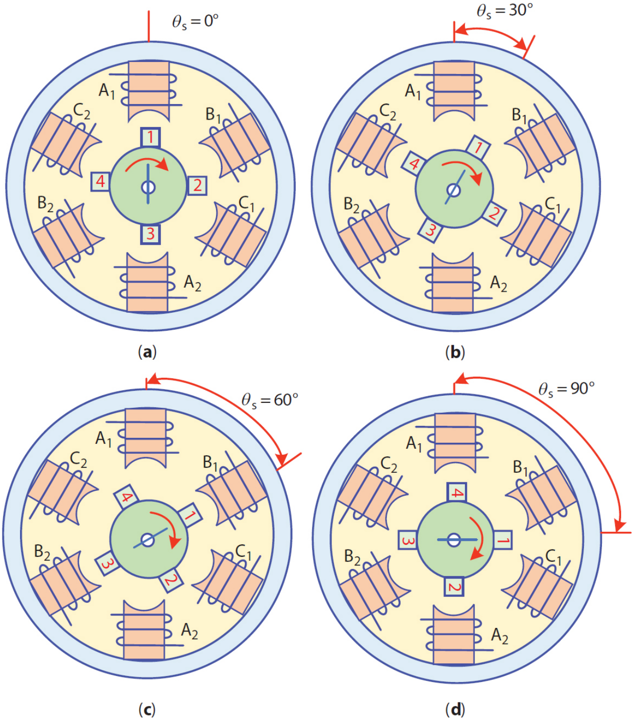

$\theta_s = 0°$
$\theta_s = 30°$
$\theta_s = 60°$
$\theta_s = 90°$
A_1
A_2
B_1
B_2
C_1
C_2
1
2
3
4
(a)
(b)
(c)
(d)

Fig. 11.41 Steps of a 6-pole stator and 4-pole variable reluctance stepper motor.

Table 11.2 Truth table for wave stepping with one-phase ON of VRSM.

Step	Phase A	Phase B	Phase C	Step angle (θ_s)
1	1	0	0	0º
2	0	1	0	60º
3	0	0	1	120º
4	1	0	0	180º

On and Two-Phase ON (Half-step)

This operation represents alternatively one-phase ON and two-phase ON modes. The half-step or half-stepping operation mode can be obtained by exciting the phases A, B, and C as the sequence A, AB, B, BC, C, CA, A, etc., as shown in Figs. 11.42 and 11.43, respectively. It causes the rotor to advance in steps of 15° angle. The truth table for the phase sequence in half-stepping is shown in Table 11.4.

Advantages

- It has less weight.
- Less rotor inertia.
- High stepping rate capability.
- It can be operated easily.
- It needs less maintenance and is cost-effective.
- The torque to inertia ratio is higher.
- The acceleration rate is high.
- The dynamic response is very high.
- Easy to increase resolution ($Re \propto 1/\theta_s$).

Disadvantages

- The torque is not available when the windings are de-energized.
- The step angle will decrease ($\theta_s \propto 1/T_r$) with increasing the rotor teeth.
- The output power decreases with increasing the load.

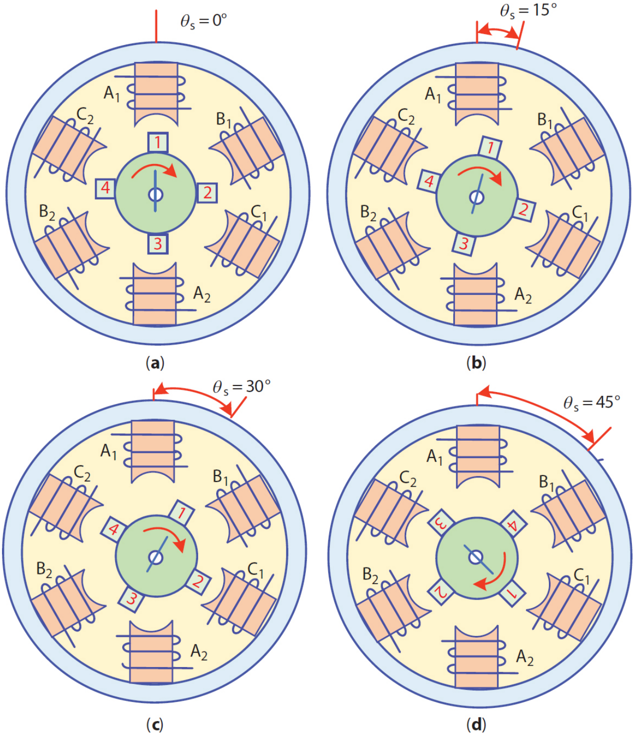

$\theta_s = 0°$
$\theta_s = 15°$
$\theta_s = 30°$
$\theta_s = 45°$
A_1
A_2
B_1
B_2
C_1
C_2
1
2
3
4
(a)
(b)
(c)
(d)

Fig. 11.42 Steps of a one- and two-phase variable reluctance stepper motor.

Table 11.3 Truth table for full stepping with two-phase ON of VRSM.

Step	**Phase A**	**Phase B**	**Phase C**	**Step angle (θ_s)**
1	1	1	0	30º
2	0	1	1	60º
3	1	0	1	90º
4	1	1	0	120º

Table 11.4 Truth table for half-stepping of VRSM.

Step	**Sequence**	**Phase A**	**Phase B**	**Phase C**	**Step angle (θ_s)**
1	A	1	0	0	0º

2	AB	1	1	0	15º
3	B	0	1	0	30º
4	BC	0	1	1	45º
5	C	0	0	1	60º
6	CA	1	0	1	75º
7	A	1	0	0	90º

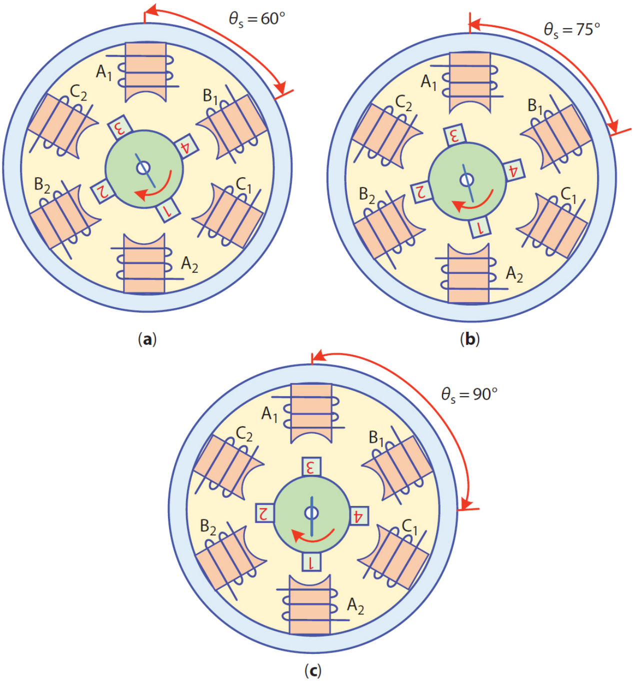


Fig. 11.43 Remaining steps of a one and two-phase variable reluctance stepper motor.

11.16 Hybrid Stepper Motor

The hybrid stepper motor is frequently chosen for a wide range of motion control precisely. This type of motor is more expensive than the permanent magnet stepper motor. However, the hybrid stepper motor provides excellent performance with respect to torque, speed and resolution.

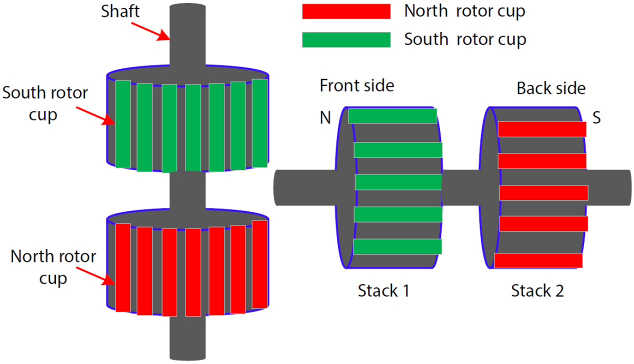


Fig. 11.44 Rotor construction of a hybrid stepper motor.

A hybrid motor is a combination of a variable reluctance motor and a permanent magnet motor. Like a permanent magnet stepper motor, the rotor of a hybrid stepper motor is magnetized axially. However, the stator of the hybrid motor is electromagnetically charged like a variable reluctance stepper motor. The rotor of the magnetic motor has two cups one for north poles and the other for south poles as shown in Fig. 11.44(a). The rotor cups are designed in such a way that they appear in an alternative way as shown in Fig. 11.44(b). To explain the working principle of a hybrid stepper motor, consider Fig. 11.45.

Here, four stator poles and six rotor teeth are considered. Coils A1 and A2 make the phase A. Similarly, coils B1 and B2 make phase B. Initially, energize phase A in such a way that makes N-pole and S-pole as shown in Fig. 11.46. In this case, rotor pole positions 1 and 4 align with stator poles due to the force of attraction between the poles and the low reluctance path. This position is known as holding position which refers to holding torque. In the second case, de-energize phase A and energize phase B (B2 by positive supply and B1 by negative supply) which creates the N-pole and S-pole as shown in Fig. 11.45(a) and the motor will rotate by a step angle of 30º ($\beta = 360/(2\times6)$) in the clockwise direction. In the third case, energize phase a as A2 is positive and A1 is negative which creates an N-pole and S-pole and the motor will rotate by 30º in the clockwise direction as shown in Fig. 11.46(b). This way the hybrid stepper can rotate continuously. When the phase or coil is de-energized, the respective phase does not hold any magnetic poles (N and S). However, the permanent magnet rotor creates the poles which develop a torque that prevents the rotation of the motor known as detent torque.

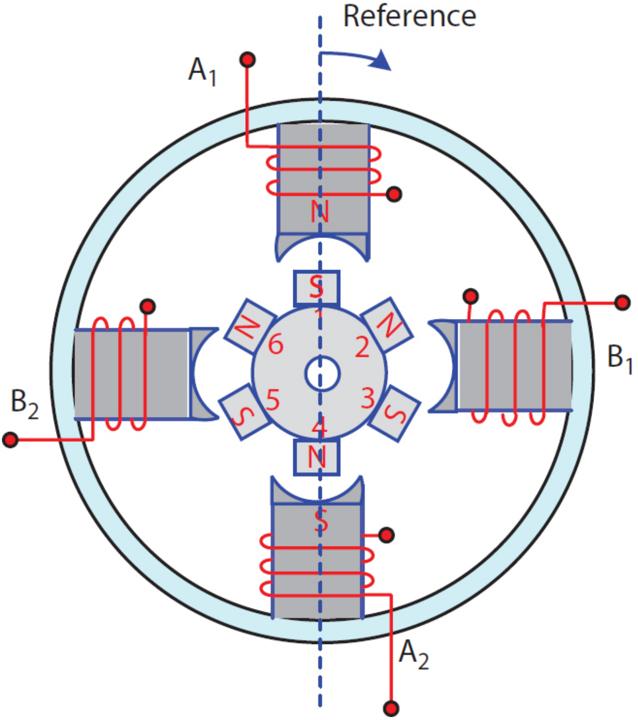

Reference
A1
N
S
N
N
6
2
5
3
S
S
4
N
S
B1
B2
A2

Fig. 11.45 Stator and rotor arrangement of a hybrid stepper motor.

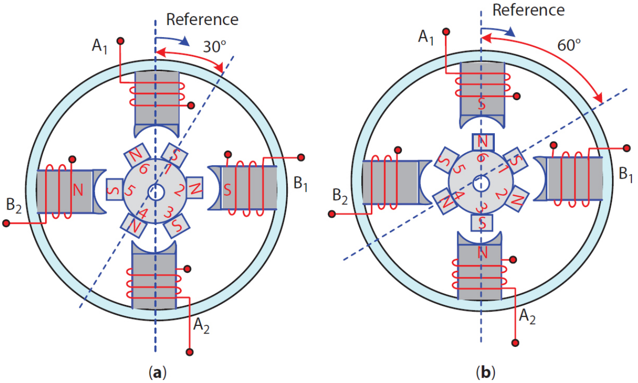

Reference
30°
A1
B1
B2
A2
N
S
(a)
Reference
60°
A1
B1
B2
A2
S
N
(b)

Fig. 11.46 Stator and rotor arrangement of a hybrid stepper motor in different positions.

Exercise Problems

11.1 The speed of a 4-pole, 50 Hz single-phase induction motor is 1400 rpm. Determine the slip due to the forward and backward fields.

11.2 The slip due to the backward field of a single-phase induction motor is 1.5. The synchronous speed of the motor is 1500 rpm. Determine the slip due to the forward field and the speed of the rotor.

11.3 A 110 V, 50 Hz, 4-pole, single-phase induction motor runs at a speed of 1450 rpm. The standstill rotor resistance is 11 Ω. Calculate the effective rotor resistance in the forward field and the backward field.

11.4 A 120 V, 60 Hz, 6-pole, single-phase motor runs at a speed of 1150 rpm. The effective rotor resistance in the forward field is 90 Ω. Calculate the standstill rotor resistance. Also, calculate the effective rotor resistance in the backward field.

11.5 The stator and the rotor impedances of a 220 V, 50 Hz, single-phase induction motor are 2 + j5Ω and 3 + j5Ω, respectively. The magnetization reactance of the motor is 50 Ω. Determine the (i) input current, (ii) input power, and (iii) power developed. Consider the slip is 3%.

11.6 The circuit parameters of a 230 V, 50 Hz, 4-pole, single-phase induction motor are $Z_1 = 3 + j2\Omega$, $Z_2 = 4 + j2\Omega$, $X_m = 80\Omega$. The motor has a core loss of 20 W and a friction and windage loss of 5 W. Determine the (i) input current, (ii) power developed, (iii) output power, (iv) efficiency, and (v) torque at a speed of 1420 rpm.

11.7 The circuit parameters of a 150 V, 4-pole, 50 Hz, single-phase induction motor are $R_1 = 3\Omega$, $X_1 = 2.5\Omega$, $R_2 = 4\Omega$, $X_2 = 3\Omega$ and $X_m = 70\Omega$. The rotational loss of the motor is 25 W and the motor runs at a speed of 1400 rpm. Determine the motor efficiency.

11.8 The circuit parameters of a 4-pole, 220 V, 50 Hz, single-phase induction motor are $Z_m = 6 + j11.5\Omega$, $Z_s = 11.5 + j6\Omega$. Determine the line current of the motor.

11.9 The starting and the main windings impedances of a 220 V, 50 Hz, capacitor-start induction motor are $Z_m = 4 + j3.5\Omega$, $Z_s = 8 + j4.5\Omega$. Find the value of the starting capacitance for getting maximum starting torque.

11.10 A split-phase, 50 Hz, single-phase induction motor is tested and the following data are obtained.
No-load test: 110 V, 4.5 A, 85 W
Blocked rotor test: 60 V, 4.75 A, 125 W
The main winding resistance is 3 Ω. Calculate the equivalent circuit parameters of the motor.

11.11 A stepper motor has 40 rotor teeth and 4 stator phases. Determine the (i) tooth pitch, and (ii) step angle.

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